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EOQ Model for Constant Demand Rate with Completely Backlogged and Shortages

R. P. Tripathi* and Neha Sang

Department of Mathematics, Graphic Era University, Dehradun, Uttarakhand, India

Abstract

This paper presents an EOQ model with constant demand rate for non deteriorating items where shortages are allowed. In this paper shortages are considered as completely backlogged. The production rate is assumed to be proportional to demand rate and finite. The optimal solution of the model has been done for finding optimal time, optimal average cost by considering four different situations. Numerical example and sensitivity analysis is given to illustrate the proposed model.

Keywords: Demand; Inventory; Shortage; Production; Economic order quantity

Introduction

In last few decades, Mathematical ideas have been developed in different areas in real life problems, particularly for controlling inventory. The most important concern of the management is to decide when and how much to order so that the total cost connected with the inventory system should be minimum. This paper deals with the problems of determining EOQ model for non deteriorating items and constant demand rate. A number of authors have discussed inventory models for non-deteriorating items.

However, there are certain substances in which deterioration plays an important role and items cannot be stored for a long time. When items are kept in stock as an inventory for fulfilling the future demand, there may be deterioration of items in the inventory system, In inventory models generally four types of demand are assumed: (a) constant demand (b) time-dependent demand (c) probabilistic demand and (d) stock-dependent demand, inventory models dealing with constant demand have more relevance and common in the present situations.

Initially, the demand rate of items was considered to be constant. However, in real life situations, demand varies with time and situations. Goyal [1], developed an economic order quantity under conditions of permissible delay in payments. In paper [1] Goyal considered that the demand rate is constant. Chung et al. [2], established the optimal inventory policies under permissible delay in payments depending on the order quantity and total minimum variable cost per unit of time is obtained. Mahata and Mahata [3], developed optimal retailer's ordering policies in the EOQ model for deteriorating items under trade credit financing in supply chain. In paper [3] minimum retailer's optimal ordering policies are obtained. Huang [4], modified the assumption that the retailers will adopt the trade credit policy to stimulate his/her customer demand to develop the retailer's replenishment model. Huang [4], assumed that the inventory lend is depleted by customer's demand alone. This assumption is valid for non permissible or non deteriorating inventory items. The demand rate for a particular item is not always constant it vary with time. Tripathi [5], established an EOQ model with time dependent demand rate and time-dependent holding cost functions. Tripathi [6], also established an optimal policy for items having constant demand and constant deterioration rate with trade credit. In this paper [6] a model for a retailer to determine its optimal price and time is obtained. Tripathi and Kumar [7], developed credit financing in economic ordering policies of time-dependent deteriorating items. This paper [7] presented the economic ordering policies in the presence of trade credit using discounted cash flows (DCF) approach.

Furthermore, when the shortages occur, it is assumed that it is either completely backlogged or completely lost. But in real life some customers are willing to wait for backorder and others would turn to buy from other sellers. Uthaya and Geetha [8], developed a replenishment policy for non-instantaneous deteriorating inventory system with partial backlogging. This paper [8] investigated an instantaneous replenishment inventory model for the non-instantaneous deteriorating items under cost minimization. Tripathi and Mishra [9], established an inventory model with shortage, time-dependent demand rate and quantity dependent permissible delay in payment. Patra et al. [10], established an order level EOQ model for deteriorating items in a single warehouse system with price dependent demand in non-linear form. In this paper [10] demand rae is considered as a function of selling price and order level inventory model for deteriorating items with single warehouse is developed where shortages are taken into consideration and it is completely backlogged. An EOQ model for permissible items with power demand and partial backlogging was developed by Singh et al. [11]. In paper [11] backlogging rate is considered as variable and dependent on the waiting time for the next replenishment. Research such as Park [12], Hollier and Mark [13], and Wee [14] considered the constant partial backlogging rate during the shortage period in their inventory models. Akad [15], established an EOQ model allowing shortage and partial backlogging. The thime proportion partial backlogging rate was developed by Chang and Dye [16], Wang [17], Teng and Yang [18], Yang [19], Wu et al [20], Dye et al. [21] and so on.

Assumption and Notation

The following assumptions and notations are used to develop the

***Corresponding author:** R. P. Tripathi, Department of Mathematics, Graphic Era University, Dehradun, Uttarakhand, India, E-mail: tripathi_rp0231@rediffmail.com

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proposed model:

- Demand rate R (t) is constant. Let it be D.
- Deterioration rate is zero.
- Production rate is $P(t) = \lambda_0 R(t)$ where (λ_0 >1) a constant therefore P (t)>R (t).
- Lead time is zero.
- Shortages are allowed and are completely backlogged.

