Journal of
Applied & Computational Mathematics

Research Article Open Access

Effects of Variable Viscosity and Thermal Conductivity on Free Convection Heat and Mass Transfer Flow through a Vertical Channel

Ajibade AO* and Ojeagbase PO

Department of Mathematics, Ahmadu Bello University, Zaria, Nigeria

Abstract

In this study, effects of variable viscosity and thermal conductivity on natural convection flow through a vertical channel are investigated. The governing equations are transformed into a set of coupled nonlinear ordinary differential equations. The transformed equations are solved using Differential Transformation Method (DTM). Results obtained were compared with exact and numerical methods. The influence of the flow parameters on fluid temperature, concentration and velocity are presented graphically. From the course of investigation, it was revealed that fluid temperature increases within the channel with increasing viscosity and thermal conductivity. In addition, results from DTM show an excellent agreement with results obtained from exact and numerical method.

Keywords: Free convection; Variable viscosity; Thermal conductivity; Incompressible fluid; Heat transfer

Introduction

Free convective flow with variable viscosity and thermal conductivity of many fluids varying with temperature have received great attention from researchers as a result of their real life applications in engineering such as chemical, geothermal systems, crude oil extraction, machinery lubrication and biochemical industries. Konch and Hazarika [1] studied the effects of variable viscosity and thermal conductivity on MHD free convective flow of dusty fluid along a vertical stretching sheet with heat generation using fourth order Runge-Kutta shooting method. Results from the above work shows that increasing variable viscosity decreases fluid viscosity while fluid velocity increases with increase in thermal conductivity. Hossain et al. [2] investigated the effect of variable viscosity and thermal conductivity on natural convection over an isothermal vertical wavy cone using a very efficient implicit finite difference method. Munir and Gorla [3] have considered a two dimensional mixed convection flow of a viscous incompressible fluid of temperature dependent viscosity and thermal conductivity past a vertical impermeable flat plate using a perturbation technique. Variable viscosity and thermal conductivity effects on heat transfer by natural convection from a cone and a wedge have been analyzed numerically using the finite difference scheme by Hassanien et al. [4]. Ozalp and Selim [5] presented the influence of variable thermal conductivity and viscosity for non-isothermal fluid flow. In the above work, the viscosity and thermal conductivity exhibit linear temperature dependence and is solved iteratively using chebyshev Pseudo spectral method. Attia [6] investigated the unsteady hydro magnetic channel flow of dusty fluid with temperature dependent viscosity and thermal conductivity numerically. Recently, free convection flow along a vertical flat plate with thermal conductivity and viscosity depending on temperature have been investigated by Nasrin and Alim [7] using implicit finite difference method with Keller-Box scheme . Choudhury and Hazarika [8] investigated the influence of variable viscosity and thermal conductivity on MHD flow due to a point sink numerically using shooting method. From the above work, the variable viscosity and thermal conductivity parameters have substantial effects on velocity field as well as on the drag and heat transfer characteristic within the boundary layer due to point sink. The effects of varying viscosity and thermal conductivity on steady MHD free convective flow and heat transfer along an isothermal plate with heat generation have been studied by Sharma and Singh [9]. They observed that the varying viscosity and thermal conductivity modifies the flow of fluid and the governing differential equations were solved using the Runge kutta fourth order technique along with shooting method. More so, Mahanti and Gaur [10] investigated the effects of linearly varying viscosity and thermal conductivity on steady free convective flow of a viscous incompressible fluid along an isothermal vertical plate in the presence of heat sink. The Runge-Kutta fourth order method with shooting technique was used to solve the nonlinear differential equation. Molla and Gorla [11] investigated natural convection laminar flow with temperature dependent viscosity and thermal conductivity along a vertical wavy surface. They considered the boundary layer regime when the Grashof number Gr is large and the governing equations were solved employing the implicit finite difference method together with Keller-Box scheme. Numerical study on a vertical plate with variable viscosity and thermal conductivity was carried out by Palani and Kim [12] using a very efficient implicit finite difference scheme known as Crank-Nicolson scheme. Ashraf et al. [13] presented a numerical solution for the problem of steady two dimensional boundary layer buoyancy flows on a vertical magnetized surface when both the viscosity and thermal conductivity are assumed to be temperature dependent. More recently, Keimanesh and Aghanajafi [14] studied the effect of temperature dependent viscosity and thermal conductivity on micro polar fluid using the shooting method and forth-order Runge-Kutta method.

Several solution techniques have been derived to solve nonlinear and coupled equations. Some of them employed numerical techniques such as Runge-kutta shooting method, finite difference and finite elements methods. Other techniques are approximate analytical solution techniques which include perturbation, Homotopy Perturbation Technique, Adomian Decomposition Technique, He-

***Corresponding author:** Ajibade AO, Department of Mathematics, Ahmadu Bello University, Zaria, Nigeria, Tel: 84752698547; E-mail: ojeagbase10@gmail.com

Received December 26, 2018; **Accepted** April 04, 2019; **Published** April 11, 2019

Citation: Ajibade AO, Ojeagbase PO (2019) Effects of Variable Viscosity and Thermal Conductivity on Free Convection Heat and Mass Transfer Flow through a Vertical Channel. J Appl Computat Math 8: 450.

