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Economic Growth Analysis of a Singapore: Simultaneous Equations Model

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Abstract

This paper has carried out an in-depth study based on the simultaneous equations model by estimating three structural equations associated to the three components of the Real Gross Domestic Product per Capita (gdp) in Singapore over the period (1991-2017), that is, the Real Gross Domestic Saving per Capita (gds), the Household Final Consumption Expenditure per Capita (hfce), the Government Final Consumption Expenditure per Capita (gfce). The primary nominal data were divided by the product of the consumer price index and the annual population for leading to real data per capita taking into account both inflation and population. The fourth equation represented the income identity expressed by equality (gdp_t=gds_t+$hfce_t$+$gfce_t$). Seven instruments variables are used to accomplish the study*:* a constant, three predetermined variables characterized by gds_{t-1}, $hfcd_{t-1}$ and $gfce_{t-1}$, three exogenous variables as real interest rate (rir_t), the real foreign direct investment per capita (fdi_t) and the real money supply per capita (m_{1t}). The study shows that the three structural equations are over-identified and by consequence; each equation is estimated using the following methods: Two-Stage Least Square estimator (2SLS), Heteroscedastic Two-Stage Least Squares (H2SLS), Limited Information Maximum Likelihood (LIML) and the Three-Stage Least Squares (3SLS) which is often more efficient than other methods and promoted by Hausman test. Finally, the performance of the estimated equations is measured comparing the fitted values with the observed values by the Mean Relative Error (MRE). The findings have shown that the MRE values are 2.46%, 1.37%, 4.9% and 1.37% for the variables gds_t, $hfce_t$ $gfce_t$ and gdp_t respectively.

Keywords: Expenditures; Gross savings; Macroeconomic determinants; Simultaneous equation modelling

Introduction

We find very useful to begin the introduction by taking a look at the beginnings of simultaneous equations system. As it is known in the literature of econometrics, when we use the simultaneous equations system, we decide to deal with several linear or dynamic regressions basing to the macro-economic theory. Thus, a simultaneous equation model (SEM) will be available provided they has been chosen in the light of the economic theory allowing a correct diagnosis of the system and reflecting the real interactions between variables which their use helps in prediction and in proper planning. For this the importance of looking to the interactions between the variables, on the one hand, to realize a correct estimate of the equations, and on the other hand, to have for the ability to interpret them. The proposed equations, which are known as structural equations, must comport with the economic theory. So, we will distinguish, firstly, between endogenous variables and exogenous variables, and secondly between the behavioural equations and the equations of balance which do not contain disturbance variables. So, the researcher should think carefully before estimating the coefficients of the system because if the ordinary least squares (OLS) method is used arbitrarily, then the estimators will be biased and inconsistent. Thus, the importance of using a suitable method to estimate the equations in the system, as we shall see in this research, as Instrumental Variables (IV) estimation, Two-Stage Least Squares (2SLS) and Three-Stage Least Squares (3SLS) estimators, etc. The necessary important points in the simultaneous equations is the sorting of exogenous and endogenous variables leading to acquire or no the correct estimation of parameters. What we would like to add in this context, the attribute exogenous or endogenous of a variable, is not an intrinsic characteristic but it is associated to the considered model. Moreover, the balance identity has an important role allowing the existence of the complete system of equations where we have equated the number of equations with the number of endogenous variables. Examining the development of these models, they started since the Thirties of the last century and they were the precedence in the birth of econometrics as a scientific and vital discipline. For example, the Dutch famous economist, Tinbergen [1], in his research entitled "for the Dutch economy in 1936", studied a Dutch system of twenty-four equations covering the all economic aspects. In his writing, the expression "Simultaneous relations" is founded and he defined it as relationships in which there is no effect of the past time, i.e. without distributed lag. Tinbergen was awarded in the first Nobel Prize in Economic Sciences with Frisch, Norwegian econometrician in recognition for their ground breaking work which led to the development of econometrics as a branch in the sciences. According to the Tinbergen's methodology, Klein published his book titled "economic fluctuations in the United States" using annual data for the period 1921-1941, see Mourad [2] for a rich explanation of these equations and their estimations with adequate methods.

From an economic perspective, it is common knowledge in the economic literature that the gross domestic product (GDP_t) is positively affected by a number of variables such as the household final consumption expenditure ($HFCE_t$), the government final consumption expenditure ($HFCE_t$), the money supply (M_{1t}), the inflation which is sometimes expressed as a consumer price index (CPI_t) and the real interest rate (rir_t) and others variables. It is also known that there is a specific identity relationship with the two components of the total expenditure and the gross domestic savings (GDS_t) as follows:

$$\text{Income identity} \rightarrow GDP_t = GDS_t + HFCE_t + GFCE_t$$

Taking into account the inflation and dividing each variable by *(CPI × POP)* we, obtain the income identity with real economic variables per capita, expressed in small letters. In the same way we have considered the real Foreign direct investment fdi_t and the real money supply m_t.

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In this research, we will attempt to focus on the simultaneous equations system consisting of four equations with four endogenous variables gds_t, $hfce_t$, $gfce_t$ and gdp_t and seven exogenous and predetermined variables Constant, gds_{t-1}, fdi_t, rir_t, $hfce_{t-1,}$ $gfce_{t-1}$ and m_t. A set of four simultaneous equations models with three equations with parameters and one equation as identity i.e. without parameters.

After the construction of the system, the identification problem will be studied hoping that over identified or just identified equations will be founded remembering that under-identified equation cannot be estimated. Each identified equation can be estimated using methods as two-stage least square estimator (2SLS) and three-stage least squares (3SLS).

This paper is divided into the following five sections:

The first section deals with a general introduction to the topic that allows for a general understanding of the problem of the study and its purpose. The second section is intended to elaborate statistical descriptions of variables. The third section briefly addresses the literature review and the implementation of SEM. Then the fourth section is devoted to the theoretical aspect intended to study the identification problem and the the estimation methods commonly used in a simultaneous equations model. The fifth section will be devoted to the analysis of the three equations. Finally, in the sixth section, conclusions are drawn focusing on the findings of the research and measuring the goodness of the four equations, gds, hfce, gfce and gdp.

Statistical Descriptions of Variables

This section is intended to illustrate graphically each of the time series and to perform a statistical description allowing to appreciate the temporal evolution of each variable. Firstly we calculte the basic statistical characteristics such as averages, standard deviations, minimums, maximums and the Compound Annual Growth Rate (CAGR) that is a specific term for the geometric progression ratio that provides a constant rate of evolution over the time period. For a time series X_t, the CAGR, between the first and end observations, is calculated by:

$$CAGR\left(1,T\right)=\left(\frac{X_T}{X_1}\right)^{\frac{1}{T-1}}-1$$

It is clear that this description would have a meaning especially if a time series reveals is considered a realization of the stationary Gaussian random process. In any case, we hope that this section will better know the temporal evolution of each of the variables and especially if we can spot breaks in time because of a political or economic intervention that had an impact on the growth of the variable. As we announced in the introduction, we have two types variables 4 endogenous variables and seven instrumental or exogenous variables. The four endogenous variables are observed over the period 1961-2017 (57 years), while the exogenous variables m_t, $fdi_{t,}$ and i_t are observed on the periods (1991-2017), (1970-2017) and (1978-2017) respectively. For this, the proposed model will be estimated over the period (1991-2017).

