

Distance Constrained Location Problems

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Abstract

Organizations are subject to decisions regarding the location of facilities and the required minimum number of the facilities to satisfy certain needs. In this regard, p-median and set covering problems are discussed in a way that they are not adequate under some real-world scenarios. Hence, this paper introduces an optimization model for providing the required number of facilities and their locations to serve different sets of nodes (i.e., departments) within a specified distance range. A deterministic integer programming model is developed and implemented on a real-world problem.

Keywords: Facility location; Optimization; P-median; Set-covering

Introduction

Facility location problems have attracted the attention of researchers resulting in hundreds of papers [1-5]. Many studies have been done on the subject of choosing facility or distribution center locations to provide a maximum coverage of service to a group of people or customers [6-9]. Aside from the need to decide on the location of a facility or distribution center, the decision of which customers will be served by the facility's coverage area (an allocation problem), is also a common problem in the public and private sectors [7].

Furthermore, facility location problems exist in the logistics, manufacturing, and healthcare industries, among others. Other location problems include placing distribution centers where they will be able to serve most of the customers. Also, for healthcare organizations, in the event of large-scale emergency situations, it is important to know how many facilities or equipment (i.e., ambulances) are needed and where to locate them [2,5,10].

In some cases, a model is required to decide on the number of facilities needed to serve the nodes within a specified distance. In other words, a model employing distance constraints to locate facilities within a specified distance, while minimizing transportation costs might be necessary.

The proposed model tries to locate the facilities near high-demand areas within a specified distance with the objective of minimizing the number of facilities and the total travel distance. The paper is organized as sections of literature review, methodology, application and results, and conclusion.

Literature Review

Facility location is an important decision problem evidenced by many studies and models introduced over the past few years. Such as the paper was published regarding reliable p-median facility location network two-stage [1]. Also, other paper was published about maximizing expected coverage location [9]. Many of these studies and models originate from an economic standpoint, where it is important to determine the number of possible facilities and their locations. The fixed cost of establishing the facility and the variable cost of shipment of items between the facility and the customer have an inverse relationship. For example, having many facilities could reduce the cost of shipments to the costumers but will increase the fixed cost of establishing new facilities or vice versa.

This section will discuss some basic common models used to

determine the locations of facilities. Where those models are well known in felid of location models and considered to be less complex compared with other location models [6]. Moreover, the deterministic case will be considered to assuming the data is known, and the one stage situation will be considered where a company provides service directly from their facilities to customers, without an intermediary between them.

P-median problem

The P-median problem is one of the most famous in the facility location domain. The model is applicable to many facility location problems under the assumption that the number of facilities P is known. The second assumption is that the nodes should have weights or demands and the facility will be among these nodes. The third assumption is that the customers should use the shortest path when travelling between those nodes or facilities [8].

The first P-median formulation formulated as an integer programming problem was by ReVelle and Swain (1970) [8].

We consider the formulation of the problem stated in [8]. A set of $i=1,2,\dots,m$ nodes (customers), and a set of $j=1,2,\dots,n$ potential facilities sites are defined. Two types of binary variables are introduced:

$$X_{ij} = \begin{cases} 1, & \text{if demand of node } i \text{ is assigned to facility } j \\ 0, & \text{otherwise} \end{cases}$$

$$Y_j = \begin{cases} 1, & \text{if a facility is located in candidate site } j \\ 0, & \text{otherwise} \end{cases}$$

In addition, the following parameters are given:

D_i = demand associated with each node i

d_{ij} = distance between node i and facility j

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p = total number of facilities to be located

$$\text{Objective:} \quad \text{Min} \quad \sum_j^m \sum_i^n D_i d_{ij} X_{ij} \quad (1)$$

$$\text{Subject to:} \quad \sum_j^m X_{ij} = 1 \quad \text{for all } i \quad (2)$$

$$X_{ij} \leq Y_j \quad \text{for all } i, j \quad (3)$$

$$\sum_j^n Y_j = p \quad (4)$$

$$X_{ij} \in \{0,1\} \quad \text{for all } i, j \quad (5)$$

$$Y_j \in \{0,1\} \quad \text{for all } j \quad (6)$$

The objective function (1) is to minimize the travel distance between node i and facility j with consideration for the total demand from the demand of the nodes (customers). Constraint (2) makes sure that each node is assigned to only one facility. Constraint (3) ensures that nodes are only assigned to working facilities. Constraint (4) indicates that the sum of facilities should be equal to P . Constraints (5) and (6) ensure that the variables X_{ij} and Y_j are binary.

