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Estimation and Simulation of Bond Option Pricing on the Arbitrage Free Model with Jump

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Abstract

Three contents for the pricing of bond options on the arbitrage-free model with jump are included in this paper. The first uses a new technique to derive a Closed-Form Solution (CFS) for bond options on Hull and White (HW) model with jump. The second deals with the pricing of bond option for Heath-Jarrow-Morton (HJM) model based on jump, and the third simulates the proposed models by the Monte Carlo Simulation (MCS). We also analyze the values obtained by the CFS and MCS. There is a substantial difference between bond option prices which are obtained by the HW model with jump and the HJM model based on jump. For this, we use the well-known Mean Standard Error (MSE) and show that lower value of Precision (PCS) in the proposed models corresponds to sharper estimates. In particular, we confirm that the PCS for the HJM based on jump is lower than that for the HW model with jump. Through the empirical simulation of our method suggested, we obtain a better accurate estimation for the pricing of bond options.

Keywords: Hull and White (HW) model with jump; Heath-Jarrow-Morton (HJM) model based on jump; Bond option pricing; Monte Carlo Simulation (MCS)

Introduction

In pricing and hedging with financial derivatives, term structure models with jump are particularly important [1], since ignoring jumps in financial prices may cause inaccurate pricing and hedging rates [2]. Solutions of term structure model under jump-diffusion processes are justified because of movements in interest rates displaying both continuous and discontinuous behaviors [3]. Moreover, to explain term structure movements used in the latent factor models, it means how macro variables affect bond prices and the dynamics of the yield curves [4]. Current research using jump-diffusion processes relies mostly on two classes of models: the affine jump-diffusion class [5] and the quadratic Gaussian [6]. We consider the classes of HW model with jump and HJM model based on jump to investigate a CFS for bond option price on the proposed models. In this paper, we show the actual proof analysis of the HJM model based on jump easily under the extended restrictive condition of Ritchken and Sankarasubramanian (RS) [7]. By beginning with certain forward rate volatility processes, it is possible to obtain classes of interest models under HJM model based on jump that closely resembles the traditional models [8]. Finally, we confirm that there is a substantial difference between bond option prices which are obtained by HW model with jump and HJM model based on jump through the empirical computer simulation which used MCS, which is used by many financial engineers to place a value on financial derivatives. For this, we use the well-known MSE. We make sure that lower value of PCS in the proposed models corresponds to sharper estimates [9]. In particular, we confirm that the PCS for the HJM based on jump is lower than the HW model with jump. These results mean an accurate estimate in the empirical computer. The structure of the remainder of this paper is as follows. In section 4, investigate the pricing of bond on arbitrage-free models with jump. In section 4, the pricing of bond option on arbitrage-free models with jump are presented. Section 6, explains the simulation procedure of the proposed models using MCS. In Section 7, the proposed models' performances are evaluated based on simulations. Finally, Section 8 concludes this paper.

The Pricing of Bond on Arbitrate-Free Model with Jump

All our models will be set up in a given complete probability space (Ω, F_t, P) and an argument filtration $(F_t)_{t\geq 0}$ generated by an Winear process and N(t) represents a Poisson process with intensity rate h and the total number of extreme shocks that occur in a financial market until time t [10]. If there is one jump during the period [t, t+dt] then dN(t)=1, and dN(t)=0 represents no jump during that period. In the same way that a model for the asset price is proposed as a lognormal random walk, let us suppose that the interest rate r and the forward rate f are governed by a SDE of the form

$$dr = u(r,t)dt + w(r,t)dW(t) + JdN(t) \quad (1)$$

$$df(t,T) = u_f(t,T)dt + \sigma_f(t,T)dW(t) + JdN(t), \quad (2)$$

where w(r, t) is the instantaneous volatility, u(r, t) is the instantaneous drift, $u_f(t,T)$ represents drift function, $\sigma^2{}_f(t,T)$ is volatility coefficient, dW(t) is the standard Wiener process, jump size $J \sim N(\mu, \gamma^2)$, and dN(t) is the Poisson process with intensity rate h. When interest rates follow the SDE (1), a bond has a price of the form V (t; T); the dependence on T will only be made explicit when necessary. To get the bond pricingequation with jump, we set up a riskless portfolio containing two bonds with different maturities T1 and T2. And then we applied the jump-diffusion version of Ito's lemma. Hence, we derive the partial differential equation (PDE) for bond pricing. Theorem 1: If r satisfies SDE (1), then the zero-coupon bondpricing equation with jumps is