Before presenting the bulk of the descriptive statistics of our variables, it would be of great interest to focus on the history of Singapore's economic development. It is well known that the Singapore economy is a highly prosperous liberal market economy, which has become a model that inspires the policies of the major countries in the world, including China. According to the Economist, there is a wide gap in Asia, between the top countries as (Singapore, Hong Kong) and the poorest performers (Bangladesh and Pakistan), noting that over the period 2009-2018, Singapore has held the first Business Environment Rankings (BER) among out of the 82 countries to do business in.

To appreciate better the transparency of the public sector, let's look at the level of corruption in Singapore using Corruption Perceptions Index 2017. In fact, among 180 countries, Singapore had the sixth rank producing a score (84/100) compared to New Zealand the first top country scoring (89/100). The unemployment rate was about 10% in 1965, the year of its independence from the United Kingdom and reached 2.2% in 2017. The labor force has gone from 701700 in 1970 to 3657000 in 2017 with CAGR from 3.575%, with female labour participation grew from 28.2% to 58.6%. The majority of the public debt is domestic. At the end of 2017, the domestic debt for more than one-year maturity reached 445.583 milliards SGD while it is about 41.8 milliards SGD for one-year maturity or less, the external debt is zero. By consequence Singapore had a public debt-to-GDP ratio at around 109% in 2017.

Let us look at the Singapore's population structure over the period 1990-2017. In Singapore, the population is composed of residents and non-residents according the following equality:

Total population = Singapore Residents + Non-Residents

= Citizens + Permanent Residents + Non-Residents

It is made up of three main ethnic groups that are Malays, Chinese and Indians whose average proportions are (13.74%), (75.91%), (8.23%) respectively. It remains 2.11% for Others (Figure 1).

On the 21 July 1964, during a peaceful procession from the Padang to Geylang, Singapore has experienced race riots between Malay participants and Chinese individuals. The ministry of education in Singapore invites the Singaporeans to continuously remember the riots during two five-day periods where property was destroyed, people were injured and some lost their lives. As a result, the country celebrates Racial Harmony Day (RHD) to commemorate the 1964 race riots in Singapore focusing on the consequences of racial disharmony. Each year, the Singaporeans recite the Declaration of Religious Harmony of Singapore during the week when Racial Harmony Day. The political decision makers ensure that the harmony will not be a word spoken hypocritically, but Harmony from the Heart!

Examining now the evolution of the main commodities associated to the Total Merchandise Exports between the two years 1976 and 2017 (Appendix 1), we find that some commodities have shown net growth, while other have revealed a decline in their ratio. To simplify the presentation, we have grouped commodities into two categories: in the first, the commodities showing a decrease in their ratio in the Total Merchandise Exports, and in the second, the commodities that reveal an increase in their ratio (Figures 2 and 3).

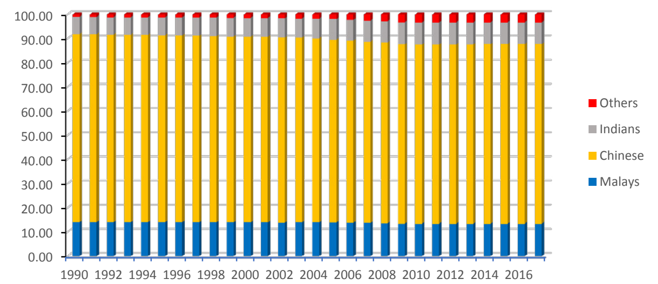

Figure 1: Main ethnic groups.

I believe interesting to focus on the commodities that has been growing significantly in time because they reflect an aspect of economic prosperity in Singapore. Those materials are: Organic Chemicals, Medicinal & Pharmaceutical Products, Essential Oils & Resinoids & Perfume Materials, Plastics in Primary Forms, Office Machines & Automatic Data-processing Machines, Electrical Machinery Apparatus, Professional Scientific & Controlling Instruments and Miscellaneous Manufactured.

Table 1 shows a very significant growth particularly in the variables Real GDP (RGDP) and gds which the CAGR are respectively (8.66%) and (15.19%). In fact, since its independence Singapore enjoyed a magnificent economic prosperity that made it an almost unique example of the success of a people made up of several ethnic groups and various regions. For all variables, the largest values of the range measured by the difference between Maximum and Minimum reflect the presence of a very increasing trend over the considered periods. This is also shown by means greater than medians and high standard deviations except for the real interest rate, the median and the mean are very close to each other. The CAGR indicator summarizes the important growth of each variable (Table 1).

We recall that the real interest rate is the interest rate that has been adjusted to remove the effects of inflation and it can be calculated using the Fisher equation, which states that the following: $1+\text{rir}=\frac{1+i_{\text{nominal}}}{1+i_{\text{inflation}}}$

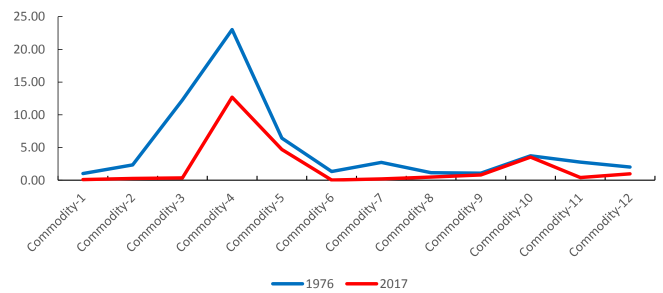

Figure 2: Commodities with decrease ratio.

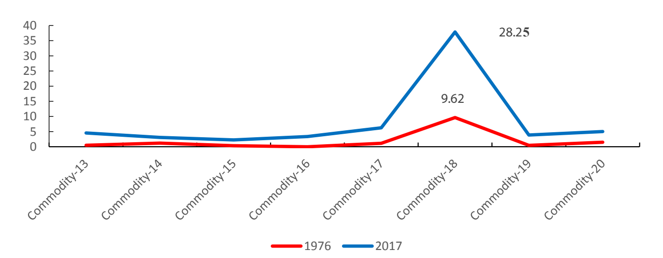

Figure 3: Commodities with increase ratio.

. If $i_{\text{nominal}} < i_{\text{inflation}}$ then rir < 0. The stock market desires negative real interest rates because they make profits by borrowing money practically free and invest and develop their businesses. If the rate of inflation is zero, then the real interest rate is equal to the nominal interest rate. According to the precise formula above, when the rate of inflation is positive then the nominal interest rate is higher than the real interest rate.

