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Distribution of Inventory Level on a Repairable Parts System under Performance-Based Contract

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Abstract

Mirzahosseinian develop a closed-loop inventory system consisting of a repair facility and a single warehouse, for a repairable parts system operating under a Performance-Based Logistics contract. They model the closed-loop inventory system as a multi-server queueing model (M/M/m queue) where component failures follow Poisson distribution and the repair times at the service facility are exponential. The model sets up the balance equations of the steady-state probability distribution of inventory level. This paper extends their model. A recursive method is utilized and an analytic steady-state probability distribution of inventory level is derived. It also demonstrates by numerical experiments that the analytic steady-state probability distribution is correct. We obtain the analytic system performances and the metrics, which contribute to system cost optimization.

Keywords: Inventory; Performance; Steady-state probability distribution; Recursive method

Introduction

The after-sales parts and services have become an important source of revenue for original equipment manufacturer (OEM) in capital-intensive industries, such as aerospace, defense, and industrial equipment. Performance-Based Logistics (PBL) contracting is replacing traditional service procurement practices. Recently, there has been an increased interest in the repairable spare parts inventory system under PBL, where maintenance and servicing is not paid according to the spare parts used, repairs or activities, but paid by how many hours the customer obtains power from the inventory system. PBL contracts have been used widely in both commercial and military sector, and quite a few articles have studied the design and implementation of PBL contract [1-5].

Recently, Mirzahosseinian et al. [1] develop a closed-loop inventory system consisting of a repair facility and a single warehouse, for a repairable parts system operating under a PBL contract. They model the closed-loop inventory system as a multi-server queueing model (M/M/m queue), where component failures and repair times follow Poisson and exponential distributions, respectively. The model provides the supplier and the customer increased flexibility to achieve target availability. They set up the balance equations of the steady-state probability distribution of inventory level. However, they do not derive an analytic steady-state probability distribution of the inventory level at the warehouse. The analytic steady-state probability distribution is required to characterize the long-term behavior of the inventory system in the warehouse and to derive the analytic system performance and metrics. Moreover, they do not provide the numerical algorithm for obtaining the system performance and the metrics (Mean Time between Failures, MTBF; Mean Time to Replace, MTTRe; Average Number of Backorders, EB; Availability), which are the base of their numerical study in section 6 and parametric analysis in section 7. This paper extends their model, aiming to derive the analytic steady-state probability distribution of inventory level and, obtain the analytic system performance and the metrics.

The rest of the paper is organized as follows. Section 2 describes the model of Mirzahosseinian et al. [1]. Section 3 presents a further work on their equations and analytic steady-state probability distribution of inventory level. Numerical experiments are provided in Section 4. Conclusions are given in the last Section.

Extended Model

Following the notations and assumptions from Mirzahosseinian et al. [1], the closed-loop inventory system consisting of a repair facility and a single warehouse is described as follows:

1) There are N independent and identical systems. Each system has a repairable identical component for support service. Suppose that each component failure follows a Poisson process with constant rate λ.

2) The failure of the N components is independent. A one-for-one base stock S replenishment policy is followed at the warehouse. When a component is failure, it will be replaced immediately by the ready-to-use one from the warehouse, and sent to repair facility for recovery. But, if there are no ready-to-use parts in the warehouse, the system will remain breaking down.

3) The failed components will be repaired as a new one. The repair facility is model as an M/M/m queueing model. There are m servers and N customers in the closed-loop repairable part support system. The failed parts arrive at the servers following Poisson process with variable rate $\lambda(z)$, where z is the number of operational systems. Repair time for each failed component follows negative exponential distribution with constant rateμ. The time between replacement of components from repair facility to the warehouse follows Exponential distribution with the variable rate $\mu(y,m)$, where y is the number of components at the repair facility. The variable failure rate $\lambda(z)$ and the variable repair rate $\mu(y,m)$can be calculated as follows, respectively:

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$$\lambda(z)=\begin{cases}(N+x)\lambda, -N+1\le x\le -1\\ N\lambda, 0\le x\le S\end{cases}, \quad (1)$$

$$\mu(y,m)=\begin{cases}mu, -N\le x\le S-m\\ (S-x)\mu, S-m+1\le x\le S-1.\\ 0, x=S\end{cases} \quad (2)$$

Mirzahosseinian et al. [1] establish a queueing model and set up the balance equations of the steady-state probability distribution of inventory level and carried out a numerical study and parametric analysis which was then coded in MATLAB software. This paper proposes a recursive method to derive the analytic steady-state probability distribution of inventory level at the warehouse. The recursive method has some advantages over the method proposed by Mirzahosseinian et al. [1]. The results derived by the recursive method can be used to compute the steady-state performance and the metrics easily, the accuracy can also be g=uaranteed.