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$$\frac{\partial V}{\partial t}+\frac{1}{2}W^2\frac{\partial^2 V}{\partial r^2}+(u-\lambda w)\frac{\partial V}{\partial r}=rV-hE\,[V(r+J,t)-V(r,t)], \qquad (3)$$

where λ(r,t) is the market price of risk. The final condition corresponds to the payoff on maturity and so V (T, T)=1. Boundary conditions depend on the form of u(r, t) and w(r, t).

The HW model with jump

Let be V (t; T) the price at time t of a discount bond. A solution of the form:

$$V(t,T)=\exp[-A(t,T)r+B(t,T)] \qquad (4)$$

can be guessed. We now consider a quite different type of random environment. In this paper, we extend jump-diffusion version of equilibrium single factor model to reflect this time dependence. This leads to the following model for r(t):

$$dr(t)=[\theta(t)-a(t)r(t)]dt+\sigma(t)r(t)^{\beta}\,dW(t)+JdN(t), \qquad (5)$$

where ϴ (t) is a time-dependent drift; σ(t) is the volatility factor; a(t) is the reversion rate; dW(t) is the standard Wiener process; dN(t) is the Poisson process with intensity rate h. We investigate the β=0 case is an extension of Vasicek's jump-diffusion model; the β=0:5 case is an extension of CIR's jump-diffusion model. Under the process specified in equation (5), r(t) is defined as:

$$r(t)=r_0(t)\{r(0)+\int_0^t r_0^{-1}(u)\theta(u)du+\int_0^t r_0^{-1}(u)\sigma(u)r(u^{\beta})\,dW(u)+\sum_{i=1}^{N(t)} r_0^{-1}(T_i)J_i\}, \qquad (6)$$

where $r_0(t)=\exp\{-\int_0^t -a(u)du\}=e^{-a(t)t}$, Ti is time that the j-th jump happens, $0<T_1<T_2<\ldots<T_N(t)<t$, and N(t) is the number of jumps happening during the period [0, t]. It can be shown that the probability density function for r(t). Therefore, the conditional expectation and variance of jump-diffusion process given the current level are

$$\mathrm{E}[r(t)]=e^{-a(t)t}r(0)+\left(\frac{\theta(t)}{a(t)}++\frac{h\mu}{a(t)}\right)(1-e^{-a(t)t}), \qquad (7)$$

and

$$Var[r(t)]=\begin{cases}\dfrac{e^{-2a(t)(e^{a(t)t}-1)^2\theta(t)\sigma(t)^2}}{2a(t)^2} & : \quad \beta=0\\[2ex] \hline \dfrac{e^{-2a(t)(e^{a(t)t}-1)^2\theta(t)\sigma(t)^2}}{2a(t)^2} & \\ \hline +\dfrac{h}{2a(t)}(\mu^2+\gamma^2)(1-e^{-2a(t)}t) & : \beta=\dfrac{1}{2}\end{cases} \qquad (8)$$

To drive the pricing of bond on the HW model with jump, we use a two-term Taylor's expansion theorem to represent the expectation terms of equation (3) is given by

$$E[V(r+Jt)-V(r,t)]=-\mu A(t,T)+\frac{1}{2}(\gamma^2+\mu^2)A(t,T)^2$$

where a jump size J~N(μ, ɣ²). Thus, we get the partial differential difference bond pricing equation:

$$[\theta(t)-a(t)r(t)-\lambda(t)\sigma(t)\,r(t)^{\beta}]\,V_r+V_t+\frac{1}{2}\sigma(t)^2V_{rr}$$
$$-rV+hV\,[-\mu A(t,T)+\frac{1}{2}(\gamma^2+\mu^2)A(t,T)^2]=0 \qquad (9)$$