Now let's consider the figures of the raw time series. The Real Gross Domestic Product (RGDP), base 1991, grew slowly from 1965 to 1984 (Export-led industrialization through multinationals) passing from 2.48 to 21.84 billion that happened the parliamentary general election which People's Action Party (PAP) realized a victory. Mr Goh Chok Tong, the chairman of the party's election committee, turned his attention to the vision for the future of which the PAP's strategy focused on the country's impressive economic growth and high standard of living. The graph shows a change in the trend over the 1985-2011, this was the period of the liberalization and the rise of modern services and it will be divided into three-sub periods, the first 1985-1997, the second 1998-2003 and the third 2004-2011. In fact, in the first period, the manufacturers in the electronic sub-sectors in Singapore were forced to focus their operation to the higher end operation of the value chain. Singapore registered the highest annual growth rate of 11.33% in terms the labor productivity compared to Malaysia, Taiwan, USA and Philippines, see Abdullah and Bin Bakar [3]. In July 1997, the Asian financial crisis started in Thailand due the float the Baht (official currency of Thailand) and affected the most Asian countries but Singapore were less affected, but suffered from a loss of demand and confidence throughout the region. The graph shows a decline in RGDP from (USD 88.29 billion) in 1997 to (83.65 billion) in 2003. Over the period 2003-2011, Singapore enjoyed remarkable economic prosperity, exception between 2008 and 2009, a small decline is taken place due to the global financial crisis, but the domestic debt grew from (SGD 163.8 billion) in 2003 to (SGD 338.735 billion) in 2011 with a CAGR of (9.51%) while the CAGR of RGDP was (11.14%), moreover, Singapore shows more erratic growth rate during this period compared with Hong Kong per example. The variance is higher, "this implies that internal and external volatilities have influenced Singapore's growth rate more than Hong Kong's" [4]. Finally, on the period 2011-2017 and according to the Monetary Authority of Singapore, this is the demographic slowdown and economic restructuring that is characterized by the overcoming of resource constraints through a significant restructuring of the economy. The Real GDP declined in 2015 compared to the latest year about 1.87% and then it rises to score USD 212.33 billion in 2017. For more information about the sources of growth by period, Khuong [5].

The gdp, hfce, gfce graphs are very similar. Almost the same phases of evolution (a little less for gfce) are observed with CAGR of 6.37%, 4.61% and 6.74% respectively over the whole period. For gds, a difference with the previous graphs characterizes the period 1997-2001

Variables	Period	Minimum	Maximum	Median	Mean	Standard Deviation	CAGR
RGDP	1961-2017	2.03	212.33	32.56	64.46	68.0472	0.0866
gdp	1961-2017	1194.11	37833.72	11107.78	15076.20	12450.71	0.0637
gds	1961-2017	7.35	20460.71	4910.02	7354.88	6892.78	0.1519
hfce	1961-2017	1017.46	13716.86	5114.89	6200.58	4352.92	0.0461
gfce	1961-2017	106.26	4107.13	1082.87	1520.74	1251.66	0.0674
fdi	1970-2017	107.44	8730.77	1602.46	2827.46	2865.87	0.0943
m	1991-2017	2783.89	15124.96	5044.72	7492.67	4470.37	0.0673
rir	1978-2017	-0.50	10.09	4.77	4.59	2.68	…

Table 1: Descriptive Statistics of the variables.

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where an annual growth rate decreases of about 9.65%. Indeed, it was a period that began with the Asian financial crisis in 1997 and ended with September 11 attacks in 2001.

The FDI reveals more erratic evolution compared with the previous variables. A decline about 40.63% between 1990 and 1992 because in the end of 1990, some international companies left Singapore, 48.47% between 1997 and 1998 due to the Asian financial crisis, 63.97% between 2001 and 2002 due to September 11 attacks, 77.27% between 2007 and 2008 due to global financial crisis. The CAGR is evaluated on 9.43% between 1970 and 2017. For more information about the foreign direct investment in Asia, see Bishwanath and Ishigami [6].

For the money supply per capita (*m*), the graph shows a decline about 20.3% between 1997 and 1998, a rise with CAGR about 10.96% between 1997 and 2011, and finally a slight growth between 2011 and 2017 with CAGR about 1.92%.

The inspection of the real interest rate (*rir*) shows three particular values: The highest values are 9.63 in 1985, 10.09 in 1999 and the lowest value is (-0.5) in 2007. The median and the mean are relatively close to each other (4.77 and 4.59) and the fluctuations oscillate around these values (Figures 4-10).

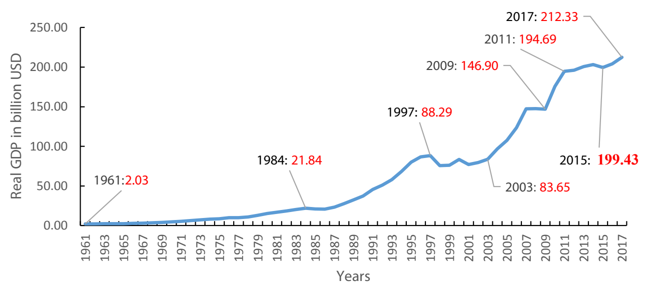

Figure 4: Singapore RGDP.

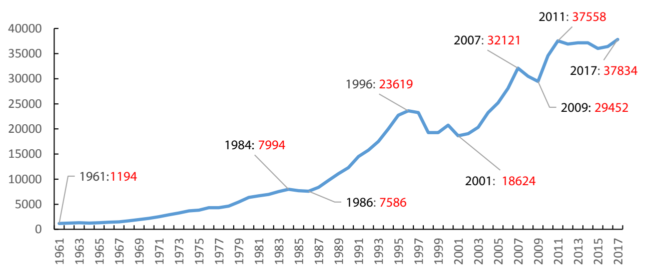

Figure 5: Singapore GDP.

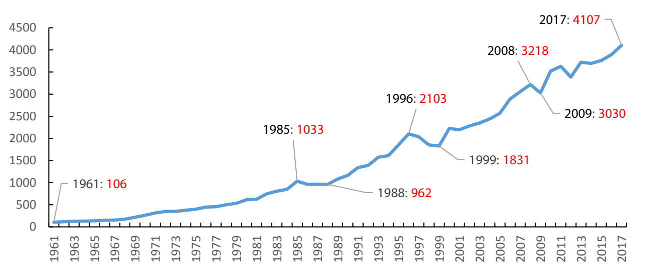

Figure 6: Singapore GFCE.

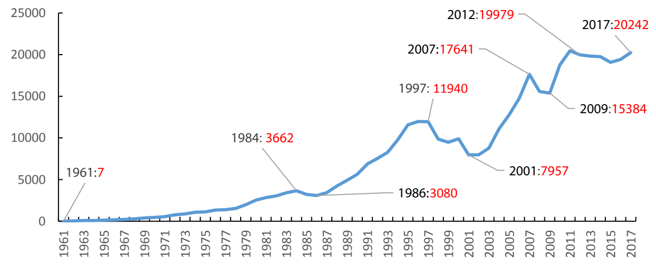

Figure 7: Singapore GDS.

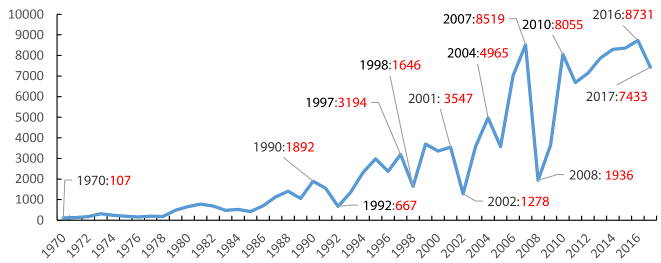

Figure 8: Singapore FDI.