Set covering problem

Set covering problem is used usually in cases that requires providing certain type of service to a potential group of people or customers. Where, a certain set of points needs to be covered with some certain point or points [3]. For example, let us say it is required to locate an emergence center in a new town and this center provides any types of urgent services such as medical or whatever. In this case, all people of that town considered to be potential customers and time of service considered to be critical in this case since it is an urgent service. Therefore, the locations should be selected carefully, and the people who are assigned to those centers should be notified. Furthermore, in those situations, cost is the main factor in deciding the locations of those centers. The model minimizes the cost of the assigned facilities regardless of the travel distance. An assumption of this method is that nodes should have weights or demands and the facility will be among these nodes.

Notation: We consider a set of $i=1,2,\dots,m$ nodes (customers) and a set of $j=1,2,\dots,n$ potential facilities sites. Based on the formulation by [3], we define one type of binary variable:

$$Y_j = \begin{cases} 1, & \text{if a facility is located in candidate site } j \\ 0, & \text{otherwise} \end{cases}$$

In addition, consider the following parameters

C_j = cost associated with facility j

a_{ij} = adjacency matrix

$$\text{Objective:} \quad \text{Min} \quad \sum_j^m C_j Y_j \quad (1)$$

$$\text{Subject to:} \quad \sum_j^m a_{ij} Y_j \geq 1 \quad \text{for all } i \quad (2)$$

$$Y_j \in \{0,1\} \quad \text{for all } j \quad (3)$$

The objective function (1) is to minimize the cost associated with facility j . Constraint (2) ensures each node i is assigned to at least one facility. Constraint (3) ensures the variable is binary.

Methodology

We consider a set i of demand points (customer locations) and a set j of possible facilities (distribution centers). From these sets, we define two types of binary variables:

Decision variables

$$X_{ij} = \begin{cases} 1, & \text{if demand of node } i \text{ is assigned to facility } j \\ 0, & \text{otherwise} \end{cases}$$

$$Y_j = \begin{cases} 1, & \text{if a facility is located in candidate site } j \\ 0, & \text{otherwise} \end{cases}$$

Demand parameters

D_i = demand associated with each node i

d_{ij} = distance between node i and facility j

In addition, we consider the following parameters:

d_{ij} = distance between node i and facility j

Z = maximum travel distance

From the above information, it is now possible to formulate a location model to locate distribution centers between the nodes. We also consider the walking distance (d_{ij}) between facilities and nodes. The model employs a distance constraint for decision makers to allocate resources within a range Z .

$$\text{Objective:} \quad \text{Min} \quad \sum_j^m \sum_i^n Y_j D_i d_{ij} \quad (1)$$

$$\text{Subject to:} \quad \sum_j^n X_{ij} = 1 \quad \text{for all } i \quad (2)$$

$$\sum_j^m X_{ij} d_{ij} \leq Z \quad \text{for all } i \quad (3)$$

$$X_{ij} \leq Y_j \quad \text{for all } i, j \quad (4)$$

$$X_{ij} \in \{0,1\} \quad \text{for all } i, j \quad (5)$$

$$Y_j \in \{0,1\} \quad \text{for all } j \quad (6)$$

Our objective function (1) minimizes number of facilities Y_j times weighted distance. Constraint (2) makes sure that each node is assigned to only one facility. Constraint (3) shows that the travel distance from node i to facility j cannot exceed Z . Constraint (4) ensures that nodes i are assigned only to working facilities. Constraints (5) and (6) ensure that the variables X_{ij} and Y_j are binary.

Application

Location models usually have many applications, such as locating emergency phones within a specific range or area in the public and private sectors. They also find optimum locations for distribution centers to cover demand points. However, it is important to choose an area (specified distance range) that serves as many customers as possible and considers the cost of traveling within that range.

Introduction: Self-drive service at company x campus

The self-drive service provides cars to the company's employees. The cars are typically returned at the end of the day or the end of a trip. Most of the employees are able to use this service. The pools (this term refers to offices concerned with delivering and receiving keys from the employees) should be located in such a way as to encourage the