Bond price derivatives can be calculated from (4), and then the substitution of these derivatives into (9). Thus, equating powers of r(t) yields the following equations for A and B

Theorem 2: (The Equation (9) with β=0)

$$-\frac{\partial A}{\partial t}+a(t)A-1=0 \qquad (10)$$

$$-\frac{\partial B}{\partial t}+\phi(t)A+\frac{1}{2}\sigma(t^2)A^2+h[-\mu A+\frac{1}{2}(\gamma^2+\mu^2)A^2]=0 \qquad (11)$$

where $\phi(t)=\theta t-\lambda(t)\sigma(t)$ and all coefficients is constants.

Theorem 3: (The Equation (9) with _=0:5)

$$-\frac{\partial A}{\partial t}+\psi(t)A+\frac{1}{2}\sigma(t^2)A^2-1=0 \qquad (12)$$

And

$$\frac{\partial B}{\partial t}-(\theta(t+h\mu)A+\frac{1}{2}h[(\gamma^2+\mu^2)A^2]=0 \qquad (13)$$

where $\psi(t)=a(t)+\lambda(t)\sigma(t)$ and all coefficients is constants. From theorem 2 and 3, to satisfy the final data that V (T, T)=1 we must have A(T, T)=0 and B(T, T)=0:

The HJM model based on jump

We denote as f(t, T) the instantaneous forward rate at time t for instantaneous borrowing at time T(≥ t). Then the price at time t of a discount bond with maturity T, is defined as

$$V(t,T)=\exp\left(-\int_t^T f(t,s)ds\right) \qquad (14)$$

We consider the one-factor HJM model with jump under the corresponding risk-neutral measure Q, and we obtain the SDE is given by

$$df(t,T)=\sigma_f^{\,*}(t,T)dt+\sigma_f(t,T)dW^Q(t)+JdN(t), \qquad (15)$$

Where $\sigma_f^{\,*}(t,T)=\sigma_f(t,T)\int_t^T\sigma_f(t,s)\,ds$, $dW^Q(t)$ is a standard Wiener process generated by the risk-neutral measure Q, and dN(t) is the Poisson process with intensity rate h. In similar way as before, therefore, the conditional expectation and variance of the SDE (15) given the current level are

$$E[f(t,T)]=f(0,T)+E\left[\int_0^t\sigma_f^{\,*}(s,T)ds\right]+\mu ht \qquad (16)$$

and

$$Var[f(t,T)]=E[f(t,T)-E[f(t,T)])^2]$$
$$\int_0^t\sigma f^2(s,t)ds+\gamma^2ht+\mu ht \qquad (17)$$

In the study, we use the relation between short rate and forward rate process to obtain the formula of bond price under the extended restrictive condition of RS.

Theorem 4: Let be the jump-diffusion process in short rate r(t) is the equation (5). Let be the volatility form is

$$\sigma f(t,T)=\sigma_r(t)(r(t))^{\beta}\eta(t,T) \qquad (18)$$

with $\eta(t,T)=\exp\left(-\int_t^T a(s)ds\right)$ are deterministic functions. We know the SDE for forward rate (15). Then we obtain theequivalent model is

$$f(0,T)=r(0)\eta(0,T)+\int_0^T\theta(t)\eta(s,T)ds-\int_0^T\sigma_r^2(s)r(s)^{2\beta}\eta(s,T)\int_s^T\eta(s,u)\,duds \qquad (19)$$

that is, all forward rates are normally distributed. By the theorem 4, we derive the relation between short rate and forward rate. Using the equation (19), we obtain bond pricing equation as follows;

Corollary 1: Let be the HJM model based on jump is the SDE (15). Then discount bond price V (t, T) for forward rate is given by

$$V(t,T) = \exp\{-\frac{1}{2}\left(\frac{\int_t^T \sigma_f(t,s)ds}{\sigma_f(t,T)}\right)^2 \int_0^t \sigma_f^2(s,t)ds - \frac{\int_t^T \sigma_f(t,s)ds}{\sigma_f(t,s)}[f(0,t)-r(t)]\}\frac{V(0,T)}{V(0,T)} \quad (20)$$

with the equation (19).