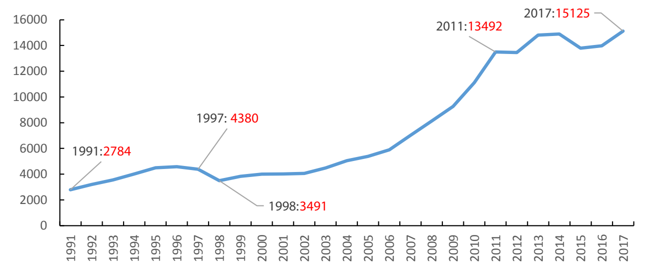

Figure 9: Singapore M.

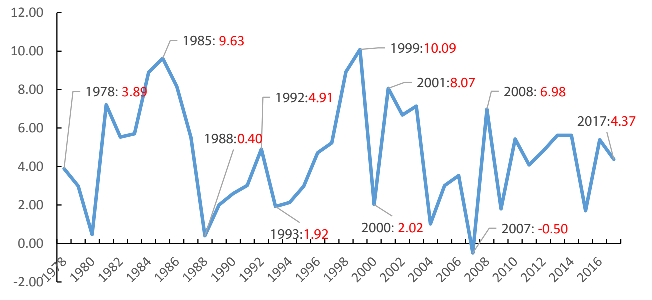

Figure 10: Singapore real interest rate.

Literature Review

The theoretical and applied literature is full in studies dealing with Simultaneous-Equation Models. Let us first mention the interesting books that develop well the simultaneous equation models. We start with Kmenta and Ramsey [7], Johnston [8], Judge et al. [9], Baltagi [10], Greene [11], Jeffrey [12] and Mourad [2]. Fair [13] discussed various methods for the estimation of simultaneous equation models with lagged endogenous variables and first order serially correlated errors. Hausman [14] analysed the SEM when errors in variables are present in the exogenous variables. Lee [15] proposed an econometric approach for estimating the simultaneous equation systems consisting of two structural equations with unobservable health capital and wage rate jointly dependent. Noureddine [16] specified the relationship between demand and supply for world crude oil and natural gas markets considering a system of four structural equations associated to four endogenous variables that are crude oil demand, crude oil supply, natural gas demand, natural gas supply, and four exogenous variables (included the constant term). Fielding and Torres [17] studied the bi-directional relationships between income inequality and other indicators of social and economic development considering four endogenous variables represented by per capita income, education, health and inequality, and the estimates have been made using the (3SLS) method. Ciavolino and Dahlgaard [18] studied the relationships between management factors and enterprise performance formalizing by a Simultaneous Equation Model and estimated by the generalized maximum entropy (GME) method and others. The three considered dependent variables are Performance, Human Resources, and Strategic Planning but Leadership is taken as independent variable. The data collected from 120 Italian manufacturing companies. Matei and Anghelescu [19] considered the simultaneous equation systems consisting in 17 equations and used the Two Stage Least Squares method to estimate the parameters. Tudorel, Matei and Oancea [20] suggested the simultaneous equation models to analyse some important issues related to the progress of the reform process in the public health system in Bucharest. They had the opportunity to use the OLS and (2SLS) methods to estimate the parameters of the structural equations. Tajdini, Ghajebeigloo and Ronnie [21] focused in the simultaneous equation models to describe the relation between supply and demand of the wood veneer industry in Iran using the iterative 3 stage least squares (I3SLS) to estimate his structural parameters. An excellent recent paper for Hsiao and Qiankun [22] considered the asymptotic properties of the GMM estimator for a structural equations in a panel dynamic simultaneous equations model and showed that the consistency of the GMM estimator need N (number of individuals) much larger than T (size of time series), i.e. $\frac{T}{N} \underset{N\to\infty}{\to} 0$. However, the GMM estimator is still asymptotically biased as long as $\frac{T^3}{N} \underset{N\to\infty}{\to} \kappa \neq 0 < \infty$. Thus, the authors suggested a jackknife procedure showing that the JIVE is asymptotically normal without an asymptotic bias. Sudhanshu [23] exploited the simultaneous equations model to specify the inter-connections among the measures of globalization, measures of democracy, human development, corruption perception index and per capita income. The author used the (2SLS) and alternatively its modified form to estimate the system. Finally, Mourad and Trabulsi [24] carried out an in-depth study based on the simultaneous equations model by considering three structural equations associated to the three components of the Real Gross Domestic Product per Capita in Singapore over the period (1991-2017).

Econometric Methodology

This section is dedicated to study the possibility of estimating the parameters in the structural equations where the dependent variable is located at the left hand side while the other endogenous variables and the instruments constituted from the homogenous or predetermined variables are logged at the right hand side. The econometric analysis is taken from the book of Mourad [2].

Inspired by economic theory, the system is described by:

Four endogenous variables (M=4): gds_t, $hfce_t$, $gfce_t$, gdp_t

Seven instruments (K=7): β_0, gds_{t-1}, fdi_t, i_t, $hfce_{t-1}$, $gfce_{t-1}$, m_t

We consider the following equations:

$$gds_t = \beta_{01} + \varphi_{41} gdp_t + \beta_{11} gds_{t-1} + \beta_{21} fdi_t + \beta_{31} rir_t + \beta_{41} hfce_{t-1} + \varepsilon_{1t}$$

$$\varphi_{41} > 0 \; \beta_{11} > 0 \; \beta_{21} > 0 \; \beta_{31} < 0 \; \beta_{41} < 0$$

$$hfce_t = \beta_{02} + \varphi_{12} gds_t + \varphi_{42} gdp_t + \beta_{12} gds_{t-1} + \beta_{32} rir_t + \beta_{42} hfce_{t-1} + \varepsilon_{2t}$$

$$\varphi_{12} < 0 \; \varphi_{42} > 0 \; \beta_{12} < 0 \; \beta_{32} < 0 \; \beta_{42} > 0$$

$$gfce_t = \beta_{03} + \varphi_{42} gdp_t + \beta_{23} fdi_t + \beta_{53} gfce_{t-1} + \beta_{63} m_t + \varepsilon_{3t}$$

$$\varphi_{43} > 0 \; \beta_{23} > 0 \; \beta_{53} > 0 \; \beta_{63} > 0$$

$$gdp_t = gds_t + hfce_t + gfce_t$$

The fourth structural equation does not contain parameters; it is an identity.