Laplace technique and Differential Transformation Method. The DTM technique is accurate and more efficient and requires less computational effort in comparison to other methods mentioned above. The Differential Transformation Method (DTM) is a powerful mathematical tool for solving systems of linear and nonlinear differential equations and requires significantly less computational resources. Zhou [15] first introduced the differential transformation method for solving linear and nonlinear initial value problems in an electrical circuit theory. Chen and Ho [16] developed the differential transformation method for partial differential equations and developed a closed form series solutions for linear and nonlinear initial value problems. Umavathi and Shekar [17] studied the combined effect of variable viscosity and thermal conductivity on free convection flow of a viscous fluid in a vertical channel using the differential transformation method. Jha and Ajibade [18] investigated free convection heat and mass transfer flow in a vertical channel with Dufour effect. The investigation carried out reveals that the steady state of the problem is independent of the Dufour effect. This present work investigates the effect of temperature dependent viscosity and thermal conductivity on the steady state of Jha and Ajibade [18] with heat source and chemical reaction effects

Mathematical Formulation

The system under consideration is a laminar non-isothermal flow of an incompressible fluid between two vertical parallel plates positioned at $y'=0$ and $y'=h$ with uniform temperature T_1 (hot wall) and T_2 (cold wall). The flow is assumed to be in the x'-direction which is taken vertically upward along the vertical plates and the y'-axis is taken normal to the plates as shown in Figure 1. Since the plates are infinite in lengths, the velocity, temperature and concentration fields are function of y' only.

All fluid properties are considered constant except the influence of variable viscosity, thermal conductivity and density variation with temperature. In addition, the influence of density and expansion coefficient variation in terms in the momentum and energy equations are negligible. The steady natural convection fully developed flow in a vertical channel under the usual Boussinesq approximation is governed by the following equations:

$$\frac{1}{\rho}\frac{d}{dy'}\left(\mu\frac{du'}{dy'}\right)+g\beta(T'-T_2')+g\overline{\beta}(C'-C_2')=0 \qquad (1)$$

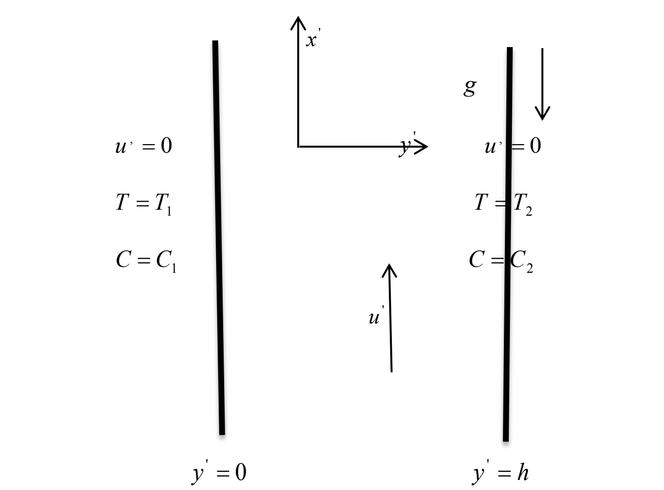

Figure 1: Schematic diagram of fluid flow.

$$\frac{1}{\rho cp}\frac{d}{dy'}\left(k\frac{dT'}{dy'}\right)+\frac{Q^*}{\rho cp}(T'-T_2')=0 \qquad (2)$$

$$D_m\frac{d^2C'}{dy'^2}-K_r'(C'-C_2')=0 \qquad (3)$$

The boundary conditions to the governing equations are:

$$u'=0, T'=T_1', C'=C_1' \quad at \quad y'=0$$
$$u'=0, T'=T_2', C'=C_2' \quad at \quad y'=h \qquad (4)$$

Introducing the following non-dimensional quantities:

$$u=\frac{u'}{u_0}, y=\frac{y'}{h}, \theta=\frac{T'-T_2'}{T_1'-T_2'}, C=\frac{C'-C_2'}{C_1'-C_2'},$$
$$K_c=\frac{K_r^*h^2}{\nu_0}, \Pr=\frac{\mu_0 cp}{k_0}, Sc=\frac{\nu_0}{D}, Q=\frac{Q_0h^2}{\nu_0\rho cp}, N=\frac{\overline{\beta}(C_1'-C_2')}{\beta(T_1'-T_2')} \qquad (5)$$
$$u_0=\frac{g\beta h^2(T_1'-T_2')}{\nu_0}$$

Where Pr is the Prandtl number, N is buoyancy parameter, Sc is the Schmidt number. K_c is the chemical reaction parameter and Q is the heat source parameter.

