

## Development of an Inspection Optimization Model Using Semi-Markov Process and Delay Time Concept

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### Abstract

Inspection in equipment maintenance is implemented to finding possible defects of the system and their repairs before becoming failures. In some of the publications, inspection intervals are obtained using Markov model uniformly. Markov model possesses many benefits of the theories; however, there are some difficulties in using Markov models including definition, measurement and computation for actual conditions in area of maintenance. Since systems get eroded over time, the uniform inspection intervals are not optimum for them. Semi Markov process makes the conditions closer to real world through considering sojourn time. So, this paper proposes a model using semi Markov process and delay-time concept in which non-uniform inspection intervals are obtained. It will be shown that the obtained non-uniform inspection intervals compared to uniform inspection intervals cause the more reduction of system unavailability time. The MATLAB optimization toolbox is applied in order to solving the proposed optimum model.

**Keywords:** Inspection intervals; Maintenance; Optimization; Semi-Markov process; Delay time

### Introduction

The main function of maintenance is controlling the status of equipment and it includes measures which must be taken for inspection, repair, replacement and correction of a component or a group of components of a system. One of the problems regarding maintenance is determination of the inspection frequencies. Defects may be produced and appeared between the inspections for which the appropriate measure is taken. Sometimes, a defect may be remained latent until its trend growth causes emergence and identification in an inspection. This paper presents an optimal inspection model using semi Markov modeling technique and through considering the concept of delay time. Our concern is if the current determined inspection interval by the managers is suitable or not, so we make a relationship between inspection interval and function measurement using the delay time concept and semi Markov process. The delay time concept was first proposed by Christer and Waller [1]. Delay time provides an opportunity within which to identify and remove the defects using inspection. This concept is similar to potential failure interval, but it provides rich modeling methods [2]. Many models and case studies have used the delay time modeling method which can be found in Wang paper [3]. The previous works in the area of delay time can be divided into single-component systems [4] or multi-component complex systems.

Many delay-time based studies consider the assumption that a defect maybe identified only in a PM inspection [5-7]. Some of the researches have mentioned opportunistic maintenance in failure or PM in order to inspection or protection of the other subsystems [8-10], but a few number of papers have addressed the delay-time based models with such policy [11,12]. This policy is named maintenance based opportunistic maintenance without considering production [12]. Some reports have addressed studying opportunistic maintenance policies. Xia et al. [12] have supposed the installation times as opportunities of maintenance.

Estimation of the modeling parameters is an important issue. Modeling parameters mostly include defect occurrence rate, distribution of the defect delay time and quality of an inspection. One of the methods in determination of these parameters is Bayesian method

[13,14]. In contrast, merely data based methods using observed data were proposed. Some case studies have applied the data based method merely in complex systems [15]. The maximum likelihood method has been used significantly for parameters estimation [15,16]. The literature of delay time models is very rich, specifically the works using objective functions, solution methods and modeling assumptions. The most common objective functions focuses on minimizing long term cost [17,18], sum of the expected reduced costs over an infinite horizon [19] or over a finite time horizon [20].

Taghipour and Banjevic [21] have proposed two optimization model of inspection with limited and unlimited time horizon for a repairable multi component system consisting potential failures. Flag [22] under consideration of the delay time concept proposed a model in which the inspections are imperfect and system includes three states perfectly functioning, defective and failed. He, moreover, mentioned that system may be thought of as a passive two-state system subject to random demands. Zhao et al. [23] for a production plant system offered a case study of delay-time based PM modeling. Liu et al. [24] presented a new inspection model based on delay time n-component parallel system and studied optimal inspection interval minimizing the long-term expected cost per unit time. Van Oosterom et al. [14] studied a delay time model under a postponed replacement for single-component system. Nazemi and Shahanaghi [25] proposed a delay-time based model for deteriorating structures in which the optimal inspection times were obtained non-uniformly.

Berrade et al. [26] studied the inspections quality in a maintenance optimization model of a standby system in which the inspections may have incorrect positive result. Markov chain may be used to describing

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the multi-state failure process with limited space [27,28] have addressed the modeling of inspection process in a multi-state parallel system using Markov chain [29,30].

Zhou [31] modeled three different failures and presented a framework for assessing the reliability under consideration of the regular inspection policy. Afterwards, Dawotola et al. [32] obtained optimal inspection intervals for risk-based maintenance. Furthermore, Liu et al. [33] have addressed obtaining the optimal policy of the condition-based maintenance for a system with several failure states. Lecchi [34] proposed a methodology using In-line inspection data. In this study, software named GADpro applying the mentioned methodology is produced to finding inspection optimal time. Many of the delay-time models have been proposed to study the optimal inspection interval of the single-component systems under two-state failure process [3]. Wang and Christer [11] extended the two-state failure process to the three-state failure process in which states are normal, initial defect and sever defect. Zhao et al. [35] developed the model proposed by Wang and Banjevic [3] and also considered age-based inspection and replacement.