Regarding to the number of variables used in the system as a whole, we have eleven variables, for this, there exists the (11 × 4) matrix $A = \begin{pmatrix} \Phi \\ \mathfrak{B} \end{pmatrix}$, where

$$A = \begin{pmatrix} -1 & \varphi_{12} & 0 & 1 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 1 \\ \varphi_{41} & \varphi_{42} & \varphi_{43} & -1 \\ \beta_{01} & \beta_{02} & \beta_{03} & 0 \\ \beta_{11} & \beta_{12} & 0 & 0 \\ \beta_{21} & 0 & \beta_{23} & 0 \\ \beta_{31} & \beta_{32} & 0 & 0 \\ \beta_{41} & \beta_{42} & 0 & 0 \\ 0 & 0 & \beta_{53} & 0 \\ 0 & 0 & \beta_{63} & 0 \end{pmatrix} = (\alpha_1 \alpha_2 \alpha_3 \alpha_4)$$

Where $\left[\alpha_i = \begin{pmatrix} \Phi_i \\ \mathfrak{B}_i \end{pmatrix} = \left(\varphi_{1i}, \ldots, \varphi_{Mi}, \beta_{1i}, \ldots, \beta_{Ki}\right)' \right]$. Φ_i is a (M × 1) of parameters associated to the endogenous variables present in the equation and $\mathfrak{B}_i$ **is** (K × 1) vector of parameters of the present exogenous variables and lagged endogenous variables figured in the equation.

Let's begin the diagnosis of the structural equations using approach proposed by Judge et al. [9] with many applications made by Mourad [2]:

First structural equation

Four variables do not exist: Two endogenous variables and two exogenous variables, thus there are four exclusive restrictions**:**

$$\Gamma_1 = \begin{pmatrix} 01000000000 \\ 00100000000 \\ 00000000010 \\ 00000000001 \end{pmatrix} \to (4 \times 11) \to \text{Rank}(\Gamma_1) = R_1 = 4$$

$$\Gamma_1 A = \begin{pmatrix} 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & \beta_{53} & 0 \\ 0 & 0 & \beta_{63} & 0 \end{pmatrix}$$

The rank of $\Gamma_1 A$ corresponds to the maximal number of linearly independent columns:

$$\mathring{A}_1 = \begin{pmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & \beta_{53} & 0 \end{pmatrix} \Rightarrow \left|\mathring{A}_1\right| = \beta_{53} \neq 0 \rightarrow \text{Rank}(\Gamma_1 A) = 3$$

Since R_1 > M-1 and Rank (Γ_1A) = M-1=3 the first structural equation is over identified.

1.1 **Second structural equation**

Four variables do not exist: One endogenous variable and three exogenous variables, thus there are four exclusive restrictions:

$$\Gamma_2 = \begin{pmatrix} 00100000000 \\ 00000010000 \\ 00000000010 \\ 00000000001 \end{pmatrix} \rightarrow (4\times 11) \rightarrow \text{Rank}(\Gamma_2) = R_2 = 4$$

$$\Gamma_2 A = \begin{pmatrix} 0 & 0 & -1 & 1 \\ \beta_{21} & 0 & \beta_{23} & 0 \\ 0 & 0 & \beta_{53} & 0 \\ 0 & 0 & \beta_{63} & 0 \end{pmatrix}$$

$$\mathring{A}_2 = \begin{pmatrix} 0 & -1 & 1 \\ \beta_{21} & \beta_{23} & 0 \\ 0 & \beta_{53} & 0 \end{pmatrix} \Rightarrow \left|\mathring{A}_2\right| = \beta_{21}\beta_{53} \neq 0\left(\beta_{21},\beta_{53} \neq 0\right) \rightarrow \text{Rank}(\Gamma_2 A) = 3a$$

R_2 > M-1 and Rank (Γ_2A) = M-1=3, this means that the second structural equation is over identified.

Third structural equation

Fifth variables do not exist: Two endogenous variables and three exogenous variables, thus there are fifth exclusive restrictions:

$$\Gamma_3 = \begin{pmatrix} 10000000000 \\ 01000000000 \\ 00000100000 \\ 00000001000 \\ 00000000100 \end{pmatrix} \rightarrow (5\times 11) \rightarrow \text{Rank}(\Gamma_3) = R_3 = 5$$

$$\Gamma_3 A = \begin{pmatrix} -1 & \varphi_{12} & 0 & 1 \\ 0 & -1 & 0 & 1 \\ \beta_{11} & \beta_{12} & 0 & 0 \\ \beta_{31} & \beta_{32} & 0 & 0 \\ \beta_{41} & \beta_{42} & 0 & 0 \end{pmatrix}$$

$$\mathring{A}_3 = \begin{pmatrix} 0 & -1 & 1 \\ \beta_{11} & \beta_{12} & 0 \\ \beta_{31} & \beta_{32} & 0 \end{pmatrix} \Rightarrow \left|\mathring{A}_3\right| = \beta_{11}\beta_{32} - \beta_{12}\beta_{31}$$

$\left(\left|\mathring{A}_3\right| \neq 0\right)$ if $\left(\frac{\beta_{11}}{\beta_{12}} \neq \frac{\beta_{31}}{\beta_{32}}\right)$. In this case, R_3 > M-1 and Rank (Γ_3A) = M-1=3 and by consequence the third structural equation is over identified.

Since the three structural equations above are over identified, they will be estimated using one of the following methods:

Two-Stage Least Squares (2SLS)

Historically, the first use of this method was done by Theil and Basmann. The (2SLS) method is a special case of the Generalized Least Squares (GLS) method for estimating the unknown parameters in a linear regression model [25]; it is used to estimate just or over identified structural equations. For more information, you can see studies as Judge et al. [9], Johnston [8], Greene [11] and López-Espín et al. [26] for a proposal algorithm to estimate the structural parameters. We mention also the Heteroscedastic Two-Stage Least Squares (H2SLS) method which is a modification of the traditional two-stage least squares used when the disturbances are heteroscedastic [27].

Limited Information Maximum Likelihood (LIML)

The (LIML) is considered the oldest method that began with Anderson and Rubin to assess the estimate one of the simultaneous equations system. It uses the instrumental variables technique and it belongs to the category (κ-class estimation). To appreciate the (LIML) estimator, it will be very useful to return to Pagan which has observed (LIML) as a seemingly unrelated regressions (SUR) allowing to consider it as a (2SLS) estimator with a correction factor. The (LIML) estimator has been performed by Johnston [8], see also, Davidson and MacKinnon [27], considering the Least Ratio Variance (LRV) to obtain $\hat{\kappa}$ and finally leading to an estimate of the structural equation parameters. For more details, Mourad [2]. Finally, we mention the following:

- When the structural equation is exactly identified, the estimators (LIML) and (2SLS) are both the same.
- The (LIML) estimator is less efficient than the (3SLS) estimator because it doesn't deal with all the information available in the system.
- An important advantage to the LIML estimator is its stability according to the normalization restriction of the structural equation, i.e. it is not affected by the choice under normalization, while it is not for the method (2SLS).
- When the instruments are only weakly correlated with the endogenous regressors, the LIML estimator performs better than the 2SLS and GMM estimators.
- For the over-identified equations, 2SLS and LIML are asymptotically identical.

Three-stage least squares (3SLS)

The three-stage least squares method developed by Zellner and Theil (ZT), using the two-stage least squares estimated moment matrix of the structural disturbances to estimate all coefficients of the entire system simultaneously at the condition the structural equations, are just or over identified. Srivastava and Tiwari generalized ZT's approach studying a large number of important and special cases confining their attention to the availability of the conditions providing the equality of the (2SLS) and (3SLS) estimators $\left(\hat{\delta}_{(2SLS)} = \hat{\delta}_{(3SLS)}\right)$. In this regard, the famous paper of Kapteyn and Fiebig [28] is mentioned to secure the necessary and sufficient conditions to equality the (2SLS) and (3SLS) estimators Mourad [2]. They concluded the special following cases (of course not alone):

1- The residuals in all structural equations are not correlated.