Following (Umavathi and Shekar [17]), the fluid viscosity (μ) is assumed to vary linearly as a function of temperature in the form:

$$\mu=\mu_0[1-a(T'-T_2')] \qquad (6)$$

In general, (a<0) for gases and (a>0) for liquids. Where a is the dimension of k^{-1} and eqn. (6) can be rewritten in the form:

$$\mu=\mu_0(1-\lambda\theta) \qquad (7)$$

Where $\lambda=a(T_1'-T_2')$.

Similarly, following (Anjali and Prakash [19]), the fluid conductivity (k) is assumed to vary linearly as a function of temperature in the form:

$$k=k_0[1+b(T'-T_2')] \qquad (8)$$

In general, (b>0) for fluids such as water and gases while (b<0) for fluids such as lubricating oils. Where b is the dimension of K^{-1} and eqn. (8) can be rewritten in the form:

$$K=k_0(1+\gamma\theta) \qquad (9)$$

Where $\gamma=b(T'_1-T'_2)$. In addition, the range of variation of γ can be taken as follows (Schlichting and Gerstenk [20]) for air $0 \le \gamma \le 6$ for water $0 \le \gamma \le 0.12$ and for lubrication oil $-0.1 \le \gamma \le 0$.μ_0 and k_0 are constants when the temperature is T_2. The reference temperature is taken to be T_2.

Using eqn. (5), eqn. (7) and eqn. (9), eqns. (1)-(4) reduces to:

$$(1-\lambda\theta)\frac{d^2u}{dy^2}-\lambda\frac{d\theta}{dy}\frac{du}{dy}+\theta+NC=0 \qquad (10)$$

$$\frac{(1+\gamma\theta)(1+\lambda\theta)}{\Pr}\frac{d^2\theta}{dy^2}+\frac{\gamma(1+\lambda\theta)}{\Pr}\left(\frac{d\theta}{dy}\right)^2+Q\theta=0 \qquad (11)$$

$$\frac{1}{Sc}\frac{d^2C}{dy^2}-K_cC=0 \qquad (12)$$

The reduced boundary conditions are:

$u=0$, $\theta=1$, $C=1$ at $y=0$

$u=0$, $\theta=0$, $C=0$ at $y=1$ (13)

Differential Transformation Method

We define the transformation of the k^{th} derivative of a function as:

$$F(K)=\frac{1}{k!}\left[\frac{d^k f(\eta)}{dn^k}\right]_{\eta=\eta_0} \quad (14)$$

Where $f(\eta)$ is the original function and $F(k)$ is the transformed function. The differential inverse transform of $F(k)$ is given by:

$$f(\eta)=\sum_{k=0}^{\infty}F(K)\left(\eta-\eta_0\right)^k \quad (15)$$

The concept of differential transformation is derived from a Taylor series expansion and in real applications, the function $f(\eta)$ is expressed by a finite series as follows:

$$f(\eta)=\sum_{k=0}^{m}F(K)\left(\eta-\eta_0\right)^k \quad (16)$$

Where the value of m is decided by the convergence of the series coefficient. The operations performed by differential Transformation Method (DTM) are listed in Table 1 (Rashidi et al. [21]).

Solution with Differential Transformation Method

Taking the differential transforms of eqns. (10)-(13), we obtain the following:

$$\overline{U}(k+2)=\frac{1}{(k+1)(k+2)}\begin{pmatrix}\lambda\sum_{r=0}^{k}(k-r+1)(r+1)\overline{U}(k-r+1)\overline{\theta}(r+1)-\overline{\theta}(k)-N\overline{C}(k)\\ +\lambda^2\sum_{r=0}^{k}\sum_{s=0}^{r}(k-r+1)(r-s+1)\overline{U}(k-r+1)\overline{\theta}(r-s+1)\overline{\theta}(s)\\ -\lambda\sum_{r=0}^{k}\overline{\theta}(k-r)\overline{\theta}(r)-\lambda N\sum_{r=0}^{k}\overline{\theta}(k-r)\overline{C}(r)\end{pmatrix} \quad (17)$$

$$\overline{\theta}(k+2)=\frac{1}{(k+1)(k+2)}\begin{pmatrix}-\gamma\sum_{r=0}^{k}(k-r+1)(r+1)\overline{\theta}(k-r+1)\overline{\theta}(r+1)-\Pr Q\overline{\theta}(k)+\\ \gamma^2\sum_{r=0}^{k}\sum_{s=0}^{r}(k-r+1)(r-s+1)\overline{\theta}(k-r+1)\overline{\theta}(r-s+1)\overline{\theta}(s)\\ -\Pr Q\gamma\lambda\sum_{r=0}^{k}\sum_{s=0}^{r}\overline{\theta}(k-r)\overline{\theta}(r-s)\overline{\theta}(s)+\\ \Pr Q(\lambda+\gamma)\sum_{r=0}^{k}\overline{\theta}(k-r)\overline{\theta}(r)\end{pmatrix} \quad (18)$$

$$\overline{C}(k+2)=\frac{1}{(k+1)(k+2)}\left(Sc.Kc\overline{C}(k)\right) \quad (19)$$

Where $\overline{U}(k)$, $\overline{\theta}(k)$ and $\overline{C}(k)$ are the differential transform of $U(y)$,$\theta(y)$ and $C(y)$ respectively. The transformed boundary conditions are:

$$\overline{U}(0)=0,\overline{U}(1)=a,\ \overline{\theta}(0)=1,\overline{\theta}(1)=b,\overline{C}(0)=1,\overline{C}(1)=c \quad (20)$$

Where a, b, and c are constants which are computed from the boundary conditions in eqn. (13). The above equations for temperature, velocity and concentration are solved and the results obtained are presented graphically from Figures 2-9, and numerically from Tables 2 and 3 for different governing parameters.