Due to the descriptions and examples mentioned above, it is turned out that with respect to importance of the topic of the equipment inspection and maintenance under deterioration on the one hand, and capability and beneficial and unique features of semi-Markov process and delay-time models on the other, it is worthy to use semi-Markov process for modeling and assessing the performance of under-deterioration equipment by taking the concept of delay time. Unlike the other researches, the proposed model addresses finding the non-uniform optimal inspection intervals.

### Mathematical Modeling

The aim of modeling is determining an optimum inspection interval so that it reduces inaccessibility of a system in long term. Note that, in the model settings of the previous section it is obvious that in cases the distribution function  $G(u)$  is exponential, in absence of the inspection operations, the optimum inspection interval will be uniform as the exponential distribution function has constant failure rate which is why inspections are implemented uniformly. In this paper, those systems are assessed facing degradation in their lifetimes, and their failure rates are not constant and increase over the time. Given the increasing failure rate, the inspection intervals which had been considered uniform are not optimum anymore, and consequently the new intervals which may be non-uniform must be considered as to obtaining the optimum inspection intervals. Our goal is to finding optimum inspection intervals. For the simplicity, it is assumed that when a failure is occurred in the time interval  $[t_{i-1}, t_i]$  it is identified and repaired immediately.

To explaining a semi Markov process properly, four characteristics are required to be defined:

- State space
- Transition probabilities
- Steady state probabilities
- Sojourn time.

#### State space

Some authors consider the system state binary before any failure in their papers. Hence, we consider the defined sample space for the model as below:

$$I = \{i \mid i = 0, 1, 2\}$$

The state 0 indicates a state in which the system performs perfectly; state 1 indicates a state in which system has defect and state 2 indicates a state in which system faces failure. Notice that a defect is only identified in the inspection, but a failure is identified immediately upon the occurrence.

#### Transition probabilities

$P_{ij}$  shows the transition probability of the system state from the state  $i$  to state  $j$ . Since these probabilities are nonnegative and system must be transferred to any of the defined states anyway, we have [36].

$$\sum_{j=0,1,2} P_{ij} = 1; i = 0 \quad P_{ij} \geq 0 \tag{1}$$

If  $P$  is transition probabilities matrix of the system, we have

$$P = \begin{bmatrix} P_{00} & P_{01} & P_{02} \\ P_{10} & P_{11} & P_{12} \\ P_{20} & P_{21} & P_{22} \end{bmatrix}$$

Note that the matrix  $P$  is always a probabilistic matrix as any of its rows is a probability vector, and its reason is that after each transition the system must take one of the states 0, 1, 2.

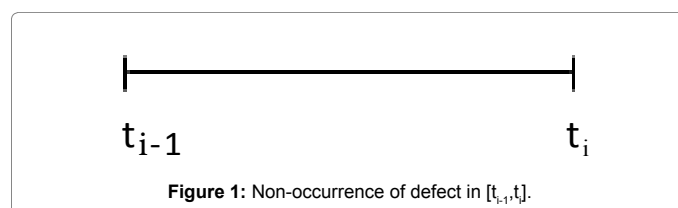
**Estimation of transition probabilities based on the delay time concept:** Formulating the above semi-Markov inspection model depends on the transition probabilities  $P_{ij}$  where  $i$  and  $j$  may be each of states 0, 1, 2. In the previous models proposed by other researchers it is assumed that elements of the transition probabilities matrix are not easily available. In practice elements of the transition probabilities matrix are not given, but it is necessary that they are estimated using available data. In this section, the parameters of the semi-Markov process depending on transition probabilities  $P_{ij}$  are estimated. Such statistical estimations play significant role in mathematical modeling.

Since the transition probabilities  $P_{ij}(t)$  indicate the probability that system is at time  $t$  is in state  $j$  where the current state of the system without inspection and repair is  $i$ , they can be obtained from delay-time concept. Here, we are interested in estimating the probabilities  $P_{00}, P_{01}, P_{02}, P_{10}, P_{11}, P_{12}, P_{20}, P_{21}, P_{22}$ .

Assuming that the subsystem has one kind of defect and our subsystem is single component one, the events which may be occurred between two inspections for a subsystem are assessed.

Firstly, the transition probability  $P_{00}$  is considered indicating the probability that  $t$  units of time from the current state of the system which is 0 is passed, and considering that no inspections and repairs are implemented within this time, the system will remain in state 0. According to the delay-time concept, this case occurs when no defects are occurred in the interval  $[t_{i-1}, t_i]$  (Figure1). In this case the transition probability  $P_{00}$  is calculated as below:

$$P_{00} = p(u \geq t_i) = 1 - \int_{t_{i-1}}^{t_i} g(u) du, \tag{2}$$



Where,  $g(u)$  is defect occurrence density function.

The transition probability  $P_{01}$  is the probability that a system faces a defect in the interval  $[t_{i-1}, t_i]$ , but it performs normally until time  $t_i$  and it does not lead to the failure (Figure 2). In this case, the transition probability  $P_{01}$  is calculated as below:

$$P_{01} = P(t_{i-1} \leq u \leq t_i, t_i - u \leq h) = \int_{t_{i-1}}^{t_i} g(u)(F(t_i - u))du \quad (3)$$

Where,  $F(u)$  is the cumulative distribution function of the delay time  $h$ .