The Pricing of Bond Options on Arbitrate-Free Model with Jump

We derive a CFS for bond options when the prices of the underlying instantaneous interest and forward rate evolve as discontinuous processes. We now consider the value of European options on discount bond equations (4) and (14). The price of a call option on the TV-maturity discount bond with exercise price K and maturity T<TV is given by

$$C = V(t,T)\,E[\max(\overline{V}-K,0)], \quad (21)$$

where E denotes expected value in a world that is forward risk neutral with respect to a zero coupon bond maturing at time $T_V; E[\overline{V}] = V(t,T_V)/V(t,T)$ and assuming the bond price is lognormally with the standard deviation of the logarithm of the bond price equal to σV, that is, $\overline{V}$ is lognormally with $\mathrm{Var}\left[\log \overline{\mathrm{V}}\right] = \sigma v^2$. Thus, the equation (21) becomes

$$C = V(t,T_V)N(d_1) - KV(t,T)N(d_2), \quad (22)$$

where $d_1 = \frac{\log([V(t,T_V)/V(t,T)]/K + \sigma v^2/2}{\sigma v}, d_2 = d_1 - \sigma v$, and N(x) is the normal cumulative density function. In similar way, we obtain the price of a put option on the discount bond.

Simulation Procedure

In this section, we explain about the simulation procedure ofthe pricing of bond options on the arbitrage-free models withjump. The MCS is actually a very general tool and its applicationsare by no means restricted to numerical integration.To execute the MCS, we divide the time interval [t, T] into mequal time steps of length Δt each. For small time steps, wecan obtain the bond price by sampling n short and forwardrates paths under the discrete version of the risk-adjusted SDEs(5) and (15). The bond price estimate is given by:

$$V(t,T) = \frac{1}{n}\sum_{i=1}^{n}\exp\left(-\sum_{i=1}^{m-1}\Phi_{ij}\,(\Delta t)\right) \quad (23)$$

where Φ means the interest rate or forward rate. Under the discrete risk-adjusted process within sample path i at time t+Δt. The current value of a European call option C(0, T, TV) can be evaluated under the risk neutral measure, where 0<t<T<TV. The option price is evaluated using formula (21), using the Euler-Maruyama scheme for the integration, as

$$C(0,T,T_V) = \frac{1}{n}\sum_{i=1}^{n}\exp\left(-\sum_{i=1}^{m-1}\Phi_{ij}\,(\Delta t)\right)\max\left(Vi\,(\Phi,0,T,T_V)-K\right) \quad (24)$$

where Vi(Φ, 0, T, T_V) is computed by the arbitrage-free models with jump. The PCS of the mean as a point estimate is often defined as the half-width of a 95% confidence interval, which is calculated as

$$\text{Precision}=1{:}96\times\text{MSE}\times 100 \quad (25)$$

where $MSE = v/\sqrt{n}$ *and* v^2 is the estimate of the variance of bond options price as obtained from n sample paths of the short and forward rates. Lower values of PCS in equation (25) correspond to sharper estimates

Estimation Results

In this section, we investigate the pricing of bond options on the arbitrage-free models with jump. We first estimate for the HW model with jump which means CFS and MCS of bond options as shown in Figure 1 and Table I. For this experiment, the parameter values are assumed to be r=0:05, a=0:56, b=0:05, ϴ= a×b, σ= 0:02, λ=-0:17, ɣ=0:02, μ=0:02, h=0:34, t=0:05, T=TV-0:5, and TV=10(the K=0:8 case is the call option, the K=1:2 case is the put option). Tables 1 and 2 represent the pricing of bond options on arbitrage-free models using the MCS. For the purpose of simulation, we conduct three runs of n=10; 000 trials per each and divide a year into m=300 time steps. We nowinvestigating the pricing of bond options on the HJM model based on jump which is shown in Figure 2 and Table 2. For this experiment, the parameter