Skin Friction

The dimensionless skin friction for the hot plate (y=0) and the cold plate (y=1) are given by:

$$\tau_0=(1-\lambda\theta)\left(\frac{du}{dy}\right)_{y=0} \text{ and } \tau_1=-(1-\lambda\theta)\left(\frac{du}{dy}\right)_{y=1} \quad (21)$$

Results for the skin friction on both boundary walls are presented on Tables 3 and 4

Heat Transfer

The dimensionless nusselt number for the hot plate (y=0) and the cold plate (y=1) are given by:

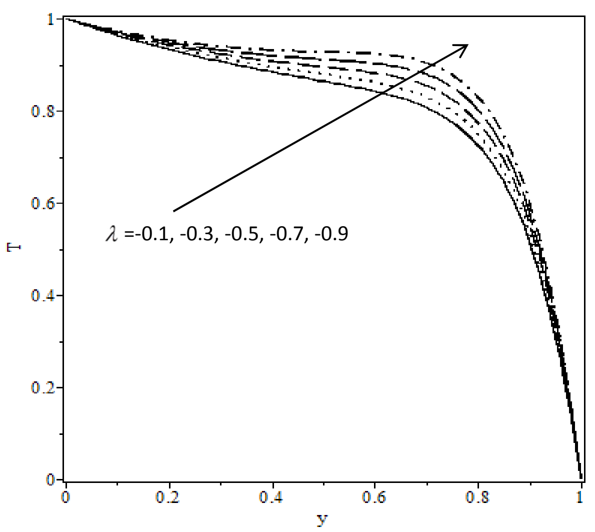

Figure 2: Temperature profile for binary mixture of carbon dioxide in air different values of viscosity parameter [γ=2, Kc=0.7, N=1, Q=0.5].

Original function	Transformed Function
1. $f(y)=g(y)\pm h(y)$	$F(k)=G(k)\pm H(k)$
2. $f(y)=\lambda g(y)$, where λ is a constant	$F(k)=\lambda G(k)$
3. $f(y)=\frac{d^n g(y)}{dy^n}$	$F(k)=(k+1)(k+2)......(k+r)G(k+r)$
4. $f(y)=g(y)h(y)$	$F(k)=\sum_{r=0}^{k}G(r)H(k-r)$
5. $f(y)=g(y)\frac{dh(y)}{dy}$	$F(k)=\sum_{r=0}^{k}(k-r+1)H(k-r+1)G(r)$
6. $F(k)=\sum_{r=0}^{k}(k-r+1)H(k-r+1)G(r)$	$F(k)=\sum_{r=0}^{k}(k-r+1)(k-r+2)H(k-r+2)G(r)$

Table 1: Operations of differential transformation method.

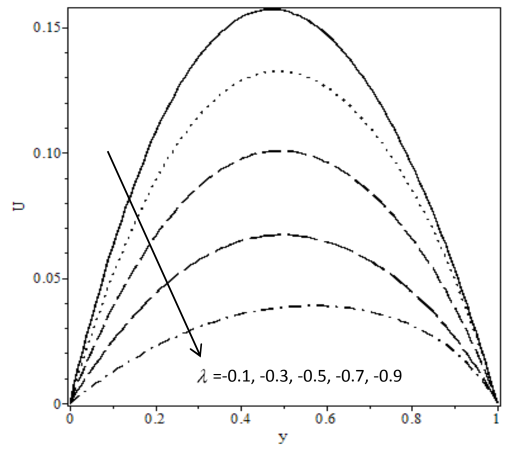

Figure 3: Velocity profile for binary mixture of carbon dioxide in air different values of viscosity parameter [γ=2, Kc=0.7, N=1, Q=0.5].