The transition probability  $P_{02}$  is the probability of a failure in the interval  $[t_{i-1}, t_i]$ . Indeed, the occurred defect fully passes its delay time in this interval and faces failure (Figure 3).

$$P_{02} = P(t_{i-1} \leq u \leq t_i, t_i - u \geq h) = \int_{t_{i-1}}^{t_i} g(u)(1 - F(t_i - u))du \quad (4)$$

Note that the following conditions shall be satisfied

$$P_{00} + P_{01} + P_{02} = 1 \quad (5)$$

The transition probability  $P_{10}$  is the probability of a system which is in defective state, and it enters the normal state within the time between two inspections. Given the model assumptions under which system is not improved in case of defect or failure, this probability is 0.

The transition probability  $P_{11}$  is the probability of a system which is in defective state, and it remains that defective state within the time between two inspections. within the time between two inspections.

The transition from state 0 to state 2 is done in two steps, at step one, system goes from state 0 to state 1, and next it enters state 1 from state 2. Consequently, using Chapman-Kolmogorov equation we have the following,

$$P_{12} = \frac{P_{02}}{P_{01}} \Rightarrow P_{02} = P_{01} * P_{12} \quad (6)$$

Given the condition  $P_{10} + P_{11} + P_{12} = 1$  and the equation (6) we have,

$$P_{11} + P_{12} = 1 \Rightarrow P_{11} = 1 - P_{12} \Rightarrow P_{11} = 1 - \frac{P_{02}}{P_{01}} \quad (7)$$

Considering the model assumptions under which the system was identified and repaired immediately within the time between two inspections in case of failure, the transition probability  $P_{20}$  equals 1, and given the condition  $P_{20} + P_{21} + P_{22} = 1$

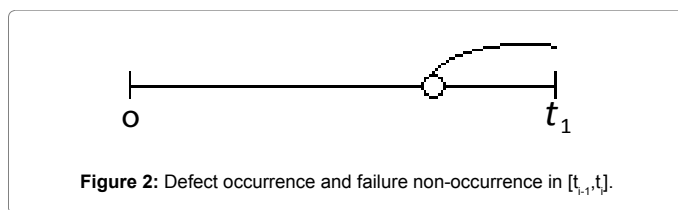


Figure 2: Defect occurrence and failure non-occurrence in  $[t_{i-1}, t_i]$ .

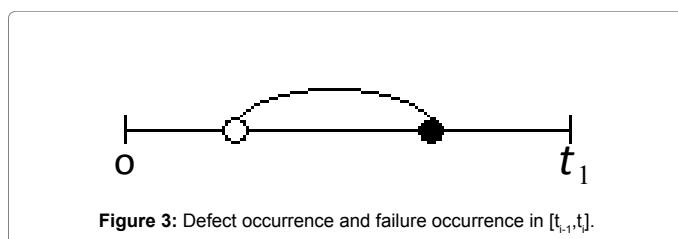


Figure 3: Defect occurrence and failure occurrence in  $[t_{i-1}, t_i]$ .

The transition probability  $P_{11}$  is the probability of a system which is in defective state, and it enters the failure state

$P_{22} = 1$ , probabilities  $p_{21}$  and  $p_{22}$  equal zero.

The transition probabilities matrix for a case in which the inspection is complete, and all of the defects are identified is

$$P_{ij} = \begin{bmatrix} P_{00} & P_{01} & P_{02} \\ 0 & P_{11} & P_{12} \\ 1 & 0 & 0 \end{bmatrix} \quad (8)$$

Transition probabilities were obtained using the distributions of the defect occurrence  $u$  and delay time  $h$ . If the aforementioned distributions could be estimated using the collected data, then the transition probabilities are obtained easily.

In case of identifying any defect or repairing the failure, system is placed in normal state. This repair is considered as minimal repair, a repair which makes the system perform normally but not as good as new system. Inspection intervals are assumed to be uniform here.

With respect to the degradation phenomenon, and increasing rate of the defect occurrence, the probability of the defect and failure occurrence increases over time, and in contrast the probability of the system to be stayed normal decreases. Hence, inspection intervals as to identifying more defects and preventing their failures must be adjusted so that they prevent failures of the made defects.

### Steady state probabilities of the embedded Markov chain

Acquiring the transition probability matrix enables us to obtain steady state probabilities of the Markov chain.

$$\begin{cases} (\pi_0, \pi_1, \pi_f) = (\pi_0, \pi_1, \pi_f) * \begin{bmatrix} P_{00} & P_{01} & P_{0f} \\ 0 & P_{11} & P_{1f} \\ 1 & 0 & 0 \end{bmatrix} \\ \pi_0 + \pi_1 + \pi_f = 1 \end{cases} \quad (9)$$

Solving the system of eqn. (8), steady state probabilities are obtained as the following,

$$\pi_0 = \frac{1 - P_{11}}{P_{01} + (1 - P_{11}) * (2 - P_{00})} \quad (10)$$

$$\pi_1 = \frac{P_{01}}{P_{01} + (1 - P_{11}) * (2 - P_{00})} \quad (11)$$

$$\pi_2 = \frac{(2 - P_{00}) * (1 - P_{11}) - (1 - P_{11})}{P_{01} + (1 - P_{11}) * (2 - P_{00})} \quad (12)$$

