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Effect of Load Eccentricity on the Strength of Concrete Columns

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Abstract

This research presents a theoretical study to determine the effect of the load eccentricity on the reinforced concrete column strength taking into account the variables: amount of eccentricity ratio (e/h=0.1 and 1.0); amount of longitudinal reinforcement ρ%=1% to 8%; concrete compressive strength $\left(f_c^{'}=21,28,35,42,63\,and\,84\,MPa\right)$; steel yielding strength $\left(f_y=414\,and\,525\,MPa\right)$; the steel reinforcement distance ratiocondition of loading (Uniaxial and Biaxial bending); shape of the cross section (rectangular and circular) and finally the distribution of the reinforcement on two opposite sides and on four sides.

Generally the strength of columns is reduced with existing the load eccentricity and amount of losses in strength increased with increasing the eccentricity amount. The average strength ratio in case of biaxial bending condition is about (82%) of the uniaxial condition in case of (e/h=0.1) and become (55%) in case of (e/h=0.1).

For uniaxial bending condition, the average relative column strength is about (75%) in case of (e/h=0.1) and (14%) in case of (e/h=0.1); while for biaxial bending condition, the ratio is (60%) in case of (e/h=0.1) and (8%) in case of (e/h=0.1). Increasing of concrete compressive strength $\left(f_c^{'}\right)$, steel yielding strength $\left(\gamma_s\right)$, steel distance ratio $\left(\gamma_s\right)$ and amount of longitudinal reinforcement $\left(\rho\%\right)$ cause increasing in column strength and reducing the losses in column strength.

Also the results show great effect of the load eccentricity ratio (e/h) and bending condition (uniaxial and biaxial) on the reduction of column strength. The distribution of the reinforcement on two opposite sides gives upper limit results and maximum column strength, compared with the case of when the reinforcement distributed on four sides and rectangular section with circular distribution of the reinforcement, while circular columns gives lower limit results and minimum column strength compared with other cases mentioned above.

Keywords: High strength concrete; Eccentricity; Column; Uniaxial and biaxial bending

Introduction

Columns are members used primarily to support axial compression loads. In reinforced concrete buildings the joints between concrete beams, floor and columns are fixed, causing some moments in the column due to end restraint. Also perfect vertical alignment of columns in a multi-storey building is not possible, causing loads to be eccentric relative to the center of columns. The eccentric loads will cause moments in columns. Therefore a column subjected to pure axial loads does not exist in concrete buildings. Concrete is used in columns because of high compressive strength and in expansive material.

Column may be classified based on loading conditions to; axially loaded columns, uniaxial loaded columns (combined axial load plus bending moment about one axis) and biaxial loaded columns (combined axial load plus bending moments about both axes). Also columns may be classified based on length of columns to; short columns (where the failure is due to the crushing of concrete or yielding of steel bars) and long columns (where buckling effect and slenderness ratio must be taken into consideration in the design). Columns may be classified according to shape of cross section, square, rectangular, circular or any other shape, also according to the type of confined reinforcement, ties or spiral [1-4].

The ratio of longitudinal steel area to gross concrete area is in the range (0.01 to 0.08), according to ACI-Code [5]. The lower limit is necessary to ensure resistance to bending moments not accounted in the analysis and to reduce the effect of creep and shrinkage of the concrete under sustained compression. Ratios higher than (0.08) are uneconomical and also cause difficulty owing to congestion of the reinforcement. Most of the columns are designed with ratios below 0.04. Larger diameter bars are used to reduce placement cases and to avoid unnecessary congestion. A minimum of four longitudinal bars is required for bars enclosed by rectangular or circular ties and six bar must be used when the bars enclosed by a continuous spiral.

Literature Review

Pharis [6] studied the behaviour and limit state performance of high strength reinforced concrete columns. Fifteen specimens were tested to failure, strength and arrangement longitudinal steel, spacing of ties, amount of load eccentricity and compressive strength of concrete are taken as a main variables of the study. He concluded that relationship between stress-strain is more linear over a greater range and can be approximated by a straight line. Also, HSC is extremely brittle; no further strain capacity can be counted on beyond the strain at peak stress. The modulus of elasticity depends on the type and quantity of coarse aggregate. Generally strain capacity is low for high strength. The strain of maximum stress may be slightly higher for high strength concrete than normal strength concrete, but the total stress at failure is normally less for HSC.

The use of rectangular stress block with maximum strain of 0.003 is not valid for high strength concrete. A triangular stress block is found to be much better approximate for high strength concrete because of the linear stress-strain characteristics of high strength concrete, maximum

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strain of 0.0025 is taken for analysis to determine ultimate strength of the columns, within a few percentage of error in the measured failure load, while using the rectangular stress block of ACI 318 resulted in overestimate of strength.

Rangar and Bisby [7] studied the effect of eccentricities on the behaviour of FRP (Fiber Reinforced Polymer) confined R.C. columns. They conclude that the strength and deformation capacity of FRP confined concrete columns under eccentric axial load is improved as compared with unconfined columns, reduction in strength is occur with increasing eccentricity.

The benefits of FRP wrapping, both in terms of peak load and lateral deformation at peak load, reduction in capacity due to load eccentricity are more pronounced for FRP confined columns. Clear evidence that axial-flexural interaction reduces the effectiveness of FRP wraps. The loop strain observed in the FRP at failure for both FRP concentric and eccentric columns was less than the failure strain in direct tensile tests on FRP coupons.

Majewski et al. [8] presents a FE (Finite Element) modelling to study failure behaviour of reinforced concrete column under eccentric compression. Concrete was described with an elasto-plastic model using isotropic; hardening and softening. The reinforcement was described with an elastic-ideally plastic constitutive law. The FE results were compared with experimental data found in previous studies and satisfactory agreement was achieved.

Lioy and Rangan [9] studied the behaviour and strength of high-strength concrete columns subjected to axial compression and uniaxial bending; they concluded that strength of columns increased by increasing the compression strength of the concrete and longitudinal reinforcement ratio. The strength is reduced with increasing the load eccentricity and mid-height deflection at failure is increased. The theory based on a simplified stability analysis and strain-stress relationship for high strength concrete predicted the strength of columns well.

Setty and Rangan [10] studied high strength concrete columns subjected to combined axial compression and bending moment. They conclude that the mode of failure of test columns was typically flexure with concrete spalling in the compression zone, the lateral reinforcement provide was adequate to prevent buckling of longitudinal bars in the compression zone. Also they proposed a simplified stability analysis to predict strength of columns and showed good correlation with test results.

Many studies [11-17] have demonstrated the economy of using high strength concrete in columns of high-rise buildings and low to mid rise buildings. In addition to reducing the column size, and producing a more durable material, the use of high strength concrete has been shown to be advantageous with regard to lateral stiffness and axial shortening and reduction in cost of forms. There is no unique definition of high strength concrete. The Australlian standard for concrete structures AS 3600-1994 [18] is limited to concrete compressive strength up to 50 Mpa, while Razvi and Soatcioglu [19], considered the strength of 41 MPa for normal weight concrete and 27 MPa for light weight concrete to be high strength concrete. This is found to be justifiable and since most of the ready-mix concrete supplied. There is no universal agreement on the applicability of ACI code requirement for calculating flexural strength of high strength concrete columns subjected to combined axial load and bending moment.

Columns are usually designed for combined for combined axial load and bending moment using the rectangular stress block. This stress block was originally derived by Mattock et al. [20].

Based on the tests of un-reinforced concrete columns loaded with axial load and moments [21], the concrete strength ranged up to 52.5 MPa, the stress block was defined by two parameters, the intensity of stress $(\alpha 1)$ nd stress block depth ratio of the neutral axis $\alpha_1=0.85$. Mattock et al. [20] proposed:

$$\alpha_1=0.85$$

$$\beta_1=1.05-0.05\left(\frac{f_c^{'}}{6.9}\right)\leq 0.85$$

Nedderman [22] proposed a lower limit on $(\beta 1)$ of 0.65 for concrete strength is excess of 55 MPa. New zealand standard and ACI-Code recommended that the currently used parameters for the equivalent rectangular concrete compressive stress block shown in Figure 1 are applicable up to $f_c^{'}=55\,MPa$. For $f_c^{'}>55\,MPa$, it is recommended that and reduced linearly with increase in to become a minimum of (0.75) at $f_c^{'}=80MPa$.

$$\alpha_1=0.85\ for\ f_c^{'}\leq 55\,MPa$$

$$\alpha_1\ =0.85\text{-}0.004(f_c^{'}-55)\geq 0.75\ for\ f_c^{'}>55\,MPa$$

$$\alpha_1=0.75\ for\ f_c^{'}=80\,MPa$$

$$\beta_1=0.65\ \ for\ f_c^{'}\leq 30\,MPa$$

$$\beta_1=0.85-0.008\left(f_c^{'}-30\right)for\ f_c^{'}>30\,MPa$$

$$\beta_1=0.65\ \ for\ f_c^{'}=55\,MPa$$

Available test data indicate that typical stress-strain curves in compression for HSC are characterized by an ascending portion that is primarily linear, with maximum strength achieved at an axial strain between (0.0024 and 0.003). Therefore it may be more appropriate to use a tri-angular compression stress block shown in Figure 1 for HSC columns when $f_c^{'}$ exceeds 70 Mpa, the intensity of compression stress equals $(0.85\,f_c^{'})$ rather than $(0.85\quad)$ or $\beta_1=0.67$ and the depth of the rectangular compression block is equal to $\beta_1=0.67$ or $\beta_1=0.67$