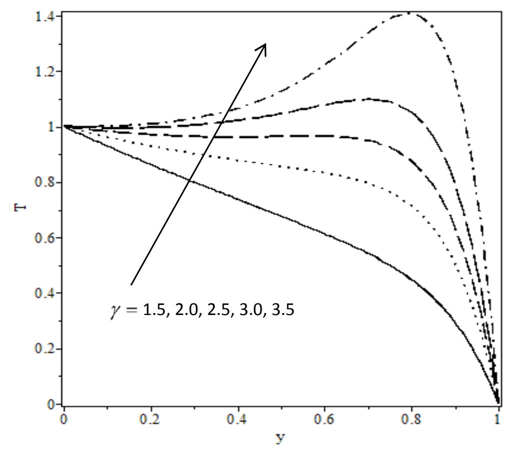

Figure 4: Temperature Profile for binary mixture of carbon dioxide in air for different values of thermal conductivity parameter [λ=-0.001, Kc=0.7, N=1, Q=0.5].

y	DTM	Exact	Numerical
0	0.0000000	0.0000000	0.0000000
0.25	0.1081620	0.1081620	0.1081620
0.5	0.1233825	0.1233825	0.1233825
0.75	0.0770296	0.0770296	0.0770296
1.0	0.0000000	0.0000000	0.0000000

Table 2: Comparison of Velocity between the present method (DTM) with Exact method and Numerical method when λ=γ=0.

$$Nu_0 = -(1+\gamma\theta)\left(\frac{d\theta}{dy}\right)_{y=0} \text{ and } Nu_1 = -(1+\gamma\theta)\left(\frac{d\theta}{dy}\right)_{y=1} \qquad (22)$$

Results for the nusselt number on both boundary walls are presented on Tables 3 and 4.

Volumetric Flow Rate

The volumetric flow rate of the fluid velocity is given by:

$$V \quad \int u(y)dy \qquad (23)$$

Also, results for the volumetric flow rate are presented on Tables 3 and 4.

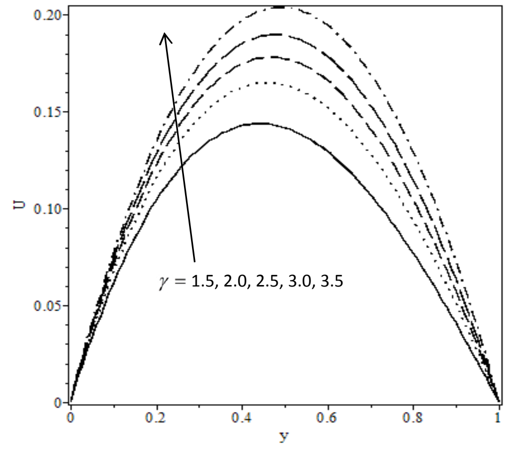

Figure 5: Velocity profile for binary mixture of carbon dioxide in air for different values of thermal conductivity parameter [λ=-0.001, Kc=0.7, N=1, Q=0.5].

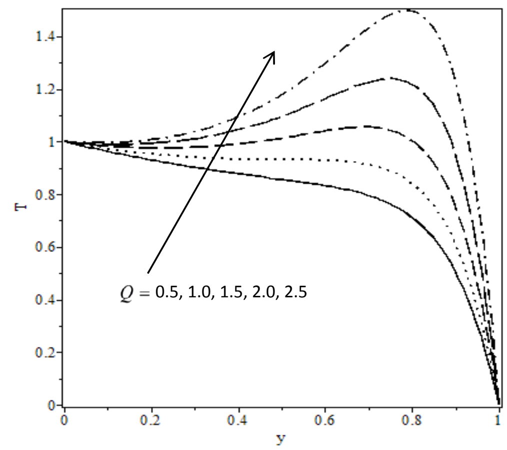

Figure 6: Temperature profile for binary mixture of carbon dioxide in air different values of heat source parameter [γ=2, Kc=0.7, N=1, λ=-0.001].

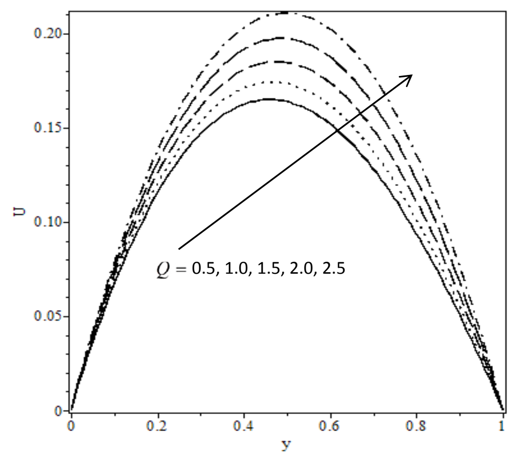

Figure 7: Velocity profile for binary mixture of carbon dioxide in air different values of heat source parameter [γ=2, Kc=0.7, N=1, λ=-0.001].

γ	DTM			Numerical		
	Temperature	Velocity	Concentration	Temperature	Velocity	Concentration
0	1.0000000	0.0000000	1.0000000	1.0000000	0.0000000	1.0000000
0.25	0.7774816	0.1088784	0.7159855	0.7774816	0.1088783	0.7159855
0.5	0.5340656	0.1244578	0.4615169	0.5340655	0.1244577	0.4615170
0.75	0.2734412	0.0778339	0.2260934	0.2734411	0.0778338	0.2260934
1.0	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000

Table 3: Comparison of present method (DTM) with Numerical method when λ=-0.001, γ=0.1, N=1, Q=0.5and Kc=0.7.