### Steady state probabilities of the embedded semi-Markov chain

Steady state probabilities of the semi-Markov process depend on the sojourn time  $\zeta_i$ . The sojourn time  $\zeta_i$  indicates the spent time in state  $i$  within a transition. Mostly, for the simplicity, the random variable  $\zeta_i$  is considered as a mean value. The sojourn time is independent from the previous states, and is defined as the spent time in the current state given the fact that it does not depend on its previous state [37].

$$P_i = \frac{\pi_i * \zeta_i}{\sum_{j \in I} \pi_j * \zeta_j} \quad (13)$$

Where,  $\zeta_i$  is sojourn time of the state  $i$ , and  $\pi_i$  is steady state

probability of the semi-Markov chain at state  $i$ , and it is the defined state space.

For modeling the maintenance inspection as a semi-Markov process we assume that inspection events are happened only at times with the same duration.

### Sojourn time

The sojourn time is the average time that system spends at time  $i$  to be transferred from state  $i$  to state  $j$ , i.e., it is the average time at which the system is in a state, and then is transferred to another state.

Generally, in the researches have been done so far, the average sojourn time would follow the exponential distribution with respect to the maintenance data. In this paper, it is tried to obtain the average sojourn time given the transfer rates from one state to another one having specific cumulative distribution.

Suppose that a system changes its states according to a Markov chain having transition probability  $p_{ij}$ , with cumulative distribution function  $F_{ij}$ . With respect to the fact that state space was assumed to be  $I$ , and given the probabilities obtained in the previous section, the sojourn time in state  $i$ , for interval  $[0, t]$  is acquired using the equation below [38],

$$\zeta_i = \int_0^t (1 - F_{ij}) \quad (14)$$

The eqn. (13) is considered for states that transfer is from state  $i$  to state  $j$ , but for states that transfer is not from state  $i$  to state  $j$  and transfer may be happened from state  $i$  to other several states the sojourn time is calculated using the equation below [38],

$$\zeta_i = \int_0^t (1 - F_{ij}) * (1 - F_{ik}) * (1 - F_{il}) * \dots \quad (15)$$

Given the state space  $I$ , the sojourn times in different states in the interval  $[t_{i-1}, t_i]$  is calculated as the following,

$$\zeta_0 = \int_{t_{i-1}}^{t_i} (1 - F_{00}) * (1 - F_{01}) * (1 - F_{02})$$

$$\zeta_1 = \left[ \int_{t_{i-1}}^{t_i} (1 - F_{11}) * (1 - F_{12}) \right]$$

With respect to the model assumptions under which the system is identified and repaired immediately after failure, the sojourn time in state 2 equals the failure repair time, so we have,

$$\zeta_1 = d_j * E[N_2(t_{i-1}, t_i)],$$

Where,  $E[N_2(t_{i-1}, t_i)]$  is the expectation value of the failures in the interval  $[t_{i-1}, t_i]$  obtained using the equation below [28],

$$E[N_2(t_{i-1}, t_i)] = \int_{t_{i-1}}^{t_i} \lambda(v) F(t_i - v) dv \quad (16)$$

Where,  $F$  is the cumulative distribution function of the delay time, and  $\lambda$  is defect occurrence rate.

### Availability optimization

The availability is evaluated when the system or the component is repairable in which case failures (reliability) and repairs as well are computed. Hence, availability is considered as a desirable computation index for repairable equipment or components.

The system availability denoted by  $A$  could be defined as a time ratio which system is in operative state [39]:

$$A = \frac{\text{Up time}}{\text{Up time} + \text{Down time}} \quad (17)$$

Given the steady state limit probabilities of semi-Markov which were obtained in the previous section, the availability of system in interval  $[t_{i-1}, t_i]$  is sum of all steady state probabilities of the states in which system is in operative state as below,

$$A_i = p_0 + p_1 \quad (18)$$

When inspection intervals are assumed as  $[t_{i-1}, t_i]$ , the equation below is used to computing the time length in which the system is available:

$$TA_i = A_i * (t_i - t_{i-1}), \quad i=1, 2, \dots, n. \quad (19)$$

Given the explained system and the proposed model in the previous section according to semi-Markov process for a system, now inspection intervals shall be obtained for maximizing the availability time. Consequently, objective function and the problem limitations are shown as below:

$$\text{Objective : Max } \sum_{i=1}^n TA_i. \quad (20)$$

Subject to:

$$t_1 > 0$$

$$t_2 \geq t_1 + d_1$$

$$t_3 \geq t_2 + d_1$$

$$L = t_{m+1} \geq t_m + d_1$$

### Computing the reliability

Reliability function is

$$R = \exp \left[ - \int_0^{\infty} h(x) dx \right], \quad (21)$$

Where,  $h(x)$  is hazard rate. Christer and Wang [40] showed using a lemma that failure occurrence trend follows a non-homogenous Poisson process (NHPP) with the rate as below:

$$v(t) = \int_{t_{i-1}}^{t_i} \lambda(v) f(t-v) dv, \quad t_{i-1} \leq t \leq t_i \quad (22)$$

By obtaining the failure occurrence rate using the eqn. (22) and substituting it in eqn. (21), the system reliability could be obtained in the interval  $[t_{i-1}, t_i]$ .