2- The all structural equations are exactly identified.

Note: In the full-information approach, we mention the Full-Information Maximum Likelihood (FIML). It is more sensitive to multicollinearity than limited information estimators.

Econometric Results and Their Implications

In order to simplify the calculation, only three estimation methods 2SLS, LIML and 3SLS will be used but focusing on the 3SLS method because all the structural equations are over identified and by consequence the entire system can be estimated simultaneously. For each estimate equation, the Ljung-Box Q (LBQ) statistic is used to test the null hypothesis that the first *m* autocorrelations are jointly zero. The value of m is chosen equal to ln(NOBS) where NOBS represents the usable observations, that is, twenty-six in our case. Finally, two tests will be used: The first is the J-test for over-identifying restrictions, and the second is the Lagrange multiplier test statistic testing that all IV's are uncorrelated with error terms *u*. Recall that two requirements must be satisfied to consider the validity of Instruments: The first is Instrument Exogeneity meaning that the valid instruments are uncorrelated with the error term, the second is Instrument Relevance which consists in considering that the valid instruments are highly correlated with the endogenous regressors. More clearly, the null hypothesis that all the instruments are likely uncorrelated with the error term will be tested versus the alternative which postulates that at least one of the instruments is likely uncorrelated with the error term.

$H_0: E(Zu_i) = 0$ Valid instruments

$H_a: E(Zu_i) \neq 0$ NO Valid instruments

The so-called J-test for over-identifying restrictions is proposed by Sergan [29], Hansen [30]. For each structural equation having p parameters, we save the residuals $\hat{u}_t$ associated to 2SLS or LIML estimators. Then we regress $\hat{u}_t$ on the all K instrument variables and we focus on the R^2. Under the null hypothesis that all IV's are uncorrelated with error terms *u*, a Lagrange multiplier statistic of the TR^2 form will not exceed the critical point on a $\chi^2_{0.05;(K-p)}$ distribution, where $(K - p)$ is the number of overidentifying restrictions (i.e. the number of excess instruments). RATS software uses the expression $J = \left(\frac{NOBS - p}{NOBS}\right)TR^2$ for 2SLS estimators and TR^2 for LIML estimator. If $J \leq \chi^2_{0.05;(K-p)}$ then we accept H_0.

To test the exogeneity of all instruments, using seemingly unrelated regressions (SUR), the estimates (2SLS) and (3SLS) will be obtained and the Hausman [31] test will be performed, see also Hausman and Taylor [32]. Under the null hypothesis, each of (2SLS) and (3SLS) is consistent but only (3SLS) estimators can be asymptotically efficient. Under the alternative hypothesis, the (2SLS) estimators are consistent but the (3SLS) estimators are inconsistent. When the simultaneous equations model is identified correctly, the (2SLS) and (3SLS) agree in consistency, in first equation per example, but (3SLS) will be more efficient than (2SLS). In this case, the difference $\left(\hat{\delta}_{2SLS} - \hat{\delta}_{3SLS}\right)$ would be very small. On the other hand, if one of the structural equations of the system is undiagnosed properly then the (3SLS) estimator will be inconsistent in all the structural equations including the first equation, while the (2SLS) estimators will remain consistent in the first equation and this is what will make the difference large between the two methods. Thus reject the null hypothesis means acceptance of the (2SLS) estimators but accept the null implies the acceptance of the (3SLS) estimators. Based on this information, it seems the importance of comparing the two estimators $\hat{\delta}_{2SLS}$ and $\hat{\delta}_{3SLS}$. The H_0 and H_a are the following:

$$H_0: \hat{\delta}_{2SLS}, \tilde{\delta}_{3SLS} \text{ consistent}, \tilde{\delta}_{3SLS} \text{ efficient}$$

$$H_a: \hat{\delta}_{2SLS} \text{ consistent}, \tilde{\delta}_{3SLS} \text{ inconsistent}$$

$$H = \left(\hat{\delta}_{2SLS} - \hat{\delta}_{3SLS}\right)' \left[V\left(\hat{\delta}_{2SLS}\right) - V\left(\hat{\delta}_{3SLS}\right)\right]^{-1} \left(\hat{\delta}_{2SLS} - \hat{\delta}_{3SLS}\right) \to \chi^2(k)$$

Where $V\left(\hat{\delta}_{2SLS}\right)$ and $V\left(\hat{\delta}_{3SLS}\right)$ are the variance–covariance matrices of coefficient vector using (2SLS) and (3SLS) respectively. When the Hausman test is performed, RATS software will drop any redundant restrictions and reduce the degrees of freedom appropriately.

$$H = \text{Chi} - \text{Squared}(5) = 7.58 < \chi^2_{0.05;5} = 11.07$$

F(5,21) = 1.51675 *p-value* = 0.1807

The decision promotes the (3SLS) method. For the finding of all estimators, see Appendix 2.

Note: If all equations are exactly identified then 2SLS = LIML = 3SLS, numerically.

Discussion and Conclusion

Let's comment now on the estimation of the system of simultaneous equations model according to the 3SLS method.

Findings of gds equation

Consider the estimate of the gds equation with 3SLS method. The coefficient of gds equation shows that gdp_t, gds_{t-1} and fdi_t have positive effect on gds_t while rir_t and $hfce_{t-1}$ have negative and significant effects on gross domestic savings per capita gds_t. These variables were statistically significant because the P values less than 0.05 except for the variable fdi_t, the p value was 0.085 leading an effect at 10% level of significance. The most significant positive influence on the gds variable comes from its past lagged one year. Indeed, a rise of one unit in gds_{t-1}, will cause an increase of 0.486 unit in gds_t, about the half of the increase in the previous year. An increase of one unit in gdp_t raises the level of gross domestic savings of 0.378 unit. To appreciate better the impact of gdp on gds in Singapore, it will be of great interest to measure the percentage of gds from gdp. It varies between a minimum of 41.88% and a maximum of 54.92% of real GDP per capita and it is dominated by a positive trend after the year 2001. The gds variable will be increased 0.193 unit, i.e. half of the effect of gdp, if an increase of one unit will take place in fdi_t. The linkage between savings and FDI has been highlighted by Chung et al. [33], Salahuddin et al. [34], Ilyas et al. [35], Aizenman [36], and supported the fact that FDI raises domestic savings. As noted above, the real interest rate has adverse effect on gross domestic saving per capita. If rir_t (resp. $hfce_{t-1}$) increases one unit then gds_t decreases about 119 unit (resp. 0.445 unit).