Pr=0.71,Sc=0.94,N=1,Q=0.5,Kc=0.7,γ=2					
λ	V_m	τ_0	τ_1	Nu_0	Nu_1
-0.1	0.10515185	0.77812318	0.59887651	1.19270141	7.65641144
-0.3	0.08954110	0.74526851	0.58266955	1.12640109	7.97885466
-0.5	0.06810047	0.63988109	0.45571188	1.06161803	8.32156071
-0.7	0.04545095	0.47002848	0.31742330	0.99852643	8.68771741
-0.9	0.02694521	0.25538257	0.28220541	0.93734380	9.08100149

Table 4: Computation of variation of mass flow rate, skin friction and Nusselt number for different value of viscosity parameter (λ).

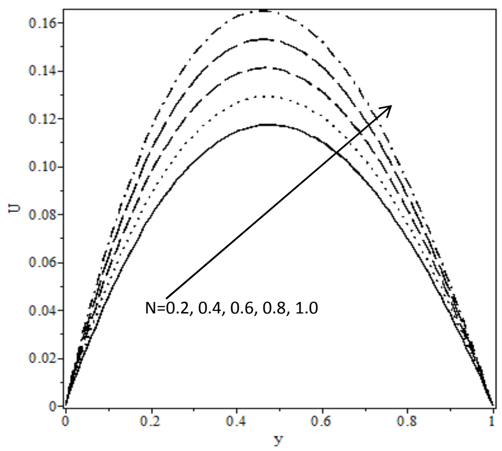

Figure 8: Velocity profile for binary mixture of carbon dioxide in air for different values of buoyancy parameter [γ=2, Kc=0.7, Q=0.5, λ=-0.001].

Validation of Result

In order to verify the accuracy of the present method (DTM), the results for the skin friction at both boundary walls are compared with those reported earlier by Jha and Ajibade [18] when d_f=λ=γ=Q=Kc=0. The results of this comparison are shown in Table 5. It can be seen from the table that excellent agreement between the results exist.

Convergence of the Differential Transformation Method (DTM)

Following Abayomi [22], Differential Transformation Method (DTM) converges to exact solution when the problem is linear. Comparison of the Differential Transformation Method (DTM) with exact and numerical methods was carried out on temperature, velocity and concentration fields for different values between the boundary walls in this work. From Table 2, results from the DTM shows a strong convergence with results obtained from both exact and numerical methods when viscosity and thermal conductivity parameters are zero. Table 3 also revealed a strong convergence to seven decimal places of the Differential transformation Method (DTM) with the numerical method for different governing parameters.

N	Jha and Ajibade [18], d_f=0		Present problem τ_1	
	τ_0	τ_1	τ_0	τ_1
0	0.33333333	0.16666667	0.33333333	0.16666667
0.25	0.41666667	0.20833333	0.41666667	0.20833333
0.5	0.50000000	0.25000000	0.50000000	0.25000000
0.75	0.58333333	0.29166667	0.58333333	0.29166667
1.0	0.66666667	0.33333333	0.66666667	0.33333333

Table 5: Comparison of skin friction between the present method (DTM) and that of Jha and Ajibade [18] when λ=γ=Q=Kc=0 for different values of buoyancy parameter (N).

Results and Discussions

This present article shows the combined effects of temperature dependent viscosity and thermal conductivity on natural convection flow in a vertical channel. The nonlinear coupled governing equations have been solved by Differential Transformation Method (DTM) to obtain results for temperature, velocity and concentration fields. In addition, the exact and numerical methods are also considered for comparison with the differential transformation method. For the purpose of discussion, the temperature, velocity and concentration fields are presented graphically for different values of the governing parameters. The values of Prandtl number and Schmidt number are chosen to be Pr=0.71 (air) and Sc=0.94 (carbon dioxide). Therefore, throughout this discussion, the fluid considered is a binary mixture of CO_2 in air.

Figures 2 and 3 shows the effect of variable viscosity parameter λ (λ<0 for gas). It is observed from these figures that increasing viscosity increases fluid temperature while fluid velocity decreases within the channel. This is expected since increasing (λ) leads to an increase in the fluid viscosity. Increasing fluid viscosity poses a hindrance to thermal diffusivity thereby causing heat accumulation leading to increase in fluid temperature.

The effect of thermal conductivity parameter (γ) is shown in Figures 4 and 5. It is noticed that fluid temperature and velocity increases with increasing values of thermal conductivity parameter within the channel. This is expected since temperature within the fluid increases as a result of increase in thermal conductivity. This causes convection currents to be strengthened so that fluid velocity increases with increase in the thermal conductivity.

The effect of heat source parameter (Q) is illustrated in Figures 6 and 7. It is observed from the figures that increasing heat source (Q) causes the fluid temperature and velocity increases within the channel. This is physically true since increasing the heat source parameter amplifies the applied temperature causing the fluid temperature to increase and also, it strengthens the convection current within the channel leading to an increase in fluid velocity.