### Numerical Study

One of the key problems in delay-time-based semi Markov inspection models is model parameters estimation for which transition probabilities, delay time and sojourn times in each state are the important parameters. For estimating the aforementioned parameters, the availability of initial time of the defect occurrence and delay time density function are required. When confronting by an industrial maintenance program, gathering information is of very high importance. Recent developments in delay time modeling show that these parameters can be estimated using failures and defects recording which have been identified in inspection process.

In this article a model is developed that is different from the former inspection models. This one deals with cases in which the maintenance

data is available and failure and inspection times are recorded. The numerical example presented in this section contributes to clarifying the following two issues:

- 1) Possibility of applying the delay time concept for systems under deterioration.
- 2) Showing strengths of applying the delay time concept for systems under deterioration.

The parameters used in this article are obtained from the recorded information in CMMS relating to an electro motor. However, some changes have been made in its information to be suitable with the conditions of the proposed model in this paper. Since, unlike the other works, the proposed model in this paper uses semi-Markov process and delay time concept simultaneously, this model is a new base model in its own area and it is not possible to compare the obtained results with the results of other models. Hence, the sensitivity analysis is applied for the model validation.

Given the recorded data in the CMMS software, delay time and defect occurrence rate follow Weibull and exponential distributions respectively. Hence, we have:

$$F(h) = \alpha\beta^{-\alpha}h^{\alpha-1}e^{-\left(\frac{h}{\beta}\right)^\alpha}$$

$$\lambda(t) = ae^{bt}$$

Where,  $\alpha$ ,  $\beta$  are the shape and scale parameters of the Weibull distribution respectively.

Since density function of the equipment is needed as to calculating transition probabilities, density function of the system is obtained using the defect occurrence rate, and then the transition probabilities are calculated. We have,

$$F(t) = 1 - \exp\left[-\int_0^t (ae^{bx}) dx\right] = 1 - e^{-\frac{a(e^{bt}-1)}{b}}$$

$$f(t) = \frac{dF(t)}{dt} = ae^{bt} \frac{a(e^{bt}-1)}{b}$$

The required parameters are given in Table 1. Given the parameters of the Table 1, defect occurrence rate plot within system life time is shown in Figure 4 and its density function is shown in Figure 5.

Now we assess the model solution results for degradation with exponential growth rate and exponential delay time function.

As it can be seen from Figure 6 by making the inspection intervals non-uniform, the trend of perfect performance probability of the system within different inspection intervals has been changed, and finally this change has made the perfect performance probability be decreased from 0.55205 to 0.532613. The reason of this decline can be verified by the following figures.

Parameter	Value
$\delta_1$	0.0625
$a_1$	0.025
$b_1$	0.018
$d_1$	1.86
$d_j$	2.38
M	15
L	120

Table 1: Required Parameters.

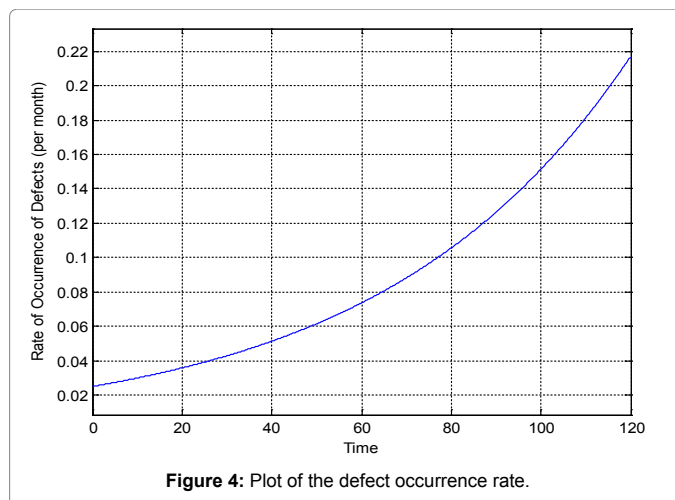


Figure 4: Plot of the defect occurrence rate.

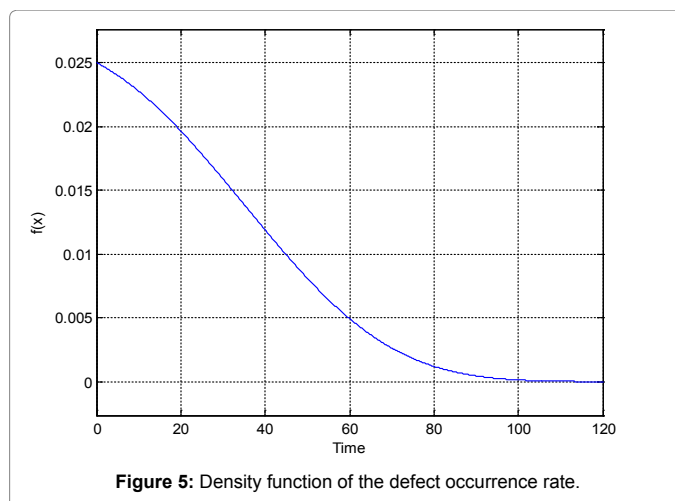


Figure 5: Density function of the defect occurrence rate.