Findings of hfce equation

Inspecting the t-statistics, there are only three significant coefficients that are the $\hat{\beta}_{02}$, $\hat{\varphi}_{12}$ and $\hat{\varphi}_{42}$. The linkage between hfce and gdp is positive but the linkage between hfce and gds is negative. The gdp were beneficial for enhancing the household final consumption expenditure hfce in a Singapore, it has positive effect on hfce and having statistical significant value. more precisely, the estimate hfce equation suggests that rise of each unit in gdp leads to a rise of 0.493 unit in hfce. This result is justified by the importance of each of the three components of gdp, where hfce is the second biggest component after gds. In fact, the real household final consumption expenditure per capita covers all expenditure made by capita to meet its everyday needs, it varies in Singapore between a minimum of 35.52% and a maximum of 46.17% of real GDP per capita and it is dominated by a negative trend over the period 1991-2017, and therefore it is an essential variable for economic analysis of demand per capita for goods and services. The gds were not beneficial for enhancing the hfce, and this is reflected by a decrease in hfce of 0.386 unit for an increase of one unit in gds, noting that the same results are found in Shaikh et al. [37]. For the other variables, gds_{t-1}, rir_t and $hfce_{t-1}$ their effects were not found to be statistically significant, except perhaps a little positive significance for the variable $hfce_{t-1}$ is observed.

Findings of gfce equation

Two variables revealed their positive effects on gfce variable. There is an effect (with 10% level of significance) of fdi_t on $gfce_t$ making an increase of one unit in its present time value leads to an increase of 0.05 unit in $gfce_t$ while a very important influence on $gfce_t$ is identified in $gfce_{t-1}$ (t-statistic around 5.85). If $gfce_{t-1}$ increases one unit then $gfce_t$ increases by 0.76 unit. This reflects the presence of a strong lag-1 autocorrelation in the variable $gfce_t$. The other variables, gdp_t, m_t, both they do not have a statistically significant effect on gfce. Indeed, the ratio of gfce to gdp presents only an average of 9.94% with a minimum of 8.05% and a maximum of 11.96% over the period 1991-2017. The real money supply per capita (m_{1t}) did not have an impact on the real government final consumption expenditure per capita. Indeed, according to the standard macroeconomic theory, in the short run, an increase in the money supply leads to more consumption but it does not always lead to an increase in the total output and spending. Regarding the money supply variable, many studies as Karras [38], Adrison [39] and Sweidan and Al-Rabbaie [40], support the idea that a decrease in the money supply reduces the output more than the monetary expansion raises it.

Finally, to measure the performance of the estimated equations, the fitted values are calculated and a comparison with the observed values is performed. This is measured by the Mean Relative Error (MRE) expressed as a percent recalling that the (MRE) expresses how large the absolute errors are compared with the observed values we are measuring. The findings in Appendix 3 shown the MRE values 2.46%, 1.37%, 4.9% and 1.37% for the variables gds_t, $hfce_t$, $gfce_t$ and gdp_t respectively.

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Supplementary

Commodities with decrease ratio		1976	2017
Vegetables & Fruit	Commodity-1	1.02	0.1
Coffee, Tea, Cocoa, Spices & Manufactures	Commodity-2	2.34	0.27
Crude Rubber	Commodity-3	12.27	0.36
Petroleum & Products & Related Materials	Commodity-4	23.02	12.68
Oil Bunkers	Commodity-5	6.43	4.7
Fixed Vegetable Fats & Oils Crude Refined or Fractionated	Commodity-6	1.34	0.03
Textile, Yarn, Fabrics Made-up Articles not elsewhere specified & Related Products	Commodity-7	2.73	0.19
Iron & Steel	Commodity-8	1.15	0.49
Manufactures of Metals not elsewhere specified	Commodity-9	1.08	0.81
Telecommunications & Sound-recording & Reproducing Apparatus & Equipment	Commodity-10	3.73	3.53
Articles of Apparel & Clothing Accessories	Commodity-11	2.76	0.41
Photographic Apparatus Equipment & Supplies & Optical Goods not elsewhere specified; Watches & Clocks	Commodity-12	2.02	0.98
Commodities with increase ratio		1976	2017
Organic Chemicals	Commodity-13	0.482188	4.057338
Medicinal & Pharmaceutical Products	Commodity-14	1.19286	1.880451
Essential Oils & Resinoids & Perfume Materials; Toilet Polishing & Cleansing Preparations	Commodity-15	0.344427	1.898162
Plastics in Primary Forms	Commodity-16	0	3.361244
Office Machines & Automatic Data-processing Machines	Commodity-17	1.135759	5.108711
Electrical Machinery Apparatus & Appliances not elsewhere specified & Electrical Parts Thereof	Commodity-18	9.626602	28.25075
Professional Scientific & Controlling Instruments & Apparatus not elsewhere specified	Commodity-19	0.455758	3.425584
Miscellaneous Manufactured Articles not elsewhere specified	Commodity-20	1.486444	3.528083

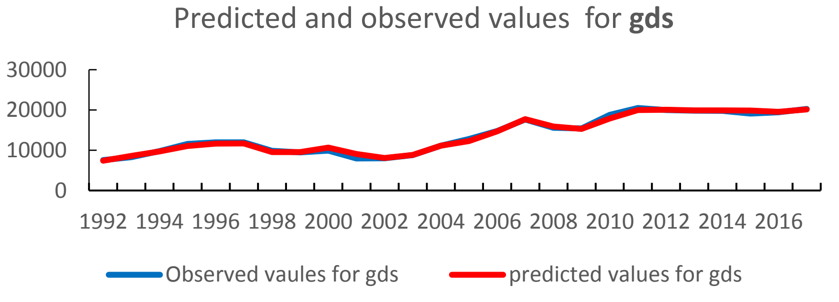
Supplementary 2

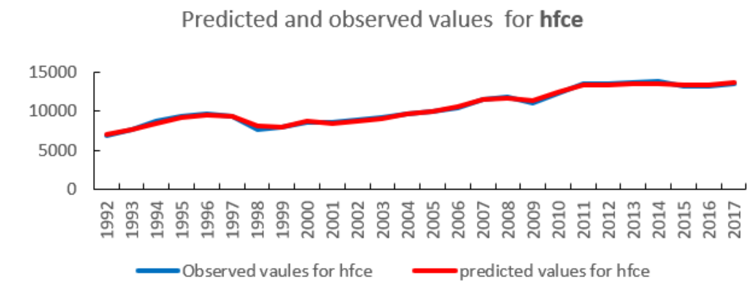
Two-Stage Least Squares (2SLS) Estimators				
gds equation				
Variable	Coefficient	Std. Error	t-Statistic	Prob
	1370.106	1288.208	1.064	0.300
	0.376	0.160	2.352	0.029
	0.489	0.179	2.735	0.013
	0.197	0.129	1.530	0.142
	-121.205	62.358	-1.944	0.066
	-0.449	0.201	-2.236	0.037
p-Value=0.341 LB(3)=7.2 J-specification(1)=0.906 86				
hfce equation				
Variable	Coefficient	Std. Error	t-Statistic	Prob
	1239.006	341.059	3.633	0.002
	-0.396	0.225	-1.760	0.094
	0.499	0.136	3.659	0.002

