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Equiprime-Semi Module over a Boolean Like Semi Ring

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Abstract

In this paper, the concepts of equiprime ideals and equiprime semi modules over Boolean like semi rings are introduced and also furnish some examples. It is proved that if P is equip rime R- Ideal of the R-module M then (P:M) is euqiprime ideal of R. Also establish if M is an equiprime R-module then (0:M) is an equiprime ideal of R. Also establish an important characterization of the equiprime ideal of R in the semi module structure over R. Using this result we can prove, If R is a Boolean like semi ring and R ≠ 0. Then R is equiprime if and only if there exists a faithful equiprime R-module M. Finally, we show that let M be an equiprime R-module and let N be an invariant sub group of R such that N is not contained in (0:M). Then M is an equiprime N-module.

Keywords: Boolean like semi ring • Semi module over boolean like semi ring • Equiprime ideal and equiprime semi module

Introduction

In 1990, Booth [1] introduced the notion of equiprime near rings and equiprime ideals and established various results. In 1992, Booth [2] introduced equiprime N-groups and characterizes the equirprime ideals of near-ring N and proved various results. In 2011, Venkateswarlu, et al. [3] introduced Boolean like semi rings and studied some of its properties. In 2012, Murthy [4] introduced a structure of semi modules over Boolean like semi rings and study some of its properties. The present other extends these concepts to semi module structures over Boolean like semi rings and proved various results and characterize the equiprime ideals of R in the semi module structure.

Literature Review

Preliminaries

The present other collects certain definitions and results concerning Boolean like semi rings and semi modules over R from [2,3]

Definition 1: A non-empty set R together with two binary operations + and satisfying the following conditions is called a Boolean like semi ring [5].

1. (R,+) is an abelian group
2. (R,.) is a semi group
3. a. (b+c)=a.b+a.c for all a,b,c ϵ R
4. a+a=0 for all a ϵ R
5. ab (a+b+ab)=ab for all a, b ϵ R. Let R be a Boolean like semi ring. Then

Lemma 1.2- For a ϵ R, a. 0=0

Lemma 1.3-For a ϵ R, $a^4=a^2$ (weak idempotent law)

Remark 1-If R is a Boolean like semi ring then, $a^n=a$ or a^2 or a^3 for any integer n>0

Definition 2: A Boolean like semi ring R is said to be weak commutative if abc=acb, for all a,b,c ϵ R.

Lemma 1.4-If R is a Boolean like semi ring with weak commutative then 0.a=0.for all a ϵ R

Lemma 1.5- Let R be Boolean like semi ring then for any a,b ϵ R and for any integers m,n>0

$$a^m a^n = a^{m+n}$$
$$\left(a^m\right)^n = a^{mn}$$
$$\left(ab\right)^n = a^n a^n$$

if R is weak commutative.

Definition 3: A non-empty subset I of R is said to be an ideal if

i) (I,+) is a sub group of (R,+),i.e. for a, b ϵ R ⇒ a+b ϵ R

ii) ra ϵ R, for all a ϵ I, r ϵ R, i.e. RI ⊆ I

iii) (r+a) s+rs ϵ I, for all r, s ϵ R, a ϵ I

Remark 2-If I satisfies (i) and (ii), I is called left ideal and If I satisfies (i) and (iii), I is called right ideal of R.

Definition 4: An element 1 ϵ R is said to be unity if a.1=1.a=a, for all a ϵ R. If a.1=a, then 1 is called right unity and if 1.a=a, then 1 is called left unity.

Proposition 1: If R is weak commutative, then ar ϵ I and ra ϵ I for all r ϵ R, a ϵ I.

Definition 5: If R is a Boolean like Semi ring, the set R/I=(x+I/x ∈ R) is called a quotient Boolean like Semi ring of R.

Definition 6: A sub set H of R is called (two sided or invariant) R-subgroup of R if.

(a) (H,+) is a sub group of (R,+)

(b) RH ⊆ H

(c) HR ⊆ H

Remark 3-In the above definition H satisfies (a) and (b), H is called left R-sub group of R and H satisfies (a) and (c), H is called right R-sub group of R [6].

Theorem 1: If a ϵ R then aR is a right R-sub group of R.