The influence of Buoyancy ratio parameter (N) and chemical reaction parameter (Kc) are depicted in Figures 8-10. It is observed from the figures that an increase in Buoyancy parameter increases fluid

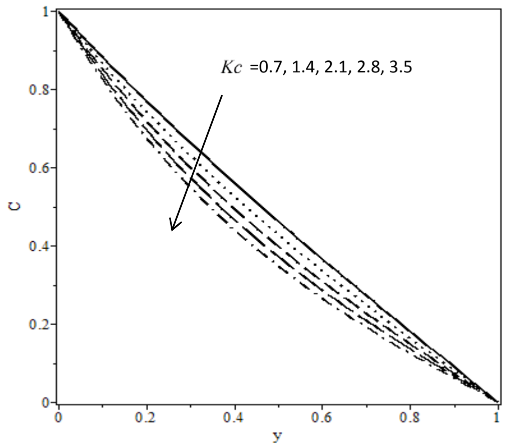

Figure 9: Concentration profile for binary mixture of carbon dioxide in air for different values of chemical reaction parameter [Q=0.5, λ=-0.001, N=1, γ=2].

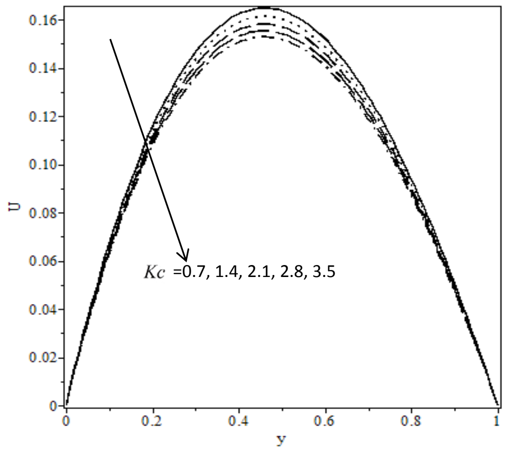

Figure 10: Velocity profile for binary mixture of carbon dioxide in air for different values of chemical reaction parameter [Q=0.5, λ=-0.001.N=1, γ=2].

velocity while increasing the chemical reaction parameter decreases both fluid velocity and temperature. The physical meaning of the observed trend is that growing *N* increases buoyancy due to mass transfer and hence, an increase in fluid velocity as shown in Figure 8. In addition, increasing the chemical reaction causes a decrease in fluid concentration and this weaken convection due to mass transfer which consequently decrease fluid velocity as shown in Figure 10.

Table 4 shows the skin friction, rate of heat transfer and the volumetric flow rate for different values of viscosity parameter($\lambda<0$). On increasing the viscosity of the fluid, volumetric flow rate as well as skin friction decreases on the surface of both channel plates. Also, the rate of heat transfer decreases at the heated wall (y=0) while it increases at the cold wall (y=1) upon increasing the fluid viscosity. This is physically true since an increase in fluid viscosity causes a decrease in velocity which decreases the volumetric flow rate within the channel. The decrease in fluid velocity caused by growing viscosity is also responsible for decrease in skin friction on the boundary wall surfaces. The rate of heat transfer that decreases with growing viscosity is traceable to the temperature increase caused by increasing viscosity which decreases the temperature difference on the heated plate as well as increasing temperature difference on the surface of the cold plate.

Table 6 shows the skin friction, rate of heat transfer and the volumetric flow rate for different values of conductivity parameter (γ). On increasing fluid conductivity, volumetric flow rate increases while the skin friction decreases at the heated wall (y=0) and cold wall (y=1). Also, the rate of heat transfer increases at the heated wall (y=0) while it decreases at the cold wall (y=1) upon increasing the fluid conductivity. This is hinged on the physical fact that an increase in thermal conductivity enhances the thermodynamics within the channel and this act to decrease heat flux into the channel from the heated boundary. It also act to increase the heat transfer from the heated fluid to the cold plate. The increase in temperature due to increase in thermal conductivity also help to strengthen the convection current and increase the velocity so that the mass flux and skin friction increase on both plates.

Pr=0.71, Sc=0.94, N=1, Q=0.5, Kc=0.7, λ=-0.001					
γ	V_m	τ_0	τ_1	Nu_0	Nu_1
0.5	0.08492942	0.67346222	0.34307393	1.17009348	1.29888514
1.0	0.08526535	0.67379917	0.34647797	1.66032898	1.44217153
1.5	0.09433024	0.70788534	0.41703410	1.82483297	4.09196587
2.0	0.10901289	0.76624386	0.52992078	1.22603721	7.50344346
2.5	0.11808503	0.80193736	0.60289333	0.73847019	9.63066481
3.5	0.13655231	0.863557248	0.80655257	0.19570537	20.194030

Table 6: Computation of variation of mass flow rate, skin friction and Nusselt number for different values of conductivity parameter (γ).