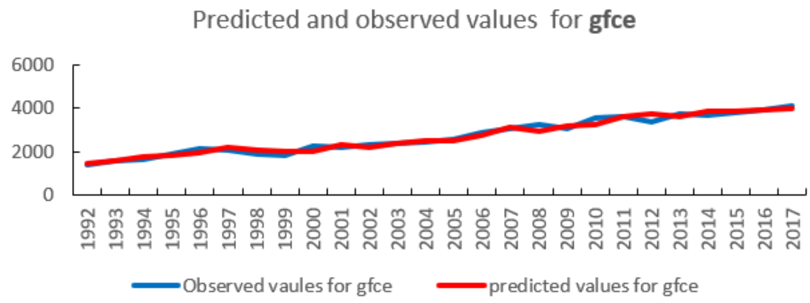
	-0.036	0.079	-0.452	0.656
	-7.578	25.135	-0.302	0.766
	0.171	0.143	1.194	0.246
p-Value=0.047 LB(3)=5.91 J-specification(1)=3.94 86				
gfce equation				
Variable	Coefficient	Std. Error	t-Statistic	Prob
	360.112	317.804	1.133	0.270
	0.001	0.026	0.026	0.979
	0.045	0.028	1.612	0.122
	0.748	0.145	5.164	0.000
	0.022	0.032	0.693	0.496
p-Value=0.676 LB(3)=2.54 J-specification(2)=0.78 86				
Heteroscedastic Two-Stage Least Squares (H2SLS) Estimators with 12 iterations				
gds equation				
Variable	Coefficient	Std. Error	t-Statistic	Prob
	1261.649	630.689	2.000	0.045
	0.383	0.092	4.172	0.000
	0.476	0.085	5.601	0.000
	0.193	0.066	2.908	0.004
	-93.245	42.811	-2.178	0.029
	-0.450	0.137	-3.277	0.001
J-specification(1)=1.475 p-Value=0.2245 LB(3)=8.15				
hfce equation				
Variable	Coefficient	Std. Error	t-Statistic	Prob
	1547.567	392.822	3.940	0.000
	-0.032	0.330	-0.096	0.924
	0.190	0.219	0.867	0.386
	-0.039	0.092	-0.422	0.673
	24.267	40.508	0.599	0.549
	0.462	0.197	2.342	0.019
p-Value=0.0723 LB(3)=3.86 J-specification(1)=3.229 86				
gfce equation				
Variable	Coefficient	Std. Error	t-Statistic	Prob
	307.937	289.260	1.065	0.287
	0.005	0.021	0.227	0.821
	0.040	0.031	1.295	0.195
	0.747	0.089	8.365	0.000
	0.019	0.026	0.735	0.463
p-Value=0.541 LB(3)=2.51 J-specification(2)=1.23 86				
LIML estimators				
gds equation				
Variable	**Coefficient**	**Std. Error**	**t-Statistic**	**Prob**
	1707.459	1526.656	1.118	0.277
	0.322	0.196	1.642	0.116
	0.542	0.215	2.519	0.020
	0.235	0.155	1.518	0.145
	-137.398	73.832	-1.861	0.078
	-0.419	0.229	-1.832	0.082
p-Value=0.2929 LB(3)=6.41 Chi-Sqaured(1)=1.106				
hfce equation				
Variable	**Coefficient**	**Std. Error**	**t-Statistic**	**Prob**
	1301.989	465.356	2.798	0.011
	-0.100	0.430	-0.232	0.819
	0.318	0.254	1.250	0.226
	-0.112	0.142	-0.790	0.439
	8.824	41.400	0.213	0.833
	0.336	0.249	1.349	0.192

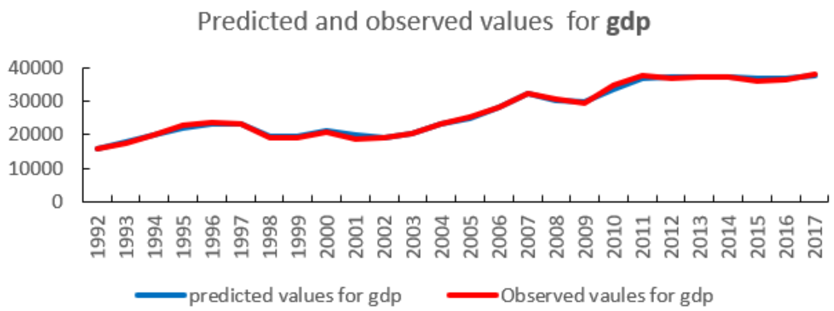
p-Value=0.0275 LB(3)=5.43 Chi-Sqaured(1)=4.86				
gfce equation				
Variable	**Coefficient**	**Std. Error**	**t-Statistic**	**Prob**
	384.398	324.924	1.183	0.250
	-0.002	0.027	-0.062	0.951
	0.046	0.028	1.639	0.116
	0.755	0.147	5.136	0.000
	0.024	0.033	0.742	0.466
Chi-Sqaured(2)=1.00 p-Value=0.6076 LB(3)=2.51				
3SLS estimators (Iterations Taken= 14)				
gds equation				
Variable	**Coefficient**	**Std. Error**	**t-Statistic**	**Prob**
	1323.333	1123.998	1.177	0.239
	0.378	0.139	2.714	0.007
	0.486	0.156	3.116	0.002
	0.193	0.112	1.721	0.085
	-119.019	54.408	-2.188	0.029
	-0.445	0.175	-2.546	0.011
LB(3)=7.23 $\leq \chi^2_{0.05;(K-p)}$ =7.82				
hfce equation				
Variable	**Coefficient**	**Std. Error**	**t-Statistic**	**Prob**
	1240.05	301.3959	4.11435	3.88E-05
	-0.38633	0.19857	-1.94559	0.051704
	0.49338	0.12043	4.0969	4.19E-05
	-0.03825	0.06966	-0.54905	0.582971
	-6.96461	22.20718	-0.31362	0.75381
	0.17614	0.12654	1.39189	0.163955
LB(3)=5.88 $\leq \chi^2_{0.05;(K-p)}$ =7.82				
gfce equation				
Variable	**Coefficient**	**Std. Error**	**t-Statistic**	**Prob**
	347.1759	285.5822	1.21568	0.224108
	6.23E-04	0.02355	0.02645	0.978899
	0.04464	0.02495	1.7891	0.073598
	0.75981	0.12997	5.84583	1E-08
	0.02029	0.0288	0.70462	0.481048
LB(3)=2.70 $\leq \chi^2_{0.05;(K-p)}$ =7.82				
Chi-Sqaured(4)=7.60 p-Value=0.1073				

Supplementary 3




Predicted and observed values for hfce
15000
10000
5000
0
1992
1993
1994
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
2008
2009
2010
2011
2012
2013
2014
2015
2016
2017
Observed vaules for hfce
predicted values for hfce


Predicted and observed values for gfce
6000
4000
2000
0
1992
1993
1994
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
2008
2009
2010
2011
2012
2013
2014
2015
2016
2017
Observed vaules for gfce
predicted values for gfce


Predicted and observed values for gdp
40000
30000
20000
10000
0
1992
1993
1994
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
2008
2009
2010
2011
2012
2013
2014
2015
2016
2017
predicted values for gdp
Observed vaules for gdp