The Canadian Code [23]; suggested the following modified rectangular stress block:

$$\alpha_1=0.85-0.0015\,f_c^{'}\geq 0.67$$

$$\beta_1=0.85\text{-}0.0025\,f_c^{'}\geq 0.6$$

Ibrahim et al. [24] compared the concrete component of the measured load and moment strength of (94) tests of eccentrically loaded columns with concrete strengths ranging up to 130 MPa and they conclude that the max. Concrete strain before spalling was greater than (0.003), and the HSC columns can be designed based on rectangular

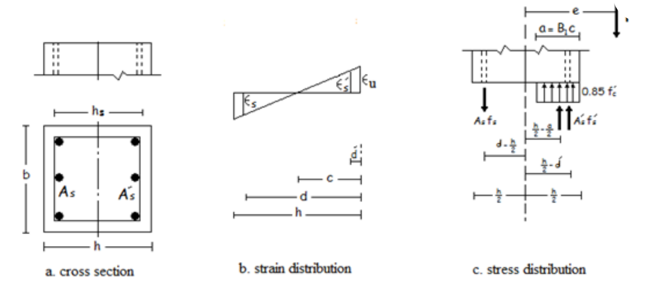

Figure 1: Column subjected to eccentric compression.

stress block with some modification of the parameters as below:

$$\alpha_1 = 0.85 - 0.00125\, f_c^{'} \geq 0.725$$

$$\beta_1 = 0.85 - 0.0025\, f_c^{'} \geq 0.6$$

Theoretical analysis

Figure 1 shows a member loaded to its axis by a compression force (P_n) at eccentricity (e) from the centreline. The assumptions taken into consideration are: the plane sections remain plane after bending and concrete strains vary linearly with distance from the neutral axis with full compatibility of deformations, the steel strains at any location are the same for concrete at the same location.

Equivalent rectangular stress block is used in the analysis with max compression strength $\left(0.85 f_c^{'}\right)$ and having $\left(0.85 \beta_c^{'}\right)$ instead of actual concrete compression stress.

Equilibrium between external and internal axis forces result the ultimate load capacity, and ultimate bending moment capacity is determined by taking moment about the centreline of the section of the internal stresses and forces.

$$Pn = 0.85\, f_c^{'}.a.b + A_s^{'} f_s^{'} - A_s f_s \tag{1}$$

$$M_n = P_n e = 0.85\, f_c^{'}.a.b\left(\frac{h}{2}-\frac{a}{2}\right) + A_s^{'} f_s^{'}\left(\frac{h}{2}-d^{'}\right) + A_s f_s\left(d-\frac{h}{2}\right)$$

These are the two basic equilibrium relations for rectangular column subjected to eccentric compression load.

where

$A_s^{'}$ =Area of tension steel bars (mm²).

$A_s^{'}$ =Area of compression steel bars (mm²).

$d^{'}$ =Effective depth of the cross section (mm).

$d^{'}$ =Location of compression steel bars (mm).

$\in_u$ =Cylinder compression strength of concrete (*MPa*).

$\in_s$ =Ultimate concrete strain (0.003).

$\in_s^{'}$ =Strain of steel in tension zone.

$f_s^{'}$ =Strain of steel in compression zone.

$f_s^{'}$ =Stress of steel in tension zone.

$f_s^{'}$ =Stress of steel in compression zone.

b =Width of the cross section (mm).

h =Depth of the cross section (mm).

P =External compression load (*N*).

P_n=Ultimate load capacity (*N*).

e =Eccentricity of the load (mm).

M_n=Ultimate bending moment capacity (mm).

c =Depth of compression zone (mm).

β_1 =Equivalent rectangular stress distribution factor.

The nominal strength of axially loaded column can be founded when the concrete crushes while the steel yields:

$$P_n o = 0.85\, f_c^{'}(A_g - A_s t) + A_s t f_y$$
$$A_s t = \left(A_s + A_s^{'}\right)\left(mm^2\right) \tag{3}$$

where

$P_n o =$ Nominal strength of axially loaded column (N).

$A_s t$ = Total steel reinforcement $\left(A_s + A_s^{'}\right)\left(mm^2\right)$.

At this stage, the steel reinforcement carries larger fraction of the load than the case at lower total load.

The maximum useful strength in tension member is the force that will just cause the stress to reach the yield point.

$$P_{nt} = A_{st}.f_y \tag{4}$$

where

$P_{nt} =$ Maximum tension load capacity (N).

$f_y =$ Yield strength of steel bars *(MPa).*

The strength interaction diagram (*M-P*) defines the failure load and moment for given column for the full range of eccentricities from zero to infinity. For any eccentricity there is a unique pair of values of $P_n \& M_n$ that pair of values can be plotted as a point on a graph relating $P_n \& M_n$ as shown in Figure 2.

A series of such calculation, each corresponding to a different eccentricity is seen. Vertical axis corresponds to (*e=0*) and P_{n0} is the capacity of the column if concentrically loaded, the horizontal axis corresponds to an infinite value of (e) i.e., pure bending at moment capacity $\left(M_0\right)$. Small eccentricities will produce failure governed by concrete compression, while large eccentricities give a failure governed by yielding of the tension steel.

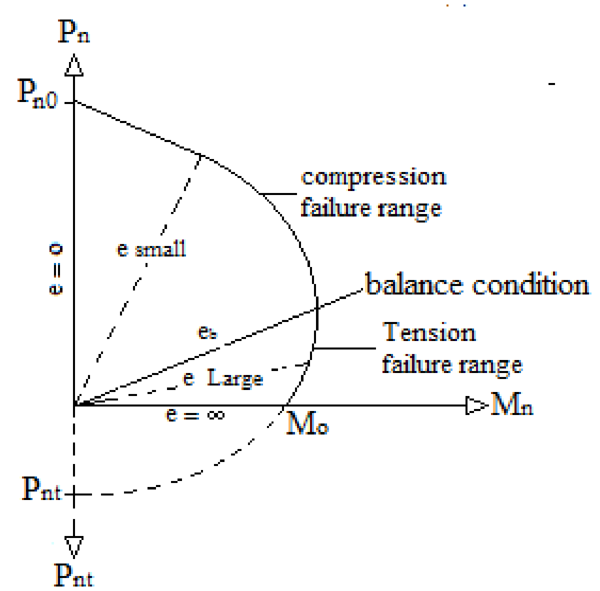

Figure 2: Combined bending and axial load interaction diagram.

For columns subjected to axial compression load and bending moments in both directions (Biaxial bending condition), Bresler's reciprocal load equation are used as shown below:

$$\frac{1}{P_n}=\frac{1}{P_{n0x}}+\frac{1}{P_{n0y}}-\frac{1}{P_0} \tag{5}$$

Where

P_{n0x} = Nominal load when only eccentricity (e_y) is present.

P_{n0y} = Nominal load when only eccentricity is present.

P_0 = Nominal load for concentrically loaded column.

P_n = Approximate value of nominal load in biaxial bending with both eccentricities $(e_x \& e_y)$.

This study presents a theoretical study to determine the effect of the load eccentricity on the column strength taking into account the following variables:

1) Amount of the eccentricity: Two ratios (*e/h*) are considered (*e/h=0.1*) and (*e/h=1.0*). The load corresponds to (*e/h=0.1*) is termed ($P_{n0.1}$) and the load correspond to (*e/h=1.0*) is termed (*e/h=1.0*) as shown in Figure 3.

a) The relative ratio with respect to pure compression, i.e., concentric condition (*e=0*) $R_{0.1}=\frac{P_{n0.1}}{P_{n0}}\times100$ and $R_1=\frac{P_{n1}}{P_{n0}}\times100$ are determined to show amount of the strength reduction and column strength due to the load eccentricity, $(f_c^{'}, f_y)$ are considered: (21,414), (28,414), (35,414), (42,414), (63,575) and (84,575) *Mpa*.

b) (*e/h=0.1 & 1.0*)

c) $\gamma_s = 0.6, 0.7, 0.8 \& 0.9$

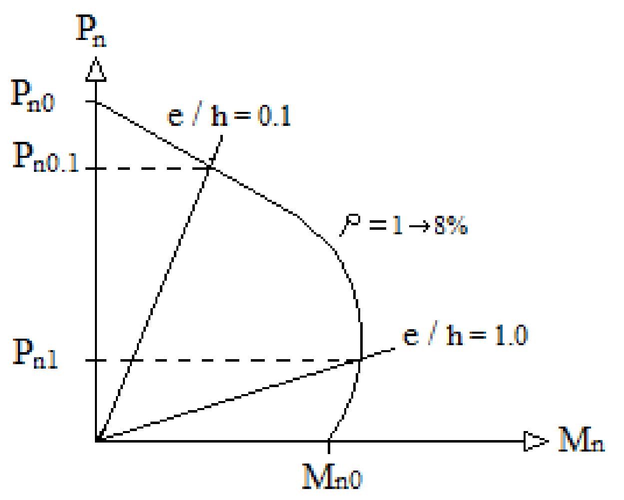

Figure 3: M_n-P_n interaction diagram for the variables of the study shown below.