Definition 7: Let R be a Boolean like semi ring and (M,+) be an abelian group then M is called a semi module over R if the mapping: M x R ⟶ M such that m(r+s)=mr+ms, m(rs)=(mr)s, for all m ∈ M, r, s ∈ R.

Definition 8: Let H be a sub group of M such that for all r ∈ R, for all h ∈

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H, we have that hr ∈ H then H is called R-sub module of M, we denote $H <_R M$

Definition 9: If R is weak commutative and M is module over R,0 ∈ M, 0r=0, for all r ∈ R.

Theorem 2: If M is semi module over R then $m_0=0$ for all m ∈ M.

Theorem 3: If M is an R-module and m ∈ M then mR is an R-sub module of M.

Definition 10: A sub group P of a module M is called R-ideal of M if for all r ∈ R, m ∈ M and n ∈ P, we have (m+n)r-mr ∈ P.

Remark 4- If M=R then R-ideals of M becomes right ideals of R and the R-sub modules of M are the right R- sub groups of R.

Theorem 4: If R is weak commutative then every R-ideal of M is an R-sub module of M.

Definition 11: Let P be an R-ideal of M then (P: M)=(r ∈ R/Mr ⊆ P)

Theorem 5: If P is an R-sub module of M then

(a) (P:M) is a right R-sub group of R.

(b) (P:M) is a left R-sub group of R if R is weak commutative.

Corollary: If R is weak commutative Boolean like semi ring and P is an R-sub module of M then (P:M) is invariant R-sub group of R.

Theorem 6: If P is an R- ideal of M then (P:M) is an ideal of R.

Theorem 7: Let M be a semi module over R and P be an R- ideal of M. Then the quotient group M/P=(m+P/m ∈ M)is semi module over R (called the quotient semi module over R) with scalar multiplication defined by (m+P) r=mr+P, for all r ∈ R, m ∈ M.

In this section, we define euqiprime ideals and equiprime semi module over R and also consider R as a weak commutative Boolean like semi ring [7,8].

Definition 12: Let I be an ideal of R. Then I is said to be equiprime if b ∈ R with b ∈ R\I, y,z ∈ R Such that yrb-zrb ∈ I, for all r ∈ R ⇒ y-z ∈ I.

Definition 13: A Boolean like semi ring is called equiprime if (0) is an equiprime ideal of R. We now furnish some examples of equiprime ideals of R (Tables 1-3).

Definition 14: Let M be an R-module and P be an R-ideal of M, then

(i) M is called equiprime if MR ≠ 0 and the following condition is satisfied, If a ∈ R with a ∈ R\(0:M) and α, β ∈ M such that αra-βra=0, for all r ∈ R ⇒ α-β=0.

(ii) P is called equiprime R-ideal of M if b ∈ R with b ∈ R\(P:M) and α, β ∈ M such that αrb-βrb ∈ P, for all r ∈ r ∈ R ⇒ α-β ∈ P.

Remark 5-In the above examples (A), (B) and (C), I is equiprime R-module.

The following theorem shows that the relation between the equiprimeness of P and (P:M).

Theorem 8: If P is equiprime R-ideal of the R-module M then (P:M) is equiprime ideal of R.

Table 1. Let R=(0,a,b,c). The binary operations "+" and "." are defined as follows.

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	0	a	a
b	0	0	b	b
c	0	a	b	c

Note: Then (R, +, .) is a Boolean like semi ring. If I=(0,a) then I is equiprime ideal of R.

Table 2. Let R= (0, x, y, z). The binary operations "+" and "." are defined as follows.

+	0	x	y	z
0	0	x	y	z
x	x	0	z	y
y	y	z	0	x
z	z	y	x	0

.	0	x	y	z
0	0	0	0	0
x	0	x	0	x
y	0	0	0	0
z	0	z	0	Z

Note: Then (R, +, .) is a Boolean like semi ring. If I= (0, x) then I is equiprime ideal of R

Table 3. Let R= (0,α, β, γ, δ, σ, τ,,1). The binary operations "+" and "." are defined as follows.