In Figure 7 the probability of the defect occurrence is more within non-uniform inspection time than uniform inspection time, as probability of the defect occurrence within the intervals which have been determined so that maximum defects are identified in them is more than the case in which intervals are selected uniformly.

In Figure 8 given the non-uniform inspection intervals instead of uniform ones, it is observed that the increasing trend of the failure occurrence probability is declined remarkably over time, and as a result this probability has reduced from 0.099659 to 0.097252. Considering the Figures 6-8, it could be seen that decreasing probability of the perfect performance of system and increasing probability of the defect occurrence of the system in non-uniform inspection intervals compared to uniform ones are due to declining the probability of the failure occurrence which had been considered as the objective function. Indeed, for optimizing the inspection intervals which have been implemented using MATLAB software, the inspection intervals have been determined so that probability of the failure occurrence and probability of the perfect performance of system are reduced, and the defect occurrence is grown which ultimately causes the increase in availability of system or decline in unavailability of it which could be observed in the next figures.

Figures shown in Figures 9-11 are related to the average sojourn

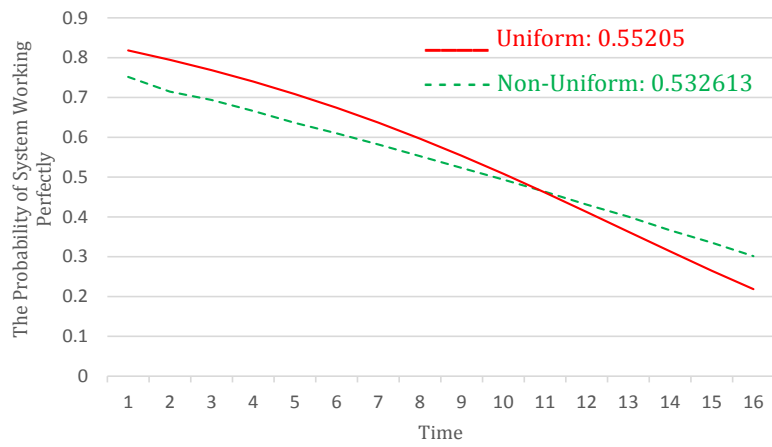


Figure 6: The probability of system working perfectly.

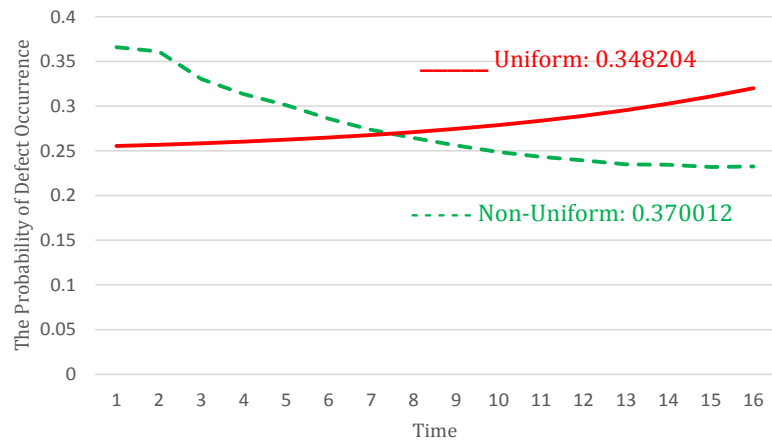


Figure 7: The probability of defect occurrence.

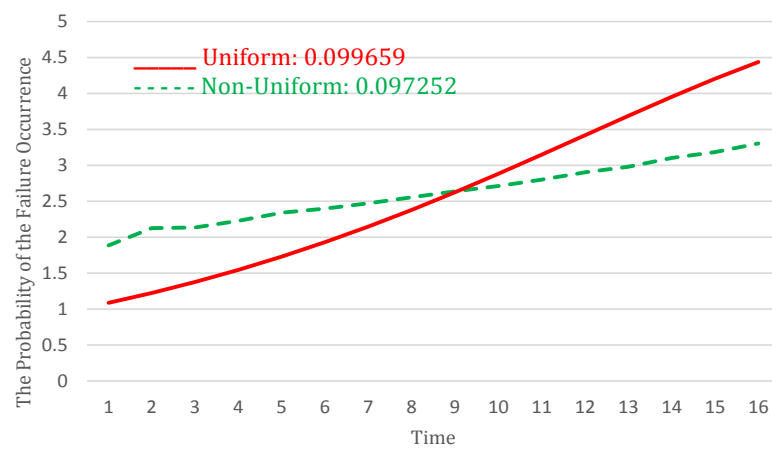


Figure 8: The probability of the failure occurrence.