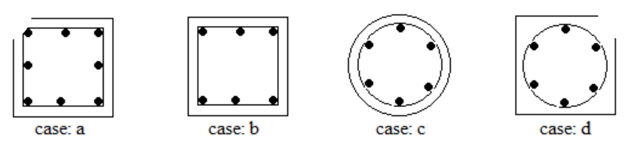

Figure 4: Distribution of the reinforcement as shown in cases.

d) $\rho\% = 1\%\, to\, 8\%$

e) Uniaxial and Biaxial conditions.

f) Distribution of the reinforcement as shown below in Figure 4.

2 Amount of longitudinal reinforcement (*e*) the values are considered.

3 Material strength, the pair of concrete compressive strength and steel yield strength are considered: (21,414), (28,414), (35,414), (42,414), (63,575) & (84,575).

4 The distance between reinforcement rows the values are: 0.6, 0.7, 0.8 & 0.9.

5 The condition of loading, uniaxial condition: combined axial load and bending moment about major axis (P_n, M_{ny}) and biaxial condition: combined axial load and bending moment about both axes (P_n, M_{nx}, M_{ny}).

6. Shape of cross section and distribution of the longitudinal reinforcement:

a) Rectangular section and distribution of reinforcement on four sides.

b) Rectangular section and distribution of reinforcement on two opposite sides.

c) Circular section and circular distribution of reinforcement.

d) Rectangular section and rectangular distribution of reinforcement.

Result and Discussion

Table 1 shows the value of column strength $\left(k=\frac{P_n}{f_c^{'}A_g}\right)$ and relative strength of column with respect to P_{n0}, that is $R_{0.1}$ and $R_{1.0}$, where $(R_{1.0})$ is the relative strength of eccentricity ratio (*e/h=1.0*) and ($R_{1.0}$) is the relative strength of eccentricity ratio (e/h=1.0) for both conditions, uniaxial and biaxial bending conditions. Concrete compressive strength $(f_c^{'}=21Mpa)$, steel yielding strength $(f_y=414\,Mpa)$ and reinforcement steel distance ratio (Υ) between (0.6 and 0.9) for rectangular column (case a), where the reinforcement distributed on four edges. For particular case , uniaxial condition and $(e/h=0.1)$, the relative column strength vary between (76.9 and 73.1%) for reinforcement index ρ between (1 & 8%), the average value is (74.742%) and the strength losses is (25.258%). For eccentricity ratio $(e/h=1.0)$, the value of strength ratio $(R_{1.0})$ vary between (9.615 and 15.126%), the average value is (14.215%) and losses (85.785%). The results show the great effect of the eccentricity on the column strength, the average strength (74.742%) reduced to (14.215%) when the eccentricity increased from (0.1) to (1.0), the strength of column reduced to about one-fifth (1/5) of its original strength. The results of column with reinforcement distance ratio $(\gamma=0.7, 0.8\, and\, 0.9)$ are also shown in Table 1, and some conclusions are obtained.

The results of columns with material strength pair concrete compressive strength and steel yielding strength $(f_c^{'}, f_y)$: (28, 414),

γ = 0.6		Uni-axial				Bi-axial			
% Rho	Ko (e/h=0)	K(e/h=0.1)	K(e/h=1)	R(e/h=0.1)	R(e/h=1)	K(e/h=0.1)	K(e/h=1)	R(e/h=0.1)	R(e/h=1)
1	1.04	0.8	0.1	76.923	9.615	0.65	0.052525	62.5	5.051
2	1.24	0.94	0.16	75.806	12.903	0.756883	0.085517	61.039	6.897
3	1.42	1.08	0.2	76.056	14.085	0.871364	0.107576	61.364	7.576
4	1.61	1.2	0.24	74.534	14.907	0.956436	0.129664	59.406	8.054
5	1.8	1.34	0.28	74.444	15.556	1.067257	0.151807	59.292	8.434
6	2	1.48	0.32	74	16	1.174603	0.173913	58.73	8.696
7	2.19	1.6	0.34	73.059	15.525	1.260432	0.184307	57.554	8.416
8	2.38	1.74	0.36	73.109	15.126	1.371258	0.194727	57.616	8.182
Average		-	-	74.742	14.215	-	-	59.688	7.663
% Loss		-	-	25.258	85.785	-	-	40.312	92.337
γ = 0.7		Uni-axial				Bi-axial			
% Rho	Ko (e/h=0)	K(e/h=0.1)	K(e/h=1)	R(e/h=0.1)	R(e/h=1)	K(e/h=0.1)	K(e/h=1)	R(e/h=0.1)	R(e/h=1)
1	1.04	0.81	0.12	77.885	11.538	0.663307	0.063673	63.78	6.122
2	1.24	0.96	0.19	77.419	15.323	0.783158	0.102882	63.158	8.297
3	1.42	1.1	0.24	77.465	16.901	0.897701	0.131077	63.218	9.231
4	1.61	1.24	0.27	77.019	16.77	1.008283	0.147356	62.626	9.153
5	1.8	1.38	0.32	76.667	17.778	1.118919	0.17561	62.162	9.756
6	2	1.52	0.37	76	18.5	1.225806	0.203857	61.29	10.193
7	2.19	1.66	0.41	75.799	18.721	1.336544	0.226171	61.029	10.327
8	2.38	1.8	0.44	75.63	18.487	1.447297	0.242407	60.811	10.185
Average		-	-	76.735	16.752	-	-	62.259	9.158
% Loss		-	-	23.265	83.248	-	-	37.741	90.842
γ = 0.8		Uni-axial				Bi-axial			
% Rho	Ko (e/h=0)	K(e/h=0.1)	K(e/h=1)	R(e/h=0.1)	R(e/h=1)	K(e/h=0.1)	K(e/h=1)	R(e/h=0.1)	R(e/h=1)
1	1.04	0.82	0.125	78.846	12.019	0.676825	0.066496	65.079	6.394
2	1.24	0.97	0.2	78.226	16.129	0.796556	0.108772	64.238	8.772
3	1.42	1.11	0.26	78.169	18.31	0.911098	0.143101	64.162	10.078
4	1.61	1.25	0.33	77.64	20.497	1.021574	0.183841	63.452	11.419
5	1.8	1.4	0.38	77.778	21.111	1.145455	0.212422	63.636	11.801
6	2	1.53	0.42	76.5	21	1.238866	0.234637	61.943	11.732
7	2.19	1.68	0.45	76.712	20.548	1.362667	0.250763	62.222	11.45
8	2.38	1.83	0.5	76.891	21.008	1.486485	0.279343	62.457	11.737
Average		-	-	77.595	18.828	-	-	63.399	10.423
% Loss		-	-	22.405	81.172	-	-	36.601	89.577
γ = 0.9		Uni-axial				Bi-axial			
% Rho	Ko (e/h=0)	K(e/h=0.1)	K(e/h=1)	R(e/h=0.1)	R(e/h=1)	K(e/h=0.1)	K(e/h=1)	R(e/h=0.1)	R(e/h=1)
1	1.04	0.83	0.13	79.808	12.5	0.69056	0.069333	66.4	6.667
2	1.24	0.98	0.22	79.032	17.742	0.810133	0.120708	65.333	9.735
3	1.42	1.13	0.28	79.577	19.718	0.938363	0.155313	66.082	10.938
4	1.61	1.28	0.35	79.503	21.739	1.062268	0.196341	65.979	12.195
5	1.8	1.42	0.41	78.889	22.778	1.172477	0.231348	65.138	12.853
6	2	1.58	0.45	79	22.5	1.305785	0.253521	65.289	12.676
7	2.19	1.72	0.51	78.539	23.288	1.41609	0.288605	64.662	13.178
8	2.38	1.86	0.56	78.151	23.529	1.526483	0.317333	64.138	13.333
Average		-	-	79.062	20.474	-	-	65.378	11.447
% Loss		-	-	20.938	79.526	-	-	34.622	88.553

Table 1: Relative column strength for columns fc'=21 MPa, fy=414 MPa (Distributed reinforcement on 4 edges).

(35, 414), (42, 414), (63, 575), and (84, 575) *Mpa* are shown in Tables 2, 3, 4, 5, and 6 respectively.

For biaxial bending condition, the column strength ratio $(R_{0.1})$ vary between (62.5 and 57.616%) for reinforcement index (ρ) between (1 & 8%) for eccentricity ratio $(e/h=1.0)$, the average column strength ratio is (59.688) and losses (40.312%). At eccentricity ratio $(e/h=1.0)$, the column strength ratio $(R_{1.0})$ vary between (5.057 and 8.182%) for same reinforcement index (ρ) between (1 and 8%), the average column strength ratio is (7.663%) and the losses (92.337%). The results show that the biaxial condition is more dangerous on the column strength compared with uniaxial bending condition, the column lost about (20%) of its strength at $(e/h=0.1)$, and lost about (46%) at $(e/h=1.0)$ when the bending condition changed from uniaxial to biaxial condition the effect of biaxial condition is more at high level of load eccentricity. Also in biaxial bonding condition the effect of eccentricity is more compared with uniaxial bending condition, when the eccentricity ratio increased from $(e/h=1.0)$ to $(e/h=1.0)$, the average column strength reduced to about one-eighth of its original strength. For this reason the designer engineers should take enough care to the conditions of the bending and