The analytic solutions

As in Mirzhosseinian et al. [1], let $\pi_x(x=-N,\cdots 0,\cdots,S)$ be the steady-state probability distribution of inventory level (x) at the warehouse. The balance equations in Mirzhosseinian et al. [1] are laid out as follows:

$\pi_x\,(z\lambda+[Min(y,m)])=(D)\times\pi_{x-1}(Min(y+1,m)\mu)+(F)\times\pi_{x-1}(Min(z+1,N)\lambda)$, for $-N\le x\le S$,

$$D=\begin{cases}1 \; if\; -N+1\le x\le S\\ 0\; otherwise\end{cases}, \qquad F=\begin{cases}1\; if\; -N\le x\le S-1\\ 0\; otherwise\end{cases}.$$

Where, D and F are binary variables. The balance equations can be formulated by the fact that the average rate of transition out of a state is equal to transition into a state for a Markov process. The left side of the equation represents the transition out of state x, and the right side of the equation represents the transition into state x. The above equation can be extended as following (3-6).

$$[Min(S+N, m)]\mu\pi_{-N}=\lambda\pi_{-N+1}, \quad (3)$$

$$\{(x+N)\lambda+[Min(S-x, m]\mu\}\pi_x=(x+N+1)\lambda\pi_{x+1}+[Min(S-x+1,m)]\mu\pi_{x1,} N+1\le x\le -1 \quad (4)$$

$$\{N\lambda+[Min(S-x, m]\mu\}\pi_x=N\lambda\pi_{x+1}+[Min(S-x+1, m)]\mu\pi_{x1,} 0\le x\le S-1, \quad (5)$$

$$N\lambda\pi_s=[Min(1, m)]\mu\pi_{s-1}. \quad (6)$$

For example, if the inventory level state x lines within the range $-N+1\le x\le -1$, then the equation is presented in (4). There are only two possible ways of reaching state x. When x is within this range, there are no ready-to-use parts in the warehouse, and then transition into this state can be only due to either a component fails or the replenishment components arrive at the warehouse, which is presented on the right-hand side of (4). The above equations (3-6) characterize the long-term behavior of the inventory system at the warehouse. This paper proposes a recursive method for deriving the analytic steady-state probability distribution of inventory level from equations (3-6).

Obviously, the number of server should be less than the total number of system components. That's to say $1\le m\le S+N$ and $-N\le S-m\le S-1$.

(1) When $x\le S-m$, we get $m\le S-x$ and $Min[m, (S-x)]=m$.

(2) When $x>S-m$, we get $m>S-x$ and $Min[m, (S-x)]=S-x$.

Now, we consider the following occasions.

(1) When $S-m\ge 0$, the corresponding transition diagram of the inventory system can be shown in Figure 1.

We can rewrite the balance equations (3-6) as equations (7-11).

$$m\mu\pi_{-N}=\lambda\pi_{-N+1}, \quad (7)$$

$$\{(x+N)\lambda+m\mu\}\pi_x=(x+N+1)\lambda\pi_{x+1}+m\mu\pi_{x-1}, -N+1\le x\le -1, \quad (8)$$

$$[N\lambda+m\mu]\pi_x=N\lambda\pi_{x+1}+m\mu\pi_{x-1}, 0\le x\le S-m, \quad (9)$$

$$[N\lambda+(S-x)\mu]\pi_x=N\lambda\pi_{x+1}+(S-x+1)\mu\pi_{x-1}, S-m+1\le x\le S-1, \quad (10)$$

$$N\lambda\pi_s=\mu_{s-1}. \quad (11)$$

From equation (7), we get

$$\pi_{-N+1}=\frac{m}{\rho}\pi_{-N}, \text{ Where } \rho=\lambda/\mu. \quad (12)$$