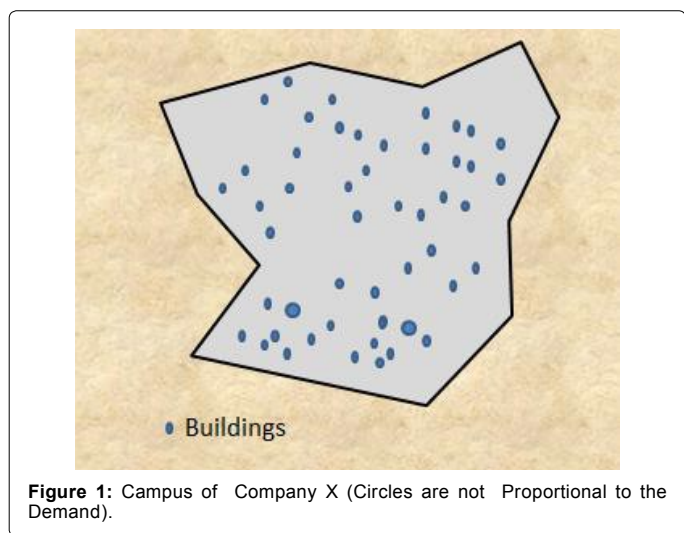
employees to use the service and increase their satisfaction, since the company goal is to be accessible by everyone on the campus (Figure 1).

Problem statement

The buildings are spread all over the campus, which required a network of pools to be efficient at delivering the required service with minimum cost. The maximum travel distance should not be more than 300 meters or a five minute walking distance since the weather in that area is usually sunny and hot during the day. Therefore, the company decided to set the range at a five minute walking distance in order to motivate their employees to walk to the pools and use the service.

Solution method

This specific type of problem is not discussed frequently, especially since it is a distance-constrained location problem. Furthermore, in most other similar cases, the P-median or set-covering problem is used.



Model	Objective function	Number of variables	Number of constrains
P-median	Minimization	2550	2551
Set covering	Minimization	50	50
Proposed model	Minimization	2550	2600

Table 1: Comparison between the models based on provided numbers from the problem i = j = 50.

Model	Outcome
P-median	It will give a network of pools based on the number of P chosen by the decision maker. Therefore, the decision maker will constantly increase the value of P every time to find the optimum number of P that satisfies the requirement. Furthermore, the value of P usually determined by previous experience or try and error method. In addition, maintaining a specific travel distance will require an extra constraint. Therefore, it is necessary to run the model several times.
Set covering	It will give a network of pools, but it will be difficult to maintain a specific travel distance.
Proposed model	It will give a network of pools based on the specific travel distance set by the decision maker, which is what's needed.

Table 2: Outcome of different models.

1	0	6	0	11	1	16	1	21	0	26	0	31	0	36	1	41	1	46	0
2	0	7	0	12	0	17	1	22	1	27	1	32	1	37	0	42	0	47	1
3	1	8	0	13	0	18	0	23	0	28	0	33	0	38	0	43	0	48	0
4	0	9	1	14	1	19	0	24	0	29	0	34	0	39	0	44	0	49	0
5	1	10	1	15	1	20	1	25	0	30	0	35	1	40	0	45	1	50	0

Table 3: Assigned buildings (1 if the building assigned as a pool, 0 if not).

Table 1 shows a comparison of these two methods, and the proposed method, according to their objective function, number of variables, and number of constraints (Table 1).

In order to know which model fits on the current problem, Table 2 shows a comparison of the expected outcome of each model. Where the maximum travel distance is the only decision maker.

Therefore, the proposed model is the most suitable method to solve this problem.

Results

The model was formulated using AMPL employing LPSOLVE Version 5 as the solver. The result after running the proposed model showed that only 18 pools are needed to provide the necessary service to satisfy the requirements of the company, where travel distance no greater than a five minute walk from the pool (almost 300 meters) with a total travel distance of 6256 meters. Table 3 shows the assigned buildings.

Conclusion

The proposed model required only a single run to provide the minimum total travel distance solution. On other hand, the P-median model needed multiple runs until it reached the required solution of having pools located at a maximum five minute walking distance from one another.

Also, the proposed model could be modified as needed. A new constraint could be added to limit the capacity of the pools. Therefore, the load on pools could be controlled and maintained. The constraint could be written as follows:

$$\sum_i^n X_{ij} = S_j \quad \text{for all } j$$

Where S_j is the maximum number of nodes facility j can serve.

In addition, the objective function could be modified to force the model to focus either on the travel distance or population instead of having both in the objective function without changing the main purpose of the model. Furthermore, if the main purpose was to consider the population or demand of the node, the objective function could be written as follows:

$$\min \sum_j^m Y_j P_j$$

Where P_j is the ratio between the total demands over the demand of facility j .

Finally, the travel distance could replace the population as the main aim to minimize in the same way:

$$\min \sum_j^m \sum_i^n Y_j d_{ij}$$

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