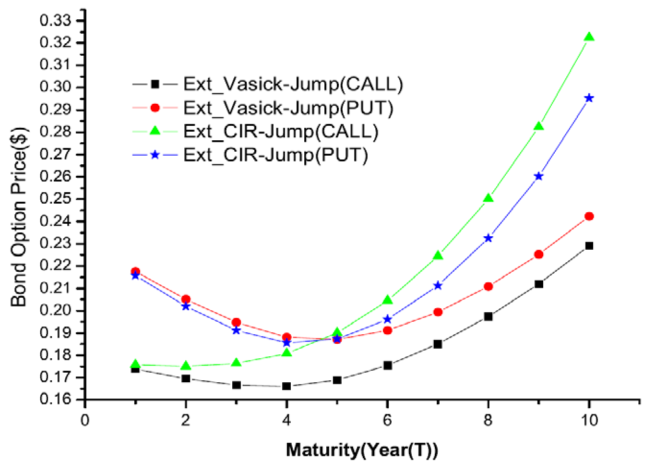

Figure 1: The pricing of bond options on the HW model with jump.

	Extended Vasicek		Extended CIR	
	CALL	PUT	CALL	PUT
CFS	0.173808	0.217562	0.175745	0.218716
MCS	0.171202	0.219311	0.171164	0.219686
CFS-MCS	0.002606	0.001749	0.004581	0.00097
PCS(%)	0.006913	0.008733	0.00665	0.00436

Table 1: The pricing of bond options on the HW model with jump is estimated by the MCS.

	HJM-Jump(CALL)		HJM-Jump(PUT)	
	Ext-Vasicek	Ext-CIR	Ext-Vasicek	Ext-CIR
CFS	0.172544	0.172545	0.218716	0.218716
MCS	0.170176	0.170046	0.219699	0.219686
CFS-MCS	0.002368	0.002499	0.000983	0.00097
PCS(%)	0.003377	0.00385	0.004488	0.00436

Table 2: The pricing of bond options on the HJM model based on jump is estimated by the MCS.

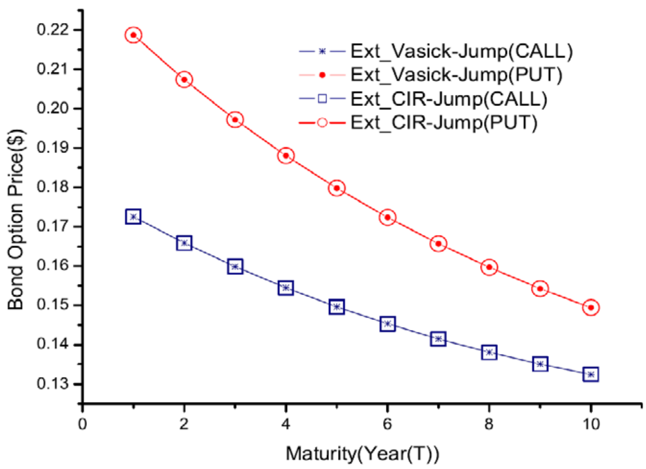

Figure 2: The pricing of bond options on the HJM model based on jump.

values are assumed as before. We then examine the pricing of bond options on the HJM model based on jump by the MCS as represented in Table 2. In empirical computer simulation Tables 1 and 2, we show that the lower values of PCS in the proposed models correspond to sharper estimates using the Mathematica [11].

Conclusion and Future Research

After investigating the models which allow the short term interest and the forward rate following a jump-diffusion process, we obtained the closed-form solutions on jump models, which are more useful to evaluate the accurate estimate for the values of bond options in the financial market. Through the MCS simulation of these solutions with jump, the price of the expected stable figure like right-downward flow as maturity increases while the graph of bond options on the HW-Jump model with the short term interest rate is humped. We need further investigation on this difference which can be caused by performing jump term simulation of different interest rate cases. Also, we obtained the more accurate estimate in empirical computing by showing the fact that the PCS for the HJM based on jump is lower than that for the HW model withjump. There are still problems remained for further research. Some of them, for instance, are (i) using the MCS to simulate more complicated two factors of the proposed models; (ii) considering a dynamic algorithm to predict the bond option prices using actual data set of bond.

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