γ = 0.6									
% Rho	Ko (e/h=0)	Uni-axial				Bi-axial			
		K(e/h=0.1)	K(e/h=1)	R(e/h=0.1)	R(e/h=1)	K(e/h=0.1)	K(e/h=1)	R(e/h=0.1)	R(e/h=1)
1	1	0.79	0.09	79	9	0.652893	0.04712	65.289	4.712
2	1.125	0.87	0.14	77.333	12.444	0.709239	0.074645	63.043	6.635
3	1.27	0.96	0.17	75.591	13.386	0.771646	0.091097	60.759	7.173
4	1.41	1.06	0.2	75.177	14.184	0.849205	0.107634	60.227	7.634
5	1.55	1.15	0.24	74.194	15.484	0.914103	0.13007	58.974	8.392
6	1.7	1.25	0.26	73.529	15.294	0.988372	0.140764	58.14	8.28
7	1.84	1.36	0.29	73.913	15.761	1.078621	0.157404	58.621	8.555
8	1.975	1.45	0.31	73.418	15.696	1.1455	0.168201	58	8.516
Average		-	-	75.269	13.906	-	-	60.382	7.487
% Loss		-	-	24.731	86.094	-	-	39.618	92.513
γ = 0.7		Uni-axial				Bi-axial			
% Rho	Ko (e/h=0)	K(e/h=0.1)	K(e/h=1)	R(e/h=0.1)	R(e/h=1)	K(e/h=0.1)	K(e/h=1)	R(e/h=0.1)	R(e/h=1)
1	1	0.78	0.09	78	9	0.639344	0.04712	63.934	4.712
2	1.125	0.88	0.15	78.222	13.333	0.722628	0.080357	64.234	7.143
3	1.27	0.98	0.2	77.165	15.748	0.797821	0.108547	62.821	8.547
4	1.41	1.08	0.235	76.596	16.667	0.875172	0.128182	62.069	9.091
5	1.55	1.19	0.275	76.774	17.742	0.965707	0.150885	62.304	9.735
6	1.7	1.29	0.31	75.882	18.235	1.039336	0.17055	61.137	10.032
7	1.84	1.4	0.34	76.087	18.478	1.129825	0.187305	61.404	10.18
8	1.975	1.5	0.37	75.949	18.734	1.209184	0.20412	61.224	10.335
Average		-	-	76.835	15.992	-	-	62.391	8.722
% Loss		-	-	23.165	84.008	-	-	37.609	91.278
γ = 0.8		Uni-axial				Bi-axial			
% Rho	Ko (e/h=0)	K(e/h=0.1)	K(e/h=1)	R(e/h=0.1)	R(e/h=1)	K(e/h=0.1)	K(e/h=1)	R(e/h=0.1)	R(e/h=1)
1	1	0.78	0.1	78	10	0.639344	0.052632	63.934	5.263
2	1.125	0.89	0.17	79.111	15.111	0.736213	0.091947	65.441	8.173
3	1.27	1	0.22	78.74	17.323	0.824675	0.120431	64.935	9.483
4	1.41	1.1	0.26	78.014	18.44	0.901744	0.143203	63.953	10.156
5	1.55	1.22	0.3	78.71	19.355	1.005851	0.166071	64.894	10.714
6	1.7	1.32	0.345	77.647	20.294	1.078846	0.19198	63.462	11.293
7	1.84	1.43	0.38	77.717	20.652	1.169422	0.211879	63.556	11.515
8	1.975	1.535	0.42	77.722	21.266	1.255331	0.234986	63.561	11.898
Average		-	-	78.208	17.805	-	-	64.217	9.812
% Loss		-	-	21.792	82.195	-	-	35.783	90.188
γ =0.9		Uni-axial				Bi-axial			
% Rho	Ko (e/h=0)	K(e/h=0.1)	K(e/h=1)	R(e/h=0.1)	R(e/h=1)	K(e/h=0.1)	K(e/h=1)	R(e/h=0.1)	R(e/h=1)
1	1	0.79	0.1	79	10	0.652893	0.052632	65.289	5.263
2	1.125	0.9	0.18	80	16	0.75	0.097826	66.667	8.696
3	1.27	1.02	0.24	80.315	18.898	0.852237	0.132522	67.105	10.435
4	1.41	1.13	0.29	80.142	20.567	0.942781	0.161621	66.864	11.462
5	1.55	1.24	0.33	80	21.29	1.033333	0.184657	66.667	11.913
6	1.7	1.34	0.38	78.824	22.353	1.105825	0.213907	65.049	12.583
7	1.84	1.45	0.42	78.804	22.826	1.196413	0.237055	65.022	12.883
8	1.975	1.56	0.48	78.987	24.304	1.289121	0.273199	65.272	13.833
Average		-	-	79.509	19.53	-	-	65.992	10.884
% Loss		-	-	20.491	80.47	-	-	34.008	89.116

Table 2: Relative column strength for columns fc'=28 MPa, fy=414 MPa (Distributed reinforcement on 4 edges).

amount of load eccentricity when they choose the building system and arrangement of columns and beams.

Figure 5 shows the variation of $(R_{0.1}\ and\ R_{1.0})$ with the steel reinforcement index $(\rho\%)$, for concrete compressive strength $(f_y=414Mpa)$, steel yielding strength $(f_y=414Mpa)$, and reinforcement steel ratio $(\rho\%)$ as shown the effect of the reinforcement index generally is small. Value of $(\rho\%)$ increased with increasing the reinforcement index $(\rho\%)$, while $(R_{0.1})$ decreased, because of high eccentricity stress in the reinforcement are increased and reaches to the yielding strength and became more effective.

The results show that increasing of the eccentricity from (0.1 to 1.0) the column strength ratio reduced from about (75% to 14%) and losses in column strength increased from (25% to 86%) for the uniaxial bending condition, while in biaxial bending condition the column strength ratio reduced from about (4% to 8%) and losses increased from (60% to 92%) for same eccentricity values (0.1 and 1.0).

More detail graph is shown in Figure 6 for column with eccentricity

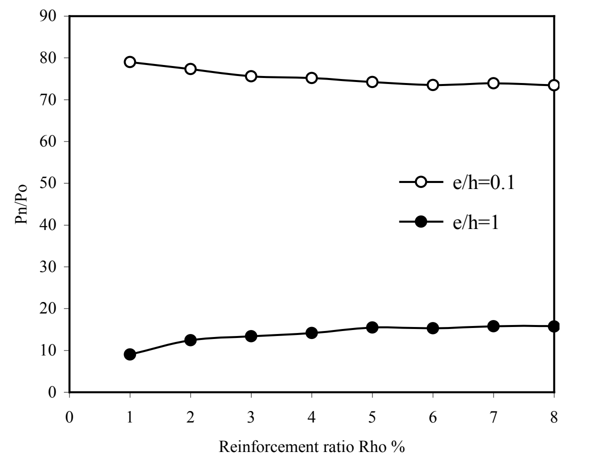

Figure 5: Relative strength of column, fc'= 28 Mpa, fy=414 MPa, hs/h=0.6.

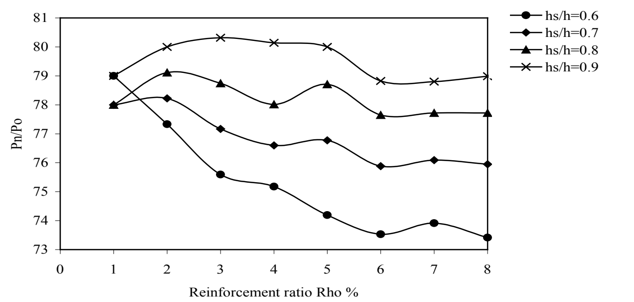

Figure 6: Relative strength of column, fc'= 28 MPa, fy= 414 MPa, e/h= 0.1.

ratio $(e/h=0.1)$ as shown the column strength ratio (ρ) decreased with increasing the reinforcement index (ρ) for columns with reinforcement steel distance ratio $(\gamma=0.8\,\&\,0.9)$, while in columns with $(\gamma=0.8\,\&\,0.9)$ the effect of (γ) is small and changing in the strength ratio is small, also the column strength ratio increased with increasing the reinforcement distance ratio (γ). In the same column with eccentricity ratio $(e/h=1.0)$ the value of column strength ratio $(R_{1.0})$ is increased with increasing $(R_{1.0})$ in all values of $(\gamma=0.6, 0.7, 0.8\, and\, 0.9)$, also column strength $(R_{1.0})$ is increased with increasing (γ) from (0.6 to 0.9), as shown in Figure 7.

Average of column strength ratio and losses of all material strengths pair (f_c', f_y) which are shown in Tables 1-6 for $(R_{0.1}\, and\, R_{1.0})$ are summarized in Table 7 for (case a), where the reinforcement is distributed on all four sides for both uniaxial and biaxial bending condition, as shown for all concrete and steel yielding strength, the value of column strength ratio $(R_{0.1}\, and\, R_{1.0})$ are increased with increasing the value of steel distance ratio (γ) for both bonding conditions (uniaxial and biaxial). The column strength is increased with increasing the concrete compressive strength (f_c') as shown in Figure 8 for uniaxial bending condition and $(e/h=0.1)$ and Figure 9 for both uniaxial and biaxial bending condition $(f_c'\, and\, f_y)$, the figure shows the great difference between the results of uniaxial and biaxial bending condition. The same conclusions are obtained with columns with eccentricity ratio $(f_c'\, and\, f_y)$ as shown in Figure 10.

Generally the strength of the columns can be increased by increasing the material strength $(f_c'\, and\, f_y)$ amount of the reinforcement (γ) and the distance between reinforcement bars (γ). At the end row of the Table 7, the average of column strength ratio is determined for all concrete compressive and steel yielding strengths for column (case a). The average strength ratio is about (78% and 15%) for eccentricity ratio $(e/h=0.1\,\&\,1.0)$ respectively for uniaxial condition, while its about (64% and 8%) for biaxial condition for both $(e/h=0.1\,\&\,1.0)$ respectively.