From equation (8), we apply recursive method and get

$$\pi_x==\frac{(\frac{m}{\rho})^{N+x}}{(N+x)!}\pi_{-N}, -N+1\le x\le 0, \quad (13)$$

Likewise, we obtain (14-15) from (9-11).

$$\pi_x=\frac{(\frac{m}{\rho})^{N+x}}{N!N^x}\pi_{-N}, 1\le x\le S-m+1, \quad (14)$$

$$\pi_x=\frac{m!m^{N+S-m}}{(S-x)!N!N^x\rho^{N+x}}\pi_{-N}, S-m+2\le x\le S. \quad (15)$$

Now, we insert (13-14) into the normalizing constraint $\sum_{x=-N}^{S}\pi_x=1$, and get

$$\pi_{-N}=\left[1+\sum_{x=-N+1}^{0}\frac{(\frac{m}{\rho})^{x+N}}{(x+N)!}+\sum_{x=1}^{S-m+1}\frac{(\frac{m}{\rho})^{x+N}}{N!N^x}+\sum_{x=S-m+2}^{S}\frac{m!m^{N+S-m}}{(S-x)!N!N^x\rho^{x+N}}\right]^{-1}. \quad (16)$$

(2) When $-N\le S-m<0$, the transition diagram of the inventory system can be shown in Figure 2.

The balance equations (3-6) can be written as equations (17-21).

$$m\mu\pi_{-N}=\lambda\pi_{-N+1}, \quad (17)$$

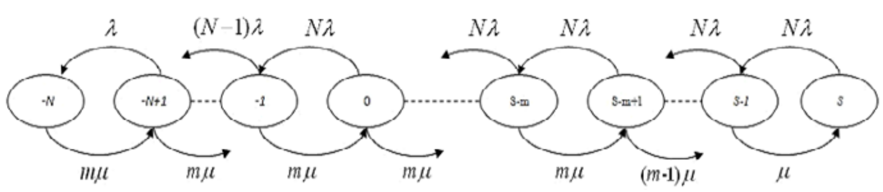

Figure 1: The transition diagram of the inventory system.

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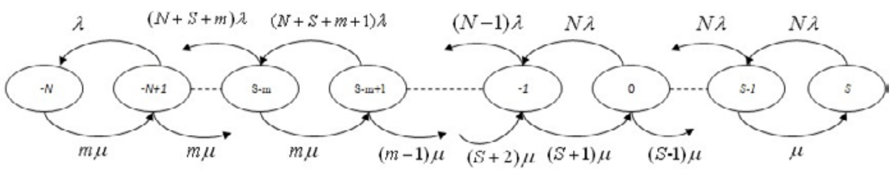

Figure 2: The transition diagram of the inventory system.

$$\{(N+x)\lambda + m\mu\}\pi_x = (x+N+1)\lambda\pi_{x+1} + m\mu\pi_{x-1},\ -N+1 \le x \le S-m, \quad (18)$$

$$[(N+x)\lambda + (S-x)\mu]\pi_x = (N+x+1)\lambda\pi_{x+1} + (S-x+1)\mu\pi_{x-1},\ S-m+1 \le x \le -1, \quad (19)$$

$$[N\lambda + (S-x)\mu]\pi_x = N\lambda\pi_{x+1} + (S-x+1)\mu\pi_{x-1},\ 0 \le x \le S-1, \quad (20)$$

$$N\lambda\pi_s = \mu\pi_{s-1}. \quad (21)$$

The steady-state probability distribution of inventory level are as follows:

$$\pi_{-N} = \left[1 + \sum_{x=-N+1}^{S-m+1} \frac{(\frac{m}{\rho})^{x+N}}{(x+N)!} + \sum_{x=S-m+2}^{0} \frac{m!m^{N+S-m}}{(N+x)!(S-x)!\rho^{N+x}} + \sum_{x=1}^{S} \frac{m!m^{N+S-m}}{(S-x)!N!N^x\rho^{N+x}}\right]^{-1}, \quad (22)$$

$$\pi_x = \frac{(\frac{m}{\rho})^{N+x}}{(N+x)!}\pi_{-N},\ -N+1 \le x \le S-m+1, \quad (23)$$

$$\pi_x = \frac{m!m^{N+S-m}}{(N+x)!(S-x)!\rho^{N+x}}\pi_{-N},\ S-m+2 \le x \le 0, \quad (24)$$

$$\pi_x = \frac{m!m^{N+S-m}}{(S-x)!N!N^x\rho^{N+x}}\pi_{-N},\ 1 \le x \le S. \quad (25)$$