+	0	α	β	γ	δ	σ	τ	1
0	0	α	β	γ	δ	σ	τ	1
α	α	0	γ	β	σ	δ	1	τ
β	β	γ	0	α	τ	1	δ	σ
γ	γ	β	α	0	1	τ	σ	δ
δ	δ	σ	τ	1	0	α	β	γ
σ	σ	δ	1	τ	α	0	γ	β
τ	τ	1	δ	σ	β	γ	0	α
1	1	τ	σ	δ	γ	β	α	0

.	0	α	β	γ	δ	σ	τ	1
0	0	0	0	0	0	0	0	0
α	0	0	0	0	α	α	α	α
β	0	0	β	β	0	0	β	β
γ	0	0	β	β	α	α	γ	γ
d	0	0	0	0	δ	δ	δ	δ
σ	0	α	0	α	δ	σ	δ	σ
τ	0	0	β	β	δ	δ	τ	τ
1	0	α	β	γ	δ	σ	τ	1

Note: T hen R is a Boolean like semi ring. If I= (0, α, β, γ) then I is equiprime ideal of R.

Proof (i) if P is R-ideal of R-module M then (P:M) is an ideal of R, by theorem 1.27.

Let $a \in R$ with $a \in R\backslash(P:M)$ and y, z $\in$ R such that yra-zra $\in$ (P:M) for all $r \in R$

$\Rightarrow$ M(yra-zra) P $\Rightarrow$ m(yra-zra) $\in$ P, for all m $\in$ M $\Rightarrow$ (my)ra-(mz) ra $\in$ P, for all m $\in$ M

$\Rightarrow$ my-mz $\in$ P m(y-z) $\in$ P, for all m $\in$ M $\Rightarrow$ M(y-z) $\subseteq$ P $\Rightarrow$ y-z $\in$ (P: M) hence (P:M) is equiprime ideal of R.

Proposition 2: If M is an equiprime R-module then (0:M) is an equiprime ideal of R.

Proof (ii)-If M is an equiprime R-module then (0:M) is an ideal of R by theorem 1.27. Let b $\in$ R with b $\in$ R\(0:M), for any x,y $\in$ R such that xrb-yrb $\in$ (0:M), for all r $\in$ R. Now for all r $\in$ R, xrb-yrb $\in$ (0:M)

$\Rightarrow$ m(xrb-yrb)=0, for all m $\in$ M

$\Rightarrow$ (mx)rb-(my)rb=0, for all m $\in$ M $\Rightarrow$ mx-my=0 (since M is equiprime)

$\Rightarrow$ m (x-y)=0, for all m $\in$ M $\Rightarrow$ x –y $\in$ (0:M)

Hence (0:M) is an equiprime ideal of R.

Theorem 9: Let R be a Boolean like semi ring and P is ideal of R with P $\neq$ R. Then the following are equivalent.

(i) P is an equiprime ideal of R

(ii) There exists an equiprime R-module M such that P=(0:M)

Proof (i) $\Rightarrow$ (ii)

We assume that P is equiprime ideal of R, then P is an ideal of R.

Let M=R/P, Then M is semi module over R by theorem 1.28.

We first show that P=(0:M)

Let x $\in$ P, r $\in$ R then xr $\in$ P $\Rightarrow$ rx+P=P $\Rightarrow$ (r+P) x=0+P, 0+P is the zero element in M x $\in$ (0:M), hence P $\subseteq$ (0:M).

Conversely, let r $\in$ (0:M) $\Rightarrow$ (x+P) r=0, for all x $\in$ R

$\Rightarrow$ xr+P=0+P, for all x $\in$ R $\Rightarrow$ xr $\in$ P, for all x $\in$ R $\Rightarrow$ xR $\subseteq$ P $\Rightarrow$ xRx $\subseteq$ P $\Rightarrow$ x $\in$ P, since P is equiprime. Hence (0:M) $\subseteq$ P, therefore P=(0:M).

We now finally show that M is equiprime R-module.

If MR=0 then (x+P) R=0, for all x $\in$ R $\Rightarrow$ (x + P)r=0+P, for all x, r $\in$ R

$\Rightarrow$ xr+P=0+P, for all x, r $\in$ R $\Rightarrow$ xr $\in$ P for all x, r $\in$ R

RR $\subseteq$ P $\Rightarrow$ R $\subseteq$ P. Hence P=R which is contradiction to P $\neq$ R. Therefore MR $\neq$ 0.