The results show that the column strength ratio $(R_{1.0})$ at load eccentricity $(e/h=0.1)$ reduced from (78% to 64%) and lost its strength about (18%) when the bending condition changed from uniaxial to biaxial condition, while the ratio $(R_{1.0})$ changed from (15% to 8%), the reduction about (47%) at high level of eccentricity (f_c', f_y). This means that biaxial bending condition generally is more dangerous on the column strength than the uniaxial condition and this effect becomes more are high level of load eccentricity. Similar tables same as Tables 1- 6 are constructed for columns with reinforcement distributed on two sides (case b), circular column (case c) and rectangular column with circular distributed reinforcement (case d) ; the total number of tables are 18 which are the tables of the same format of Tables 1-6, but the details of these tables are not shown here to save the number of pages of this research, only the final average of the column strength of all cases of material strength pair (f_c', f_y) and reinforcement distance $(R_{0.1}\, and\, R_{1.0})$ are determined and summarized and tabulated in Tables 8, 9 and 10 for cases (b, c, and d) respectively for eccentricity ratio $(R_{0.1}\, and\, R_{1.0})$ and uniaxial and biaxial bending conditions. The same conclusions are obtained in column cases (b, c and d) as in (case a), the column strength ratios $(R_{0.1}\, and\, R_{1.0})$ at eccentricity ratios $(e/h=0.1\,\&\,1.0)$ are increased with increasing the concrete compressive, steel yielding strengths and reinforcement distance (γ) for both uniaxial and biaxial bending conditions.

Increasing of concrete compressive strength (f_c') causes significant increasing of relative column strength ratio $(R_{0.1}\,\&\,R_{1.0})$ and reducing the losses in column strength as shown in Figures 8, 9, and 10. The same behaviour is obtained in case of biaxial bending condition when compared with uniaxial bending condition, for both eccentricity ratios $(e/h=0.1\,\&\,1.0)$. Significant reduction in column strength is occurred in case of biaxial bending condition compared with uniaxial bending condition, The final average of column strength ratio $(R_{0.1}\,\&\,R_{1.0})$ at eccentricity ratio $(e/h=0.1\,\&\,1.0)$ for all column cases (a, b, c and d) are determined and summarized in Table 11 for both uniaxial and biaxial bending conditions, considering all variables, concrete compressive strength (f_c'), steel reinforcement yielding strength (f_y), reinforcement index (ρ) and reinforcement distance ratio (γ). As shown in Table 11, the column (case b), where the reinforcement distributed on

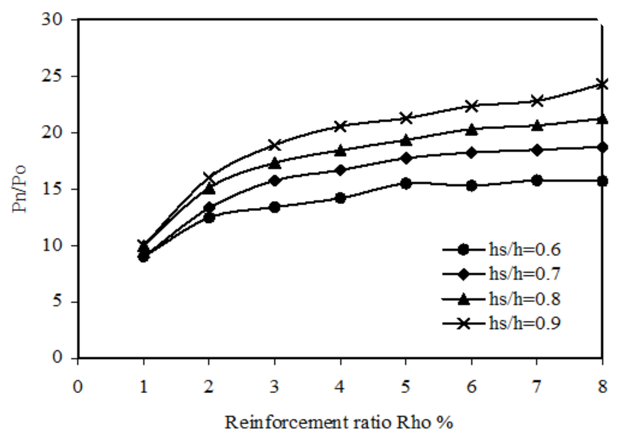

Figure 7: Relative strength of column, fc'=28 MPa, fy=414 Mpa, e/h=1.

γ =0.6		Uni-axial				Bi-axial			
% Rho	**Ko (e/h=0)**	**K(e/h=0.1)**	**K(e/h=1)**	**R(e/h=0.1)**	**R(e/h=1)**	**K(e/h=0.1)**	**K(e/h=1)**	**R(e/h=0.1)**	**R(e/h=1)**
1	0.96	0.76	0.064	79.167	6.667	0.628966	0.033103	65.517	3.448
2	1.07	0.84	0.104	78.505	9.72	0.691385	0.054656	64.615	5.108
3	1.18	0.91	0.14	77.119	11.864	0.740552	0.074414	62.759	6.306
4	1.3	0.99	0.16	76.154	12.308	0.799379	0.085246	61.491	6.557
5	1.4	1.075	0.18	76.786	12.857	0.872464	0.096183	62.319	6.87
6	1.52	1.15	0.2	75.658	13.158	0.924868	0.107042	60.847	7.042
7	1.63	1.23	0.22	75.46	13.497	0.987635	0.117961	60.591	7.237
8	1.74	1.3	0.24	74.713	13.793	1.037615	0.128889	59.633	7.407
Average		-	-	76.695	11.733	-	-	62.221	6.247
% Loss		-	-	23.305	88.267	-	-	37.779	93.753
γ =0.7		Uni-axial				Bi-axial			
% Rho	**Ko (e/h=0)**	**K(e/h=0.1)**	**K(e/h=1)**	**R(e/h=0.1)**	**R(e/h=1)**	**K(e/h=0.1)**	**K(e/h=1)**	**R(e/h=0.1)**	**R(e/h=1)**
1	0.96	0.76	0.0706	79.167	7.354	0.628966	0.036648	65.517	3.817
2	1.07	0.85	0.13	79.439	12.15	0.705039	0.069204	65.891	6.468
3	1.18	0.92	0.165	77.966	13.983	0.753889	0.088702	63.889	7.517
4	1.3	1	0.2	76.923	15.385	0.8125	0.108333	62.5	8.333
5	1.4	1.08	0.22	77.143	15.714	0.87907	0.11938	62.791	8.527
6	1.52	1.16	0.25	76.316	16.447	0.937872	0.136201	61.702	8.961
7	1.63	1.25	0.28	76.687	17.178	1.013682	0.153154	62.189	9.396
8	1.74	1.315	0.306	75.575	17.586	1.056859	0.16775	60.739	9.641
Average		-	-	77.402	14.475	-	-	63.152	7.833
% Loss		-	-	22.598	85.525	-	-	36.848	92.167
γ =0.8		Uni-axial				Bi-axial			
% Rho	**Ko (e/h=0)**	**K(e/h=0.1)**	**K(e/h=1)**	**R(e/h=0.1)**	**R(e/h=1)**	**K(e/h=0.1)**	**K(e/h=1)**	**R(e/h=0.1)**	**R(e/h=1)**
1	0.96	0.77	0.0814	80.208	8.479	0.642783	0.042502	66.957	4.427
2	1.07	0.85	0.14	79.439	13.084	0.705039	0.0749	65.891	7
3	1.18	0.93	0.19	78.814	16.102	0.767413	0.103318	65.035	8.756
4	1.3	1.02	0.23	78.462	17.692	0.839241	0.12616	64.557	9.705
5	1.4	1.105	0.256	78.929	18.286	0.912684	0.140881	65.192	10.063
6	1.52	1.186	0.29	78.026	19.079	0.972341	0.160291	63.97	10.545
7	1.63	1.267	0.32	77.73	19.632	1.036232	0.177415	63.573	10.884
8	1.74	1.355	0.35	77.874	20.115	1.109506	0.194569	63.765	11.182
Average		-	-	78.685	16.559	-	-	64.867	9.07
% Loss		-	-	21.315	83.441	-	-	35.133	90.93
γ =0.9		Uni-axial				Bi-axial			
% Rho	**Ko (e/h=0)**	**K(e/h=0.1)**	**K(e/h=1)**	**R(e/h=0.1)**	**R(e/h=1)**	**K(e/h=0.1)**	**K(e/h=1)**	**R(e/h=0.1)**	**R(e/h=1)**
1	0.96	0.77	0.087	80.208	9.063	0.642783	0.045565	66.957	4.746
2	1.07	0.855	0.145	79.907	13.551	0.711946	0.077769	66.537	7.268
3	1.18	0.942	0.2	79.831	16.949	0.783893	0.109259	66.432	9.259
4	1.3	1.03	0.24	79.231	18.462	0.852866	0.132203	65.605	10.169
5	1.4	1.116	0.28	79.714	20	0.927791	0.155556	66.271	11.111
6	1.52	1.2	0.314	78.947	20.658	0.991304	0.175084	65.217	11.519
7	1.63	1.29	0.35	79.141	21.472	1.06736	0.196048	65.482	12.027
8	1.74	1.372	0.384	78.851	22.069	1.132486	0.215814	65.085	12.403
Average		-	-	79.479	17.778	-	-	65.948	9.813
% Loss		-	-	20.521	82.222	-	-	34.052	90.187

Table 3: Relative column strength for columns fc'=35 MPa, fy=414 MPa (Distributed reinforcement on 4 edges).