Conclusion

In this study, the influence of temperature dependent viscosity and thermal conductivity on natural convection flow through a vertical channel was considered. The governing equations for temperature, velocity and concentration fields were solved analytically using the differential transformation method. The results were verified with results from the exact and numerical methods, excellent agreement was observed. The results of fluid flow within the channel reveals the following:

1. Increasing fluid viscosity increases fluid temperature within the channel while it decreases fluid velocity.
2. Skin friction increases with increase in thermal conductivity of the working fluid.
3. At the cold wall, the rate of heat transfer increases with increase in fluid viscosity.
4. The volumetric flow rate can be controlled effectively using the variations in viscosity as well as thermal conductivity of the working fluid.

References

1. Konch J, Hazarika GC (2016) Effects of Variable and Thermal Conductivity on MHD Free Convection Flow of Dusty Fluid along a Vertical Stretching Sheet with Heat Generation. International Research Journal of Engineering and Technology 3: 1029-1038.
2. Hossain MA, Munir MS, Pop L (2001) Natural convection with variable viscosity and thermal conductivity from a vertical wavy cone. International Journal of Thermal Sciences 40: 437-442.
3. Munir M, Gorla SRR (2002) Combined convection from a vertical flat plate with temperature dependent viscosity and thermal conductivity. International Journal of Fluid Mechanics 29.
4. Hassanien IA, Essawy AH, Mousy NM (2003) Variable viscosity and thermal conductivity effect on heat transfer by natural convection from a cone and a wedge in porous media. Archives of Mechanics 55.

5. Ozalp AP, Selim D (2005) Influence of variable thermal conductivity and viscosity for non-isothermal fluid flow. Physics of Fluids 17.

6. Attia A (2006) Unsteady hydromagnetic channel flow of dusty-fluid with temperature dependent viscosity and thermal conductivity. Heat and Mass Transfer 42: 779-787.

7. Nasrin R, Alim MA (2009) MHD free convection flow along a vertical flat plate with thermal conductivity and viscosity depending on temperature. Journal of Naval Architecture and Marine Engineering 6: 72-83.

8. Choudhury M, Hazarika GC (2008) The effects of variable viscosity and thermal conductivity on MHD flow due to a point sink. Matematicas: Ensenanza Universitaria, pp: 21-28.

9. Sharma PR, Singh G (2009) Effects of varying viscosity and thermal conductivity on steady MHD free convective flow and heat transfer along an isothermal plate with internal heat generation. International Journal of Numerical Methods for Heat and Fluid Flow 19: 78-92.

10. Mahanti NC, Gaur P (2009) Effects of varying viscosity and thermal conductivity on steady free convective flow and heat transfer along an isothermal vertical plate in the presence of heat sink. Journal of applied Fluid Mechanics 2: 23-28.

11. Molla M, Gorla MAR (2009) Natural convection laminar flow with temperature dependent viscosity and thermal conductivity along a vertical wavy surface. International Journal of Fluid Mechanics Research 36.

12. Palani G, Kim KY (2010) Numerical study on a vertical plate with variable viscosity and thermal conductivity. Archive of Applied Mechanics 80: 711-725.

13. Ashraf M, Chamka AJ, Iqbal S, Masud A (2016) Effects of temperature dependent viscosity and thermal conductivity on mixed convection flow along a magnetized vertical surface. International Journal of Numerical Methods for Heat and Fluid Flow 26: 1580-1592.

14. Keimanesh R, Aghanajafi C (2017) The effect of temperature dependent viscosity and thermal conductivity on micropolar fluid over a stretching sheet. International Journal of Numerical Methods For Heat & Fluid Flow 24: 371-378.

15. Zhou JK (1986) Differential transformation and its applications for electrical circuits. Hauzhong University Press Wuhan china.

16. Chen CK, Ho SH (1999) Solving partial differential equations by two dimensional differential transformation method. Appl math Comput 147: 547-567.

17. Umavathi JC, Shekar M (2016) Combined effects of variable viscosity and thermal conductivity on free convection flow of a viscous fluid in a vertical channel. Emerald Group Publishing Limited 51: 71-86.

18. Jha BK, Ajibade AO (2010) Free convection heat and mass transfer flow in a vertical channel with the Dufour effect. Journal of Process Mechanical Engineering 224: 91-101.

19. Anjali SP, Prakash M (2015) Temperature dependent viscosity and thermal conductivity effects on hydo magnetic flow over a slandering stretching sheet. Journal of the Nigerian Mathematical Society 3: 318-330.

20. Schlichting H, Gerstenk K (2000) Boundary layer theory. Physics and astronomy, springer.

21. Rashidi MM, Kavyani N, Abelman S, Uddin MJ, Fredoonimehr N (2014) Double Diffusive Magnetohyrodynamic (MHD) Mixed Convective Slip Flow along a Radiating Moving Vertical Flat Plate With Convective Boundary Condition. PLoS One 9: 10.

22. Abayomi O (2017) Convergence of Differential Transformation Method For Ordinary Differential Equations. Journal of Advances in Mathematics and Computer Science 24: 1-17.