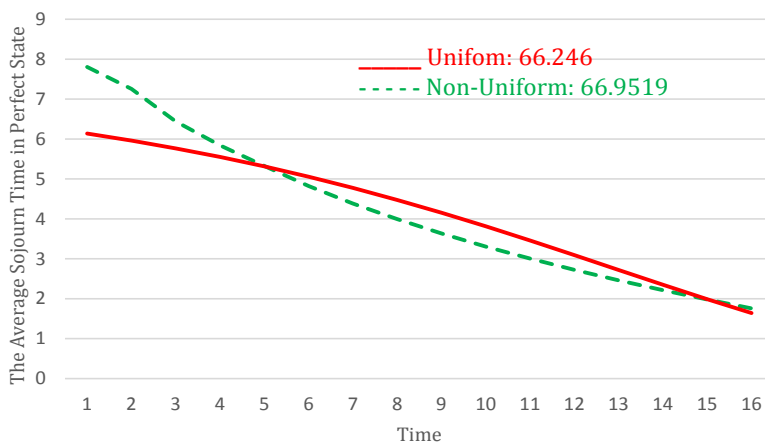


Figure 9: The average sojourn time in perfect state.

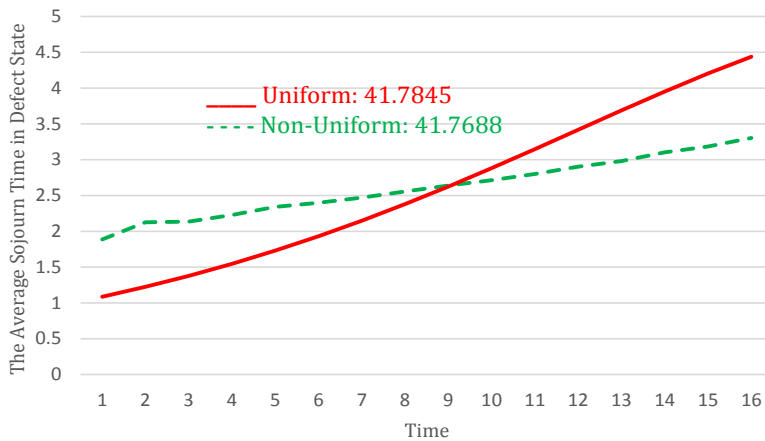


Figure 10: The average sojourn time in defect state.

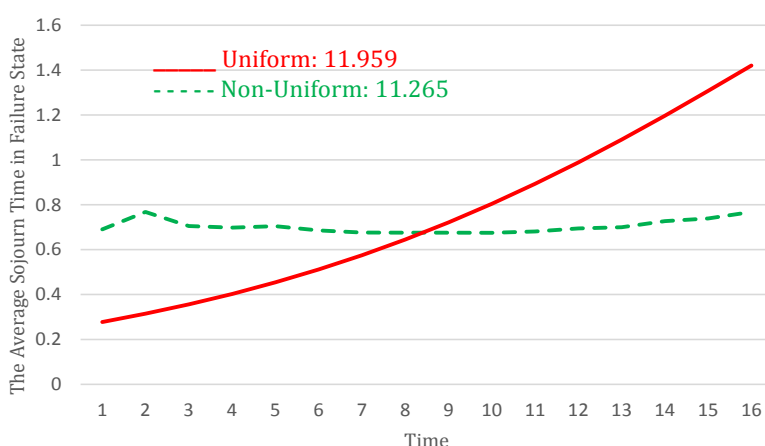


Figure 11: The average sojourn time in failure state.

times in states 1, 0 and 2 respectively which their trends are the same as trends of the plots described above.

Plots of the availability and unavailability times of the system are shown in Figures 12 and 13, respectively. The goal of modeling in this paper is increasing system availability within specified time period through determination of appropriate inspection times. As it

could be seen in Figure 12, system availability is very much during implementation of the uniform inspections, but if follows a decreasing trend over time. However, during implementation of the non-uniform inspections this trend possesses far less changes compared to uniform inspection, and availability improvement is seen from 114.7002 to 115.6078.

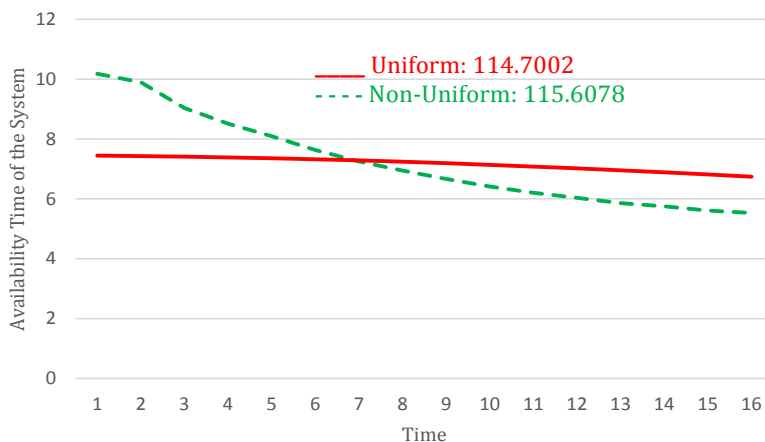


Figure 12: Availability time of the system.