Citation: Agha AZS, Rashid MHF (2018) Effect of Load Eccentricity on the Strength of Concrete Columns. J Civil Environ Eng 8: 308. doi: 10.4172/2165-784X.1000308

γ =0.6		Uni-axial				Bi-axial			
% Rho	Ko (e/h=0)	K(e/h=0.1)	K(e/h=1)	R(e/h=0.1)	R(e/h=1)	K(e/h=0.1)	K(e/h=1)	R(e/h=0.1)	R(e/h=1)
1	0.94	0.74	0.06	78.723	6.383	0.610175	0.030989	64.912	3.297
2	1.03	0.8	0.104	77.67	10.097	0.653968	0.054765	63.492	5.317
3	1.12	0.86	0.135	76.786	12.054	0.697971	0.071829	62.319	6.413
4	1.22	0.93	0.156	76.23	12.787	0.751391	0.083327	61.589	6.83
5	1.31	1	0.177	76.336	13.511	0.808642	0.094912	61.728	7.245
6	1.4	1.05	0.2	75	14.286	0.84	0.107692	60	7.692
7	1.49	1.115	0.22	74.832	14.765	0.890804	0.118768	59.786	7.971
8	1.58	1.2	0.24	75.949	15.19	0.967347	0.129863	61.224	8.219
Average		-	-	76.441	12.384	-	-	61.881	6.623
% Loss		-	-	23.559	87.616	-	-	38.119	93.377
γ =0.7		Uni-axial				Bi-axial			
% Rho	Ko (e/h=0)	K(e/h=0.1)	K(e/h=1)	R(e/h=0.1)	R(e/h=1)	K(e/h=0.1)	K(e/h=1)	R(e/h=0.1)	R(e/h=1)
1	0.94	0.74	0.073	78.723	7.766	0.610175	0.037975	64.912	4.04
2	1.03	0.81	0.115	78.641	11.165	0.66744	0.0609	64.8	5.913
3	1.12	0.875	0.14	78.125	12.5	0.717949	0.074667	64.103	6.667
4	1.22	0.94	0.177	77.049	14.508	0.764533	0.095422	62.667	7.821
5	1.31	1.01	0.2	77.099	15.267	0.821801	0.108264	62.733	8.264
6	1.4	1.073	0.23	76.643	16.429	0.869832	0.125292	62.131	8.949
7	1.49	1.146	0.25	76.913	16.779	0.931047	0.136447	62.486	9.158
8	1.58	1.208	0.27	76.456	17.089	0.977787	0.147612	61.885	9.343
Average		-	-	77.456	13.938	-	-	63.215	7.519
% Loss		-	-	22.544	86.062	-	-	36.785	92.481
γ =0.8		Uni-axial				Bi-axial			
% Rho	Ko (e/h=0)	K(e/h=0.1)	K(e/h=1)	R(e/h=0.1)	R(e/h=1)	K(e/h=0.1)	K(e/h=1)	R(e/h=0.1)	R(e/h=1)
1	0.94	0.75	0.075	79.787	7.979	0.623894	0.039058	66.372	4.155
2	1.03	0.82	0.117	79.612	11.359	0.681129	0.062023	66.129	6.022
3	1.12	0.89	0.17	79.464	15.179	0.73837	0.091981	65.926	8.213
4	1.22	0.96	0.2	78.689	16.393	0.791351	0.108929	64.865	8.929
5	1.31	1.03	0.22	78.626	16.794	0.848616	0.120083	64.78	9.167
6	1.4	1.106	0.245	79	17.5	0.91405	0.134247	65.289	9.589
7	1.49	1.17	0.28	78.523	18.792	0.963149	0.154519	64.641	10.37
8	1.58	1.245	0.303	78.797	19.177	1.027206	0.167567	65.013	10.606
Average		-	-	79.062	15.397	-	-	65.377	8.381
% Loss		-	-	20.938	84.603	-	-	34.623	91.619
γ =0.9		Uni-axial				Bi-axial			
% Rho	Ko (e/h=0)	K(e/h=0.1)	K(e/h=1)	R(e/h=0.1)	R(e/h=1)	K(e/h=0.1)	K(e/h=1)	R(e/h=0.1)	R(e/h=1)
1	0.94	0.75	0.08	79.787	8.511	0.623894	0.041778	66.372	4.444
2	1.03	0.825	0.14	80.097	13.592	0.688057	0.075104	66.802	7.292
3	1.12	0.9	0.18	80.357	16.071	0.752239	0.097864	67.164	8.738
4	1.22	0.97	0.21	79.508	17.213	0.805034	0.114888	65.986	9.417
5	1.31	1.04	0.25	79.389	19.084	0.862278	0.138186	65.823	10.549
6	1.4	1.12	0.28	80	20	0.933333	0.155556	66.667	11.111
7	1.49	1.19	0.31	79.866	20.805	0.990559	0.172996	66.48	11.61
8	1.58	1.266	0.34	80.127	21.519	1.056114	0.190496	66.843	12.057
Average		-	-	79.891	17.099	-	-	66.517	9.402
% Loss		-	-	20.109	82.901	-	-	33.483	90.598

Table 4: Relative column strength for columns fc'=42 MPa, fy=414 MPa (Distributed reinforcement on 4 edges).

γ =0.6		Uni-axial				Bi-axial			
% Rho	**Ko (e/h=0)**	**K(e/h=0.1)**	**K(e/h=1)**	**R(e/h=0.1)**	**R(e/h=1)**	**K(e/h=0.1)**	**K(e/h=1)**	**R(e/h=0.1)**	**R(e/h=1)**
1	0.92	0.72	0.06	78.261	6.522	0.591429	0.031011	64.286	3.371
2	1	0.78	0.09	78	9	0.639344	0.04712	63.934	4.712
3	1.07	0.825	0.12	77.103	11.215	0.671293	0.063564	62.738	5.941
4	1.15	0.88	0.14	76.522	12.174	0.712676	0.074537	61.972	6.481
5	1.22	0.93	0.15	76.23	12.295	0.751391	0.079913	61.589	6.55
6	1.3	0.97	0.16	74.615	12.308	0.77362	0.085246	59.509	6.557
7	1.37	1.025	0.18	74.818	13.139	0.818805	0.096328	59.767	7.031
8	1.45	1.07	0.19	73.793	13.103	0.847814	0.101661	58.47	7.011
Average		-	-	76.168	11.219	-	-	61.533	5.957
% Loss		-	-	23.832	88.781	-	-	38.467	94.043
γ =0.7		**Uni-axial**				**Bi-axial**			
% Rho	**Ko (e/h=0)**	**K(e/h=0.1)**	**K(e/h=1)**	**R(e/h=0.1)**	**R(e/h=1)**	**K(e/h=0.1)**	**K(e/h=1)**	**R(e/h=0.1)**	**R(e/h=1)**
1	0.92	0.73	0.062	79.348	6.739	0.605045	0.032081	65.766	3.487
2	1	0.79	0.093	79	9.3	0.652893	0.048768	65.289	4.877
3	1.07	0.84	0.125	78.505	11.682	0.691385	0.066377	64.615	6.203
4	1.15	0.89	0.145	77.391	12.609	0.725887	0.077378	63.121	6.729
5	1.22	0.95	0.165	77.869	13.525	0.777852	0.088484	63.758	7.253
6	1.3	1	0.18	76.923	13.846	0.8125	0.096694	62.5	7.438
7	1.37	1.055	0.2	77.007	14.599	0.857774	0.107874	62.611	7.874
8	1.45	1.11	0.215	76.552	14.828	0.899162	0.116108	62.011	8.007
Average		-	-	77.824	12.141	-	-	63.709	6.484
% Loss		-	-	22.176	87.859	-	-	36.291	93.516
γ =0.8		**Uni-axial**				**Bi-axial**			
% Rho	**Ko (e/h=0)**	**K(e/h=0.1)**	**K(e/h=1)**	**R(e/h=0.1)**	**R(e/h=1)**	**K(e/h=0.1)**	**K(e/h=1)**	**R(e/h=0.1)**	**R(e/h=1)**
1	0.92	0.735	0.066	79.891	7.174	0.611946	0.034228	66.516	3.72
2	1	0.795	0.11	79.5	11	0.659751	0.058201	65.975	5.82
3	1.07	0.847	0.13	79.159	12.15	0.70092	0.069204	65.507	6.468
4	1.15	0.9	0.16	78.261	13.913	0.739286	0.085981	64.286	7.477
5	1.22	0.96	0.18	78.689	14.754	0.791351	0.097168	64.865	7.965
6	1.3	1.02	0.21	78.462	16.154	0.839241	0.114226	64.557	8.787
7	1.37	1.075	0.22	78.467	16.058	0.884535	0.119603	64.565	8.73
8	1.45	1.13	0.245	77.931	16.897	0.925706	0.133804	63.842	9.228
Average		-	-	78.795	13.512	-	-	65.014	7.274
% Loss		-	-	21.205	86.488	-	-	34.986	92.726
γ =0.9		**Uni-axial**				**Bi-axial**			
% Rho	**Ko (e/h=0)**	**K(e/h=0.1)**	**K(e/h=1)**	**R(e/h=0.1)**	**R(e/h=1)**	**K(e/h=0.1)**	**K(e/h=1)**	**R(e/h=0.1)**	**R(e/h=1)**
1	0.92	0.74	0.07	80.435	7.609	0.618909	0.036384	67.273	3.955
2	1	0.8	0.115	80	11.5	0.666667	0.061008	66.667	6.101
3	1.07	0.86	0.15	80.374	14.019	0.718906	0.080653	67.188	7.538
4	1.15	0.92	0.18	80	15.652	0.766667	0.097642	66.667	8.491
5	1.22	0.975	0.21	79.918	17.213	0.811945	0.114888	66.553	9.417
6	1.3	1.03	0.23	79.231	17.692	0.852866	0.12616	65.605	9.705
7	1.37	1.09	0.26	79.562	18.978	0.90503	0.143629	66.061	10.484
8	1.45	1.145	0.28	78.966	19.31	0.946011	0.154962	65.242	10.687
Average		-	-	79.811	15.247	-	-	66.407	8.297
% Loss		-	-	20.189	84.753	-	--	33.593	91.703

Table 5: Relative column strength for columns fc'=63 MPa, fy=525 MPa (Distributed reinforcement on 4 edges).

