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Divisor Graphs that are Complements of Bipartite Graphs

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Abstract

In this paper, we will study some bipartite graphs whose complements are divisor graphs like paths and caterpillars, counterexample for a tree whose complement is not divisor will be presented and powers of some types of trees that have their complements divisor graphs will be classified.

Keywords: Divisor graph; Complement of a graph; Bipartite graph; Path; Caterpillar power of a graph and tree

Introduction

A finite graph G is called divisor graph if there is a finite set of positive integers $\{x_1,x_2,...,x_n\}$ such that $V(G)=\{x_1,x_2,...,x_n\}$ and $\{x_i,x_j\}\in E(G)$ if x_i/x_j or x_j/x_i.

It is known that bipartite graphs are divisor graphs but what about their complements? Are they all divisor graphs? The answer is no. we will study some special cases of bipartite graphs that have their complements divisor graphs like paths and caterpillars, while other types like some trees are not.

We know that every tree is a divisor graph [1]. The question that arises here, is the complement of a tree is also a divisor graph?

Characterization of block graphs that are divisor graphs are given [1].

Definitions in terms of transmitter, receiver and transitive vertices of a divisor orientation of a graph G are given [2].

The characterization of powers of paths and powers of cycles which are divisor graphs was given [2-4]. While, a characterization of nontrivial connected divisor graphs in terms of the upper orientable hull number was obtained [5].

It was shown that no divisor graph contains an induced odd cycle of length greater than 3. Also, it was proved that every induced subgraph of a divisor graph is a divisor graph [6-8].

Complete graphs, bipartite graphs, complete multipartite graphs, and joins of divisor graphs are divisor graphs.

The length of a longest path [9-15]. While divisor graphs with triangles [16], where a forbidden subgraph characterization for all divisor graphs containing at most 3 triangles was obtained.

Lemma

The complement of a path is a divisor graph (Figure 1).

Proof

Consider the path $P_{n=}\{v_1,v_2,...,v_n\}$ with $\{v_k,v_{k+1}\}\in E(P_n)$ for $k=1,2,...,n-1$.Let

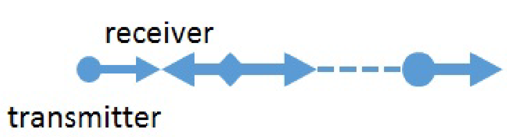

Figure 1: A complement divisor graph.

$\{p_1,p_2,...,p_n\}$ be a set of distinct prime numbers. Label v_1 by p_1,

v_2 by p_2 and

v_k by $p_1p_2p_3p_4\cdots p_{k-2}p_k$, $k=1,2,...,n$.

This recurrence relation gives an orientation for $\overline{P}_n$, the complement of P_n (Figure 2).

Example

C_n: For odd n ≥ 5 is not a divisor graph.

Example

K_n: is a divisor graph (any complete graph is a divisor graph).

Example

Every bipartite graph is a divisor graph.

Lemma

The complement of a caterpillar is a divisor graph.

Proof

$P_{n=}\{v_1,v_2,...,v_n\}$ with $\{v_k,v_{k+1}\}\in E(P_n)$ for $k=1,2,...,n-1$.

Assume that the vertex v_k is adjacent to the vertices $\{v_{k1},v_{k2},...,v_{kj}\}$ that are not adjacent to any other vertex in P_n. Then the new graph G is a caterpillar with $n+j$ vertices.

Label v_1 by p_1, v_2 byp_2 and v_k by $p_1p_2p_3\cdots p_{k-2}p_k$, $k=1,2,...,n$.

Label v_k1 by $p_1\,p_2\,p_3\cdots p_{k-2}\,p_{k-1}$, v_{k2} by v_{k2} by $p_1\,p_2\,p_3\,p_4\,p_{k-2}\,p_{k-1}\,p_k$, v_{k3} by $p_1\,p_2\,p_3\cdots p_{k-2}\,p_{k-1}\,p_k\,p_{k+1}$,

... and v_{kj} by $p_1\,p_2\,p_3\cdots p_{k-2}\,p_{k-1}\,p_k\,p_{k+1}\cdots p_{k+j-2}$.