So far, we have derived the analytic steady-state probability distribution of inventory level at the warehouse which is essential for calculating the inventory cost. When $S - m \ge 0$, the analytic steady-state probability distribution is presented by equations (13-16). When $-N \le S - m < 0$, the analytic steady-state probability distribution is presented by equations (22-25). Steady-state probability distribution can be of use to easily yield interesting steady-state system performance measures, such as average inventory level, customer service level, and out-of-stock probability. All the steady-state performance and metrics (MTBF, MTTRe, EB, Availability) in Mirzahosseinian et al. [1] can then be obtained based on the analytic steady-state probability distribution of inventory level. Based on the explicit or implicit relationships between the steady-state distribution and the system control parameters, analytical or numerical methods can then be developed to optimize the corresponding system control parameters.

Numerical Illustration

The basic parameters of the system in Table 1 are adopted from Mirzahosseinian et al. [1]. The critical components of engine, propeller, avionics computer are used to support 40 air vehicles for one year. Cases 1, 2 and 3 refer to engine, propeller and avionics computer respectively. For example, when an engine fails, it incurs 123h of repair time on average. The mean time between failures of an engine is expected to be 750h. We use the basic parameters in Table 1 to check our analytic steady-state probability distribution of inventory level.

Table 2 illustrates the computational results of the above three cases. We sum all $\pi_x, (x=-N,\cdots,0,\cdots S)$ at the bottom of Table 2. As we can see, the sum of the analytic steady-state probability distribution in each case is equal to 1 and the iterative method proposed in this note is the first try-out and success.

Case	λ/year	$\mu^{-1}h$	S	N
1	1.92	123	6	40
2	2.88	89	8	40
3	1.44	135	4	40

Table 1: The basic parameters of inventory system.

The star (*) in Table 2 indicates the corresponding number is no more than 0.0001.

As shown in Figur 3, the steady-state probability distributions of the the three cases take on different features.

(1) In Cases 1 and Case 2, the steady-state probability distribution increases to the maxmum and then decreases. While in Case 3, the steady-state probability distribution always increases.

(2) In Case 1, the probability of backorders is 0.6755. The most likely number of backorders is 3. We also find that the sum of the backorders probability from 10 to 40 is 0.1408. In Case 2, the sum of the backorders probability from 10 to 40 is 0.2638. This implies that, the probablity of serious backorders for Case 1 is less than Case 2. In Case 3, the sum of the backorders probability from 10 to 40 is olny 0.0223.

For furthermore compared with Mirzahosseinian et al. [1], we also compute the system performance and metrics (MTBF, MTTRe, Availability) by the analytic steady-state probability distribution of inventory level in Table 3. The results are the same as shown in Table 2 in Mirzahosseinian et al. [1]. The performances and metrics (average number of backorders, EB; the average inventory, EI; the average number of failed component, EF) of other three systems are the first time calculated as shown in Table 3. Among the performances and metrics of three systems, the average number of failed component is the biggest one and the average inventory is the smallest one of the three cases. For example, the average inventory is 0.88 and the average number of failed component is 9.39 in Case 1.

According to the steady-state performance and metrics, the total cost will be set up. Optimizing the total cost with availability constrained in order to obtain the optimal server repair rate, the base stock and the number of servers are an interesting area to study. The optimizing model can be solved by a real coded genetic algorithm named MI-LXPM which is introduced by Deep et al. [6]. The numerical experiments can be easily performed on MTLAB R2013a, which we ignore in this note.