γ =0.6		Uni-axial				Bi-axial			
% Rho	**Ko (e/h=0)**	**K(e/h=0.1)**	**K(e/h=1)**	**R(e/h=0.1)**	**R(e/h=1)**	**K(e/h=0.1)**	**K(e/h=1)**	**R(e/h=0.1)**	**R(e/h=1)**
1	0.9	0.715	0.05	79.444	5.556	0.593088	0.025714	65.899	2.857
2	0.96	0.745	0.07	77.604	7.292	0.608681	0.036324	63.404	3.784
3	1.01	0.78	0.115	77.228	11.386	0.635323	0.060971	62.903	6.037
4	1.07	0.82	0.128	76.636	11.963	0.664697	0.068072	62.121	6.362
5	1.12	0.86	0.14	76.786	12.5	0.697971	0.074667	62.319	6.667
6	1.17	0.89	0.155	76.068	13.248	0.718138	0.082998	61.379	7.094
7	1.23	0.94	0.165	76.423	13.415	0.760658	0.088431	61.842	7.19
8	1.28	0.97	0.17	75.781	13.281	0.780881	0.091046	61.006	7.113
Average		-	-	76.996	11.08	-	-	62.609	5.888
% Loss		-	-	23.004	88.92	-	-	37.391	94.112
γ =0.7		Uni-axial				Bi-axial			
% Rho	**Ko (e/h=0)**	**K(e/h=0.1)**	**K(e/h=1)**	**R(e/h=0.1)**	**R(e/h=1)**	**K(e/h=0.1)**	**K(e/h=1)**	**R(e/h=0.1)**	**R(e/h=1)**
1	0.9	0.72	0.055	80	6.111	0.6	0.028367	66.667	3.152
2	0.96	0.76	0.08	79.167	8.333	0.628966	0.041739	65.517	4.348
3	1.01	0.8	0.12	79.208	11.881	0.662295	0.063789	65.574	6.316
4	1.07	0.84	0.13	78.505	12.15	0.691385	0.069204	64.615	6.468
5	1.12	0.87	0.15	77.679	13.393	0.711241	0.080383	63.504	7.177
6	1.17	0.91	0.17	77.778	14.53	0.744545	0.091659	63.636	7.834
7	1.23	0.95	0.18	77.236	14.634	0.773841	0.097105	62.914	7.895
8	1.28	0.99	0.19	77.344	14.844	0.807134	0.102616	63.057	8.017
Average		-	-	78.364	11.984	-	-	64.436	6.401
% Loss		-	-	21.636	88.016	-	-	35.564	93.599
γ =0.8		Uni-axial				Bi-axial			
% Rho	**Ko (e/h=0)**	**K(e/h=0.1)**	**K(e/h=1)**	**R(e/h=0.1)**	**R(e/h=1)**	**K(e/h=0.1)**	**K(e/h=1)**	**R(e/h=0.1)**	**R(e/h=1)**
1	0.9	0.725	0.06	80.556	6.667	0.606977	0.031034	67.442	3.448
2	0.96	0.77	0.085	80.208	8.854	0.642783	0.044469	66.957	4.632
3	1.01	0.81	0.125	80.198	12.376	0.676116	0.066623	66.942	6.596
4	1.07	0.84	0.135	78.505	12.617	0.691385	0.072045	64.615	6.733
5	1.12	0.88	0.155	78.571	13.839	0.724706	0.083261	64.706	7.434
6	1.17	0.93	0.18	79.487	15.385	0.771702	0.0975	65.957	8.333
7	1.23	0.965	0.2	78.455	16.26	0.793946	0.10885	64.548	8.85
8	1.28	1	0.22	78.125	17.188	0.820513	0.120342	64.103	9.402
Average		-	-	79.263	12.898	-	-	65.659	6.929
% Loss		-	-	20.737	87.102	-	-	34.341	93.071
γ =0.9		Uni-axial				Bi-axial			
% Rho	**Ko (e/h=0)**	**K(e/h=0.1)**	**K(e/h=1)**	**R(e/h=0.1)**	**R(e/h=1)**	**K(e/h=0.1)**	**K(e/h=1)**	**R(e/h=0.1)**	**R(e/h=1)**
1	0.9	0.73	0.065	81.111	7.222	0.614019	0.033718	68.224	3.746
2	0.96	0.78	0.09	81.25	9.375	0.656842	0.047213	68.421	4.918
3	1.01	0.82	0.13	81.188	12.871	0.690167	0.069471	68.333	6.878
4	1.07	0.85	0.14	79.439	13.084	0.705039	0.0749	65.891	7
5	1.12	0.9	0.165	80.357	14.732	0.752239	0.08906	67.164	7.952
6	1.17	0.94	0.195	80.342	16.667	0.785571	0.106364	67.143	9.091
7	1.23	0.98	0.22	79.675	17.886	0.814459	0.120804	66.216	9.821
8	1.28	1.02	0.24	79.688	18.75	0.847792	0.132414	66.234	10.345
Average		-	-	80.381	13.823	-	-	67.203	7.469
% Loss		-	-	19.619	86.177	-	-	32.797	92.531

Table 6: Relative column strength for columns fc'=84 MPa, fy=525 MPa (Distributed reinforcement on 4 edges).

fc(MPa)	fy(MPa)	γ	Uni-axial		Bi-axial	
			R(e/h=0.1)	R(e/h=1)	R(e/h=0.1)	R(e/h=1)
21	414	0.6	74.74	14.21	59.69	7.66
		0.7	76.74	16.75	62.26	9.16
		0.8	77.6	18.83	63.4	10.42
		0.9	79.06	20.47	65.38	11.45
28	414	0.6	75.27	13.91	60.38	7.49
		0.7	76.83	15.99	62.39	8.72
		0.8	78.21	17.81	64.22	9.81
		0.9	79.51	19.53	65.99	10.88
35	414	0.6	76.7	11.73	62.22	6.25
		0.7	77.4	14.47	63.15	7.83
		0.8	78.69	16.56	64.87	9.07
		0.9	79.48	17.78	65.95	9.81
42	414	0.6	76.44	12.38	61.88	6.62
		0.7	77.46	13.94	63.21	7.52
		0.8	79.06	15.4	65.38	8.38
		0.9	79.89	17.1	66.52	9.4
63	525	0.6	76.17	11.22	61.53	5.96
		0.7	77.82	12.14	63.71	6.48
		0.8	78.79	13.51	65.01	7.27
		0.9	79.81	15.25	66.41	8.3
84	525	0.6	77	11.08	62.61	5.89
		0.7	78.36	11.98	64.44	6.4
		0.8	79.26	12.9	65.66	6.93
		0.9	80.38	13.82	67.2	7.47
Total Average			**77.94**	**14.95**	**63.89**	**8.13**

Table 7: Average Relative strength of column $P_n/P_{o\%}$ (Distributed reinforcement on 4 edges- Case a).

fc(MPa)	fy(MPa)		Uni-axial		Bi-axial	
			R(e/h=0.1)	R(e/h=1)	R(e/h=0.1)	R(e/h=1)
21	414	0.6	77.311	18.394	63.019	10.163
		0.7	79.149	20.639	65.495	11.554
		0.8	80.676	23.219	67.613	13.19
		0.9	82.041	25.583	69.552	14.745
28	414	0.6	77.557	17.021	63.348	9.346
		0.7	79.037	19.586	65.343	10.93
		0.8	80.221	21.556	66.977	12.174
		0.9	80.747	23.892	67.718	13.674
35	414	0.6	78.15	15.617	64.161	8.506
		0.7	79.441	18.113	65.91	10.018
		0.8	80.847	20.779	67.858	11.68
		0.9	81.919	23.384	69.376	13.361
42	414	0.6	78.36	15.493	64.424	8.443
		0.7	79.71	17.309	66.268	9.539
		0.8	80.926	19.11	67.965	10.654
		0.9	82.023	20.906	69.529	11.793
63	525	0.6	78.046	13.013	64.003	6.995
		0.7	79.393	15.165	65.833	8.256
		0.8	80.501	17.32	67.369	9.558
		0.9	81.507	19.273	68.789	10.765
84	525	0.6	78.583	11.991	64.73	6.402
		0.7	79.807	13.657	66.404	7.373
		0.8	80.799	15.5	67.787	8.471
		0.9	81.69	17.118	69.051	9.46
Total Average			**79.935**	**18.485**	**66.605**	**10.294**

Table 8: Average Relative strength of column $P_n/P_{o\%}$ (Distributed reinforcement on 2 edges- Case b).