In addition the following notations are being made throughout the paper:

C_1 Carrying cost per unit per unit time.

C_2 Shortage cost per unit per unit time.

C_3 Setup cost per production run.

C_1, C_2, C_3 are all assumed to be known and fixed during production cycle.

'AC' the total average cost for a production cycle.

Mathematical Formulation

Inventory level I(t) is zero at time t=0. The shortages starts at t-0 and accumulates up to the level A at time t=T_1. The production starts at t=T_1 and backlog is cleared at t=T_2. The stock-level reaches at level B at t=T_3.The production stopped at level t=T_3. Thus the inventory level decreases gradually due to demand and becomes zero at t=T_4.The cycle completed and repeals itself. I (t) is the inventory level at any time t (0≤t≤ T_4). The differential equations of states are given by

$$\frac{dI(t)}{dt} = -R(t) \text{, } 0 \le t \le T_1 \tag{1}$$

$$\frac{dI(t)}{dt} = K(t) - R(t) \text{ , } T_1 \le t \le T_2 \tag{2}$$

$$\frac{dI(t)}{dt} = K(t) - R(t) \text{, } T_2 \le t \le T_3 \tag{3}$$

$$\frac{dI(t)}{dt} = -R(t) \text{ , } T_3 \le t \le T_4 \tag{4}$$

With boundary conditions

$$I(0) = 0, I(T_1) = -A, I(T_2) = 0, I(T_3) = B, I(T_4) = 0 \tag{5}$$

Substituting $R(t) = D$ and $K(t) = \lambda_0 R(t)$ in the equations (1)-(4) and solving then using the boundary conditions (5), we get the following solutions:

$$I(t) = -Dt \text{ , } 0 \le t \le T_1 \tag{6}$$

$$I(t) = (\lambda_0 - 1)D(t - T_2) \text{, } T_1 \le t \le T_2 \tag{7}$$

$$I(t) = (\lambda_0 - 1)D(t - T_2) \text{ , } T_2 \le t \le T_3 \tag{8}$$

$$I(t) = D(T_4 - t) \text{, } T_3 \le t \le T_4 \tag{9}$$

Using the condition I(t)=-A and $t = T_1$ in (6) and (7) and then equating the two values of A, we get

$$T_1 = \left(\frac{\lambda_0 - 1}{\lambda_0}\right) T_2 \tag{10}$$

Again using I(t)=B and t=T_2 in (8) and (9) and then equating the two values of B, we get

$$T_3 = \frac{(\lambda_0 - 1)T_2 + T_4}{\lambda_0} \tag{11}$$

Now, we shall try to find the different costs involved in the system,

The total shortage cost in the system is

$$SC = \left[\int_0^{T_1} -I(t)dt + \int_{T_1}^{T_2} -I(t)dt\right] = \frac{C_2 D}{2}\left[T_1^2 + (\lambda_0 - 1)(T_1 - T_2)^2\right] \tag{12}$$

The total inventory holding cost in the system is

$$HC = \left[\int_{T_2}^{T_3} I(t)dt + \int_{T_3}^{T_4} I(t)dt\right] = \frac{C_1 D}{2}\left[(\lambda_0 - 1)(T_3 - T_2)^2 + (T_4 - T_3)^2\right] \tag{13}$$

Hence total average cost in the system is,

$$AC = \frac{(SC + HC + C_3)}{T_4}$$

$$= \frac{C_2 \frac{D}{2}\left[T_1^2 + (\lambda_0 - 1)(T_1 - T_2)^2\right] + C_1 \frac{D}{2}\left[(\lambda_0 - 1)(T_3 - T_2)^2 + (T_4 - T_3)^2\right] + C_3}{T_4} \tag{14}$$

For finding optimal values of T_i : i=1,2,3,4; we convert T_1 and T_3 in terms of T_2 and T_4

$$= \frac{1}{T_4}\left[\frac{C_2 D}{2}\left(1 - \frac{1}{\lambda_0}\right)T_2^2 + \frac{C_1 D}{2}\left\{\frac{(\lambda_0 - 1)}{\lambda_0^2}\left(T_4^2 - 2T_2T_4 + \lambda_0 T_2^2\right)\right\} + C_3\right] \tag{15}$$