Conclusions

Mirzahosseinian et al. [1] model a closed-loop inventory system as an M/M/m queue, and analyze determinate factors that have significant impact on the system availability. The numerical finding is

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π_x	*Case 1*	*Case 2*	*Case 3*
π_{-40}	*	*	*
π_{-39}	*	*	*
π_{-38}	*	*	*
π_{-37}	*	*	*
π_{-36}	*	*	*
π_{-35}	*	*	*
π_{-34}	*	*	*
π_{-33}	*	*	*
π_{-32}	*	*	*
π_{-31}	*	*	*
π_{-30}	*	*	*
π_{-29}	*	*	*
π_{-28}	*	*	*
π_{-27}	*	*	*
π_{-26}	*	*	*
π_{-25}	*	0.0001	*
π_{-24}	*	0.0002	*
π_{-23}	0.0001	0.0005	*
π_{-22}	0.0002	0.0009	*
π_{-21}	0.0004	0.0016	*
π_{-20}	0.0008	0.0027	*
$\pi_{-1}9$	0.0014	0.0044	0.0001
π_{-18}	0.0024	0.0069	0.0001
π_{-17}	0.0038	0.0102	0.0002
π_{-16}	0.0059	0.0146	0.0004
π_{-15}	0.0087	0.0199	0.0008
π_{-14}	0.0125	0.0262	0.0013
π_{-13}	0.0171	0.0331	0.0022
π_{-12}	0.0227	0.0405	0.0035
π_{-11}	0.0290	0.0477	0.0055
π_{-10}	0.0358	0.0543	0.0082
π_{-9}	0.0429	0.0599	0.0120
π_{-8}	0.0497	0.0639	0.0168
π_{-7}	0.0559	0.0662	0.0230
π_{-6}	0.0610	**0.0666**	0.0305
π_{-5}	0.0646	0.0650	0.0392
π_{-4}	0.0666	0.0617	0.0491
π_{-3}	**0.0668**	0.0570	0.0598
π_{-2}	0.0652	0.0513	0.0709
π_{-1}	0.0620	0.0449	0.0820
π_{0}	0.0575	0.0384	0.0923
π_{1}	0.0533	0.0328	0.1040
π_{2}	0.0494	0.0280	0.1172
π_{3}	0.0458	0.0239	0.1320
π_{4}	0.0425	0.0205	**0.1487**
π_{5}	0.0394	0.0175	——
π_{6}	0.0366	0.0149	——
π_{7}	——	0.0128	——
π_{8}	——	0.0109	——
Sum(π)	1	1	1

Table 2: The steady-state probability distribution of inventory level.

very interesting but it is not built on the optimal system operating state, because they do not derive analytic steady-state probability distribution distributions of inventory level [7-9].

This paper for the first time proposes an analytic recursive method to obtain the steady-state probability distribution distributions of inventory level in Mirzahosseinian et al. [1]. In reality, the inventory system will be operated at the optimal state. Optimizing the cost with availability constrained for the repairable inventory system in order to

Case	*MTBF*	*MTTRe*	*Availability*	*EB*	*EI*	*EF*
1	130.00	118.50	0.89	4.26	0.88	9.39
2	92.30	88.10	0.84	6.20	0.60	13.60
3	159.50	114.90	0.96	1.64	1.33	4.31

Table 3: System performance and metrics.

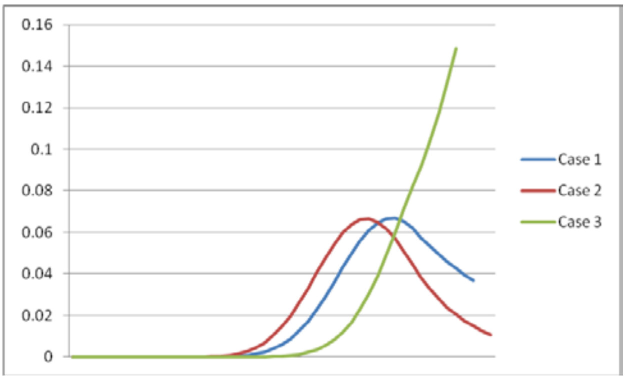

Figure 3: The probability distribution.

find the optimal failure rate, server repair rate and the number of servers will be important. The analytic steady-state probability distribution distributions will help to derive the analytic system performance and the metrics, such as the Mean time between failures (MTBF) and the Mean time to replace (MTTRe). The control polices for the analytic system performance and the metrics will invaluably contribute to future research on inventory system optimizing and designing.

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