fc(MPa)	fy(MPa)		Uni-axial		Bi-axial	
			R(e/h=0.1)	R(e/h=1)	R(e/h=0.1)	R(e/h=1)
21	414	0.6	70.584	11.901	54.588	6.33
		0.7	72.663	13.787	57.094	7.412
		0.8	74.677	15.758	59.604	8.571
		0.9	76.793	17.697	62.331	9.737
28	414	0.6	71.242	10.906	55.366	5.776
		0.7	73.077	12.565	57.588	6.719
		0.8	74.988	14.226	59.992	7.684
		0.9	76.475	15.755	61.912	8.585
35	414	0.6	72.468	10.252	56.861	5.408
		0.7	74.198	11.877	59.014	6.324
		0.8	75.783	13.54	61.025	7.285
		0.9	77.202	15.214	62.875	8.275
42	414	0.6	73.006	10.146	57.512	5.349
		0.7	74.527	11.489	59.411	6.106
		0.8	75.789	12.711	61.022	6.807
		0.9	77.382	14.003	63.112	7.563
63	525	0.6	71.238	9.229	55.384	4.842
		0.7	73.047	10.41	57.576	5.498
		0.8	74.757	11.309	59.711	6.005
			76.133	12.091	61.476	6.453
84	525	0.6	72.691	8.647	57.124	4.532
		0.7	73.925	9.186	58.657	4.83
		0.8	74.764	9.71	59.718	5.12
		0.9	75.696	10.293	60.907	5.448
Total Average			**74.296**	**12.196**	**59.161**	**6.528**

Table 9: Average Relative strength of column $P_n/P_{o\,\%}$ (Circular section- Case c).

fc(MPa)	fy(MPa)	γ	Uni-axial		Bi-axial	
			R(e/h=0.1)	R(e/h=1)	R(e/h=0.1)	R(e/h=1)
21	414	0.6	72.943	13.061	57.452	6.99
		0.7	74.653	14.901	59.599	8.059
		0.8	76.257	16.769	61.656	9.171
		0.9	78.017	18.636	63.973	10.306
28	414	0.6	73.656	12.368	58.338	6.597
		0.7	74.642	13.646	59.577	7.333
		0.8	76.126	15.001	61.485	8.128
		0.9	78.089	16.48	64.068	9.01
35	414	0.6	74.931	11.554	59.944	6.146
		0.7	76.266	12.906	61.661	6.921
		0.8	77.349	14.399	63.077	7.785
		0.9	78.341	15.773	64.401	8.597
42	414	0.6	75.777	11.352	61.032	6.026
		0.7	76.839	12.503	62.41	6.682
		0.8	77.894	13.724	63.805	7.39
		0.9	79.171	14.875	65.533	8.067
63	525	0.6	74.833	10.688	59.853	5.652
		0.7	75.984	11.671	61.325	6.208
		0.8	77.174	12.587	62.873	6.732
		0.9	78.362	13.495	64.448	7.258
84	525	0.6	76.417	9.702	61.863	5.109
		0.7	77.185	10.507	62.872	5.561
		0.8	77.863	11.112	63.771	5.903
		0.9	78.647	11.972	64.824	6.395
Total Average			**76.56**	**13.32**	**62.08**	**7.17**

Table 10: Average Relative strength of column $P_n/P_{o\,\%}$ (Rectangular section with Circular distribution reinforcement -Case d).

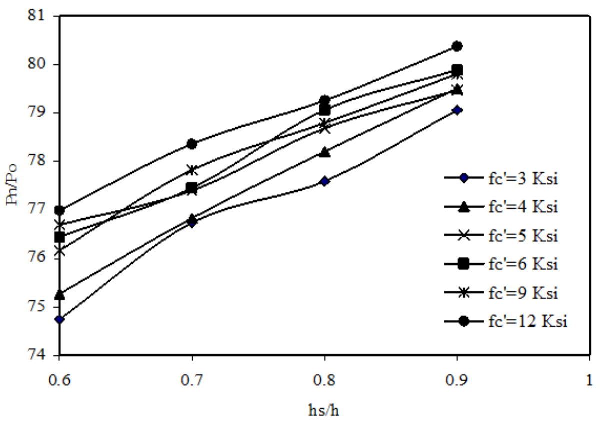

Figure 8: Relative strength of column, e/h=0.1, uni-axial bending condition.

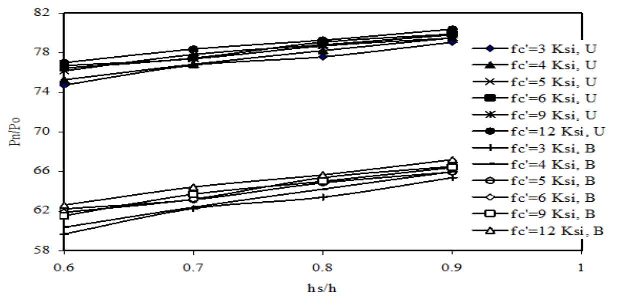

Figure 9: Relative strength of column, e/h=0.1.

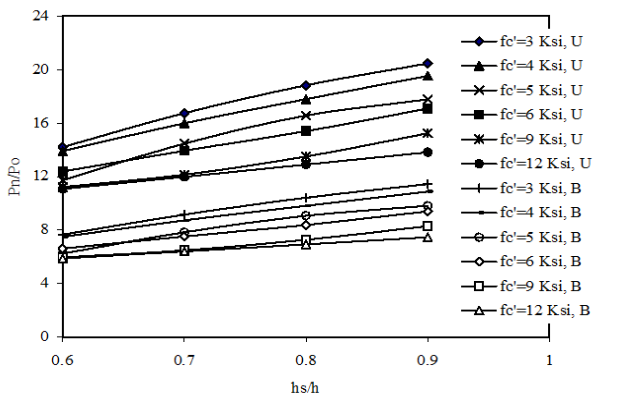

Figure 10: Relative strength of column, e/h=1.

Type of the column & reinforcement	Uni-axial		Bi-axial	
	R(e/h=0.1)	R(e/h=1)	R(e/h=0.1)	R(e/h=1)
Distributed reinforcement on 4 edges (Case a)	78	15	64	8
Distributed reinforcement on 2 edges (Case b)	80	18	67	10
Circular section (Case c)	74	12	59	7
Rectangular section with Circular distribution reinforcement (Case d)	77	13	62	7

Table 11: Effect of eccentricity on the column strength for all cases of reinforcement distribution (Average Relative strength of column $P_n/P_{o\%}$).

the opposite sides gives the maximum column strength and that is (upper limit) for both uniaxial and biaxial bending conditions and at eccentricity ratio $(e/h=0.1\,\&\,1.0)$, while circular column (case c) gives minimum values of column strength and maximum amount of losses, that is lower limit. The cases can be arranged from maximum column strength to minimum column strength as following: b, a, d, and c.

Conclusions

1. Generally, the column strength $(R_{0.1}\,\&\,R_{1.0})$ is reduced with existing the load eccentricity, and significant losses in strength occurred when the load eccentricity changed from (0.1 to 1.0)
2. The relative column strength $(R_{0.1}\,\&\,R_{1.0})$ increases with increasing the concrete compressive and steel yielding strengths.
3. The relative column strength $(R_{0.1}\,\&\,R_{1.0})$ increased with increasing the reinforcement index $(\rho\%)$.
4. Increasing of distance between reinforcement rows $\left(\gamma=\frac{h_s}{h}\right)$ cause significant increasing in column strength ratios $(R_{0.1}\,\&\,R_{1.0})$ and reducing the losses in column strength.
5. Concrete compressive strength has significant effect in increasing the column strength in case of $(e/h=0.1\,\&\,1.0)$ and in both uniaxial and biaxial bending conditions.
6. Applying load eccentricity about both axis, that is biaxial bending condition has more effect and dangerous compared with eccentricity about major axis, that is uniaxial bending condition. The table below shows the comparison between biaxial and uniaxial bending conditions for all column cases (a, b, c & d). The average column strength ratio in biaxial condition is about (82%) of the corresponding uniaxial condition for eccentricity ratio $(e/h=0.1)$ and about 55% for eccentricity ratio $(e/h=1.0)$, this means that the bending condition has more effect at high level of load eccentricity.
7. For column (case a), rectangular column with the reinforcement distributed on four edges, the average column strength ratio is about (78%) and losses 22% for eccentricity $(e/h=1.0)$ and about (15%) and losses 85% for eccentricity ratio $(e/h=1.0)$. For uniaxial bending condition, while the strength ratio is about (64% to 8%) for $(e/h=0.1\,\&\,1.0)$ respectively for biaxial bending ratio.
8. For column (case b); rectangular column with the reinforcement distributed on two opposite sides, the average column strength ratio $(R_{0.1})$ is about (80%) and losses (20%) and $(R_{1.0})$ is about (18%) and losses (82%) for $(e/h=0.1\,\&\,1.0)$ respectively for uniaxial bending condition, while the column strength ratio value are (67% and 10%) at $(e/h=0.1\,\&\,1.0)$ respectively for biaxial bonding condition.
9. For column (case c); circular column, for uniaxial bending condition, the average strength ratio values about (74% and 12%) for eccentricity ratio $(e/h=0.1\,\&\,1.0)$ respectively, while in biaxial bending condition the strength values are about (59 and 7%) for $(e/h=0.1\,\&\,1.0)$ respectively.
10. For column (case d); rectangular column with circular distribution reinforcement, the average strength ratio is about (77%) and losses (23%) for $(e/h=1.0)$ and about (13%) and losses (87%) for $(e/h=1.0)$ and uniaxial bending condition while the values of column strength ratio are about (62% and 7%) for eccentricity ratio $(e/h=0.1\,\&\,1.0)$ respectively and biaxial bending condition.
11. Case b; the reinforcement distributed on two opposite sides gives upper limit results and maximum column strength results for both bending conditions (uniaxial and biaxial) and eccentricity ratio

$(e/h=0.1\,\&\,1.0)$, while circular column (case d) gives lower limit results and minimum column strength.

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