And label the rest as follows:

v_{k+1} by $p_1\,p_2\,p_3\cdots p_{k-2}\,p_{k-1}\,p_{k+1}\cdots p_{k+j-1}$

v_{k+2} by $p_1\,p_2\,p_3\cdots p_{k-1}\,p_k\,p_{k+2}\cdots p_{k+j-1}\,p_{k+j}$

And so on till

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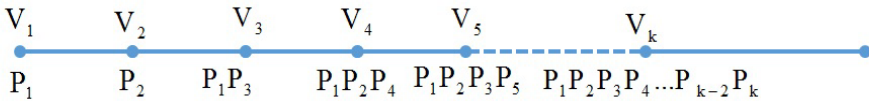

Figure 2: Recurrence Relation orientation for $\overline{P}_n$.

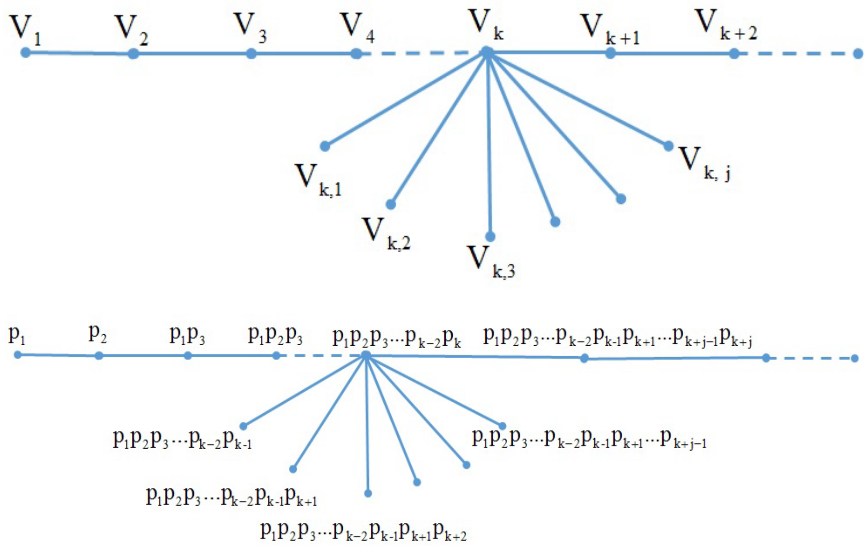

Figure 3: Recurrence orientation for $\overline{G}$.

v_n by $p_1 p_2 p_3 \cdots p_{k-1} p_k p_{k+1} \cdots p_{k+j-1} p_{k+j} p_{n-2} p_n$.

This recurrence relation gives an orientation for $\overline{G}$, the complement of G (Figure 3).

Example

Not all the trees are divisor graphs, for this purpose, consider the following tree:

This is not a divisor graph (Figure 4).

Theorem

The complement of a tree is a divisor graph only if the tree is a caterpillar.

Proof

Obvious from the above results.

The question that arises now is, are the complements of the powers of the graphs studied above also divisor graphs?

The answer is yes, which we will prove it using the following Lemma.

Lemma

The complement of the power graph of a path is a divisor graph. (For any power less than the degree of a path).

Proof

Consider the path $P_{n=}\{v_1,v_2,\ldots,v_n\}$ with $\{v_k,v_{k+1}\}\in E(P_n)$ for $k=1,2,\ldots,n-1$. Label v_1 by

p_1,

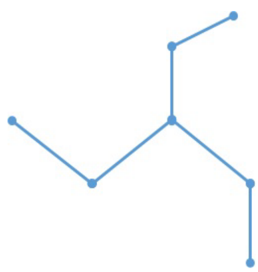

Figure 4: Trees are not divisor graphs.

v_2 by p_1, ... and

v_j by p_j. Then label v_{j+k} by $p_1 p_2 \cdots p_k p_{j+k}$, Finally, label v_n by $p_1 p_2 \cdots p_{n-j}$.

This recurrence relation gives an orientation for $(\overline{P}_n)^j$, the complement of $(P_n)^j$.

Lemma

The complement of the power graph of a caterpillar is a divisor graph.

Theorem

The complement of the power graph of a tree is a divisor graph only if the tree is a caterpillar.

Proof

Obvious from the above results.

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