The optimal value of T_2 and T_4, is obtained by solving $\frac{\partial(AC)}{\partial T_2} = 0, and \frac{\partial(AC)}{\partial T_4} = 0$ we get,

$$\lambda_0(C_1 + C_2)T_2 - C_1 T_4 = 0 \tag{16}$$

$$(\lambda_0 - 1)\left\{C_1 D T_4^{\,2} - (C_1 + C_2)D T_2^{\,2}\right\} - 2\lambda_0 C_3 = 0 \tag{17}$$

Since, $\frac{\partial^2(AC)}{\partial T_2^{\,2}} > 0, and \frac{\partial^2(AC)}{\partial T_4^{\,2}} > 0$,and $\left(\frac{\partial^2(AC)}{\partial T_2^{\,2}}\right)\left(\frac{\partial^2(AC)}{\partial T_4^{\,2}}\right) - \left(\frac{\partial^2(AC)}{\partial T_2 \partial T_4}\right)^2 > 0$

This shows that average cost (AC) will be minimum.

Solving equations (16) and (17), we get different value of T_2 and T_4 for changing different parameters.

Numerical Example

Let $\lambda_0 = 1.05, C_1 = 20, C_2 = 30, C_3 = 40, D = 100$ in appropriate units from (16) and (17), we obtain the optimal values of T_2 and T_4. Putting the optimal value of T_2 and T_4 in equation (10) and (11), we obtain the optimal values of T_1 and T_3 respectively.

We get, T_1=.02726, T_2=.5725, T_3=1.4576, T_4=1.5029 and Total average cost 'AC'= $ 602.37.

Sensitivity Analysis

All the observations from table 1 to 4 can be sum up as follows:

(i) From table 1, it can be easily seen that increase in the value of parameter C_1 will results increase in T_1, T_2 but decrease in T_3, T_4 and increase in average cost.

C_1	T_1	T_2	T_3	T_4	AC(in dollars)
20	0.0272	0.5725	1.4576	1.5029	602.37
25	0.0300	0.6320	1.4206	1.4601	623.60
30	0.0325	0.6831	1.3987	1.4345	637.91
35	0.0346	0.7274	1.3855	1.4184	648.29
40	0.0364	0.7660	1.3771	1.4077	656.38
45	0.0380	0.8000	1.3714	1.4000	662.54
50	0.3949	0.8297	1.3671	1.3940	669.27
55	0.0407	0.8559	1.3635	1.3889	672.58
60	0.0418	0.8788	1.3601	1.3842	677.02

Table 1: Variation of C_1 keeping other parameters same as mentioned in numerical example.

C_2	T_1	T_2	T_3	T_4	AC(in dollars)
30	0.0272	0.5725	1.4576	1.5029	602.37
35	0.0235	0.4941	1.3822	1.4267	632.33
40	0.0206	0.4344	1.3240	1.3685	657.51
45	0.0184	0.3876	1.2782	1.3227	678.82
50	0.0166	0.3498	1.2411	1.2857	697.25
55	0.0151	0.3187	1.2106	1.2552	713.24
60	0.0139	0.2927	1.1850	1.2296	727.20
65	0.0128	0.2706	1.1632	1.2078	739.52
70	0.0122	0.2516	1.1447	1.1891	750.55
80	0.0105	0.2206	1.1138	1.1585	769.28
90	0.0093	0.1964	1.0899	1.1345	784.56
100	0.0084	0.1770	1.0706	1.1153	797.36

Table 2: Variation of keeping other parameters same as mentioned in numerical examples.

C_3	T_1	T_2	T_3	T_4	AC(in dollars)
40	0.0272	0.5725	1.4576	1.5029	602.37
50	0.0304	0.6401	1.6300	1.6803	673.44
60	0.0333	0.7012	1.7863	1.8407	737.73
70	0.0360	0.7574	1.9295	1.9881	796.76
80	0.0385	0.8096	2.0627	2.1254	851.90
90	0.0408	0.8588	2.1879	2.2543	903.46
100	0.0430	0.9052	2.3062	2.3763	952.43
120	0.0472	0.9916	2.5532	2.6031	1043.33
140	0.0509	1.0711	2.7376	2.8117	1126.90
160	0.0545	1.1450	2.9172	3.0058	1204.74
180	0.0578	1.2145	3.0941	3.1881	1277.75
200	0.0609	1.2802	3.2615	3.3606	1346.89

Table 3: Variation of keeping other parameters same as mentioned in numerical example.

(ii) From table 2, increase in the value of parameter C_2 will results in decrease in T_1, T_2, T_3 and T_4 but increase in average cost.

(iii) From table 3, increase in the value C_3 of will results in increase in T_1, T_2, T_3, T_4 and average cost AC.

(iv) From table 4, increase in the value λ_0 of will results in decrease in T_1, T_2, T_3, T_4 and average cost AC.