We prove by the method of contrapositive, let x, y $\in$ R such that (x+P)-(y+P) $\neq$ 0

$\Rightarrow$ (x-y) +P $\neq$ 0+P $\in$ x–y is not in P.

Since P is equiprime, so there exists r $\in$ R such that xra-yra is not in P

$\Rightarrow$ (xra-yra)+P $\neq$ 0 $\in$ (xra+P)-(yra+P) $\neq$ 0 $\in$ (x+P) ra-(y+P) ra $\neq$ 0. Hence (ii) is proved (ii) $\Rightarrow$ (i)

Let us assume that (ii) holds. We shall show that P is equiprime ideal of R.

Since M is equiprime R-module then by theorem 3.3., (0:M) is equiprime ideal of R. Hence P is equiprime ideal of R.

Remark 6-If P=0, zero ideal of R then the above theorem can be stated as follows.

Theorem 10: If R is a Boolean like semi ring and R $\neq$ 0. Then R is equiprime if and only if there exists a faithful equiprime R-module M.

Theorem 11: Let M be an equiprime R-module and suppose that 0 $\neq$ N is R-sub module of M then show that

(i) $(0{:}\,M)_R=(0{:}N)_R$ (ii) N is an equiprime R-module.

(i) Let $\alpha \in (0{:}M)_R \Rightarrow m\alpha=0$, for all m $\in$ M $\Rightarrow$ n α=0, for all n $\in$ N $\subseteq$ M

$\Rightarrow \alpha \in (0{:}N)_R$. Hence $(0{:}M)_R \subseteq (0{:}N)_R$

We shall show that $(0{:}N)_R \subseteq (0{:}M)_R$

Suppose that $(0{:}N)_R$ is not contained in $(0{:}M)_R$

Then $\alpha \in (0{:}N)_R$ such that α is not in $(0{:}M)_R \Rightarrow \alpha \in R\backslash(0{:}M)_R$

Let 0 $\neq$ n $\in$ N $\subseteq$ M, since M is equiprime R-module, so r $\in$ R such that nr $\alpha \neq 0r\alpha$. But nr $\in$ N, hence N$\alpha \neq$ (0)

$\Rightarrow \alpha$ is not in (0:N), which is contradiction to $\alpha \in (0{:}N)_R$.

Hence $(0{:}N)_R \subseteq (0{:}M)_R$

Therefore $(0{:}M)_R = (0{:}N)_R$

(ii) Since M is an equiprime R-module, so MR $\neq$ 0

$\Rightarrow$R is not contained in (0:M)=(0:N). Therefore NR $\neq$ 0. Let b $\in$ R with b is not in (0:N)=(0:M).

Let x, y $\in$ N such that xrb-yrb=0, for all r $\in$ R Now b is not in (0:M), x, y $\in$ M and M is equiprime R-module Therefore xra-yra=0, for all r $\in$ R $\Rightarrow$ x–y=0

Hence N is equiprime R-sub module of M.

Theorem 12: Let M be an equiprime R-module and let N be an invariant sub group of R such that N is not contained in (0:M). Then M is an equiprime N-module.

Proof-Let M be an equiprime R-module and let N be an invariant sub group of R such that N is not contained in (0:M).Then MN $\neq$ 0.

Let a $\in$ N with a $\in$ N\ (0:M). For x, y $\in$ M such that x $\neq$ y

Since M is an equiprime R-module and a $\in$ R\ (0:M).

Then r $\in$ R such that xra-yra $\neq$ 0 $\Rightarrow$ xra $\neq$ yra $\Rightarrow$ $\exists$s $\in$ R such that (xra) sa $\neq$ (yra) sa

$\Rightarrow$ x (ras) a $\neq$ y (ras) a $\Rightarrow$ xpa $\neq$ ypa, where ras=p $\in$ R $\Rightarrow$ xpa-ypa $\neq$ 0

Therefore $\exists$ p $\in$ R such that x $\neq$ y $\Rightarrow$ xpa-ypa $\neq$ 0

Hence M is equiprime N-module.

Conclusion

This work explored several features and characterizations of semi module bases over a commutative semi ring R, as well as some equivalent criteria for a basis to be a free basis in a finitely produced free semi module over R. Parts of the results in this paper develop and generalise the related results in for commutative join-semi rings and for commutative zero sum free semi rings.

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