Conclusion

The model developed in this paper assumed that demand rate of a product is constant, shortages are allowed and replenishment rate is infinite, production rate is greater than demand rate and proportional to the demand rate. The effect of changing parameters is quite sensitive and makes significant difference to the changes. Numerical example is given to illustrate the model. Sensitivity analysis has been also given for changing the various parameters.

λ_0	T_1	T_2	T_3	T_4	AC(in dollars)
1.1	0.0186	0.3912	1.6930	1.0758	476.74
1.2	0.0124	0.2618	0.7606	0.7855	407.07
1.3	0.0097	0.2049	0.6442	0.6662	387.47
1.4	0.0081	0.1716	0.5804	0.6008	381.45
1.5	0.0071	0.1494	0.5407	0.5603	380.87

Table 4: Variation of Keeping other parameters same as mentioned in numerical example.

This model can be extended for several ways. For instance, we may extend the model for time-dependent demand rate. We could also generalised the model for cash discount, inflation etc

References

1. Goyal SK (1985) Economic order quantity under conditions of permissible delay in payments. J Oper Res Soc 36: 335-338.
2. Chung KJ, Goyal SK, Huang YF (2005) The optimal inventory policies under permissible delay in payments depending on the ordering quantity. Int J Prod Econ 95: 203-213.
3. Mahata GC, Mahata P (2009) Optimal Retailer's Ordering Policies in the EOQ Model for deteriorating items under Trade credit financing in supply chain. International Journal of Mathematical, Physical and Engineering Sciences 3: 1-7.
4. Huang YF (2003) Optimal retailer's ordering policies in the EOQ model under trade order financing. J Oper Res Soc 54: 1011-1015.
5. Tripathi RP (2011) EOQ model with time-dependent demand rate and time dependent holding cost function. International Journal of Operational Research and Information Systems 2: 79-92.
6. Tripathi RP, Misra SS (2012) An Optimal Inventory Policy for Items having constant demand and constant deterioration rate with trade credit. International Journal of Information System and Supply Chain Management 5: 89-95.
7. Tripathi RP, Kumar M (2011) Credit financing in economic ordering policies of time-dependent deteriorating items. International Journal of Business, Management and Social Sciences 2: 75-84.
8. Uthaya Kumar R, Geetha KV (2009) A replenishment policy for non-instantaneous deteriorating inventory system with partial backlogging. Tamsui Oxford Journal of Mathematical Sciences 25: 313-332.
9. Tripathi RP, Mishra SS (2011) An inventory model with shortages, time dependent demand rate and quantity dependent permissible delay in payment.
10. Patra SK, Lenka TK, Ratha PC (2010) An order level EOQ model for deteriorating items in a single warehouse system with price dependent demand in non-linear (quadratic) form. International Journal of Computational and Applied Mathematics 5: 277-288.
11. Singh TJ, Singh SR, Dutt R (2009) An EOQ model for permissible items with power demand and partial backlogging. International Journal of Operations and Quality Management 15: 65-72.
12. Park KS (1982) Inventory models with partial backorders. International Journal of system science 13: 1313-1317.
13. Hollier RH, Mark KL (1983) Inventory replenishment policies for deteriorating items in a declining market. Int J Prod Res 21.
14. Wee HM (1995) Joint pricing and replenishment policy for deteriorating inventory with declining market. Int J Prod Econ 40: 163-171.
15. Abad PL (1996) Optimal Pricing and Lot-Sizing under Conditions of Perishability and Partial Backordering. Management Science 42: 1093-1104.
16. Chang HJ, Dye CY (1999) An EOQ model for deteriorating items with time varying demand and partial backlogging. J Oper Res Soc 50: 1176-1182.
17. Wang SP (2002) An inventory replenishment policy for deteriorating items with shortages and partial backlogging. Computers and Operational Research 29: 2043-2051.

18. Teng JT, Yang HL (2004) Deterministic economic order quantity models with partial backlogging when demand and cost are fluctuating with time. J Oper Res Soc 55: 495-503.
19. Yang HL (2005) A comparison among various partial backlogging inventory lot-size models for deteriorating items on the bases of maximum profit. Int J Prod Econ 96: 119-128.
20. Wu KS, Ouang LY, Yang CT (2006) An optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand and partial backlogging. Int J Prod Econ 101: 369-384.
21. Dye CY, Ouang LY, Hsieh TP (2007) Deterministic inventory model for deteriorating items with capacity constraint and time-proportional backlogging rate. European Journal of Operational Research 178: 789-807.