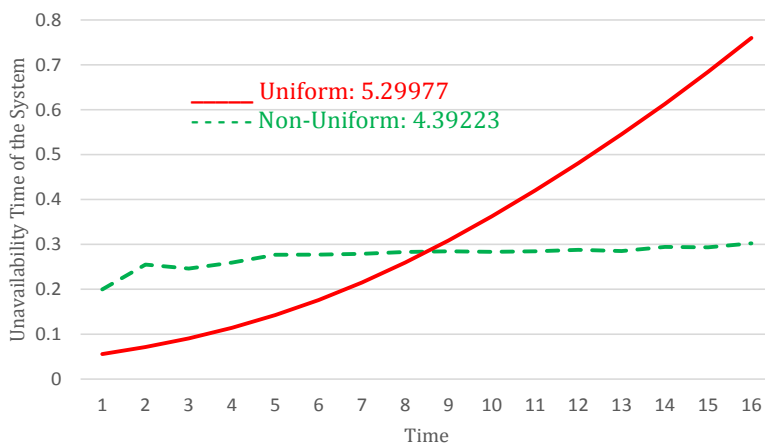


Figure 13: Unavailability time of the system.

Finally, in Figure 14 reliability variations of the system within uniform and non-uniform inspection times are shown. Applying non-uniform inspections instead of uniform inspections, with the aim of improving system availability, the system reliability is increased from 0.861119 at time of the uniform inspection to 0.873054 at the time of non-uniform inspection.

### Sensitivity analysis of the number of inspections parameter and defect occurrence rate

Number of inspections is an indicator of spending limits in the problem. The reason of this matter is that on the one hand inspections operations are costly for several systems under degradation, and they are very beneficial in system defect identification on the other. So, affordable and effective implementation of them is a necessary and rational matter. What is studied in this section is sensitivity of this parameter in the proposed model.

Figure 15 shows the variation trend of the system downtime in terms of the number of the implemented inspection operations in the system lifetime. There are two underlying points in this regard as below:

1. The graph is strictly decreasing, but it is concave up. In the other words, although increasing the number of inspections will

reduce the system downtime, but this decrease is not constant, and for many times of inspections the optimum downtime rate is declined. This problem is very applicable in view of the system maintenance decision makers as it determines that how much spending for more inspection times will improve the system condition.

2. In any inspection times, making the optimum planning policy (non-uniform) makes the system downtime to be improved since the blue curve is always laid above the red curve, but in case of many inspection times, the distance between two curves is reduced, and consequently the effect of optimization model is diminished. Hence, when the budget constraints is more serious, which implementing several inspections is not possible, applying the optimization model makes more added value.

With respect to the output of the proposed model for single-component equipment, making the inspection interval non-uniform causes the availability and reliability of the system to be improved which this amount of the improvement depends solely on the number of inspections and defect occurrence rate. Sensitivity of the aforementioned model to the number of inspections and defect occurrence rate is shown in Figures 15 and 16.



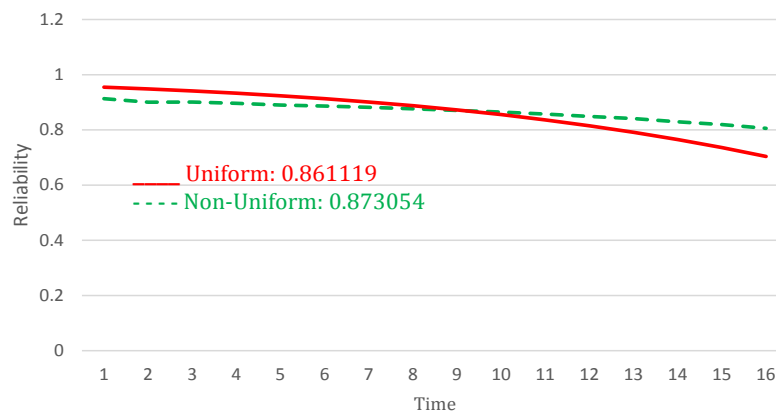


Figure 14: System reliability.

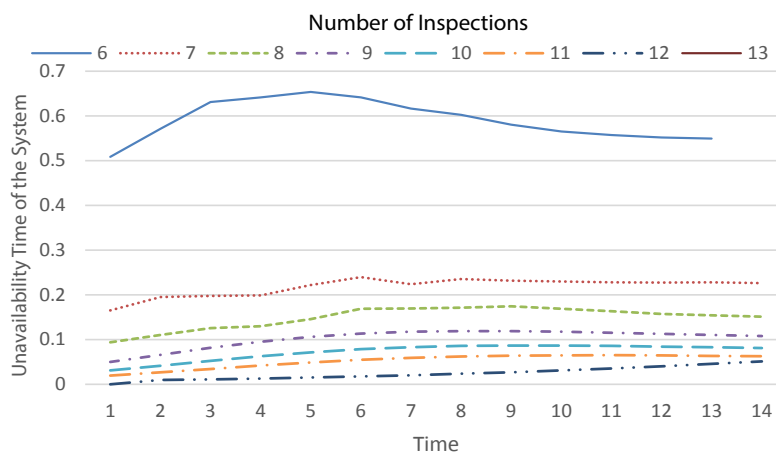


Figure 15: Sensitivity of the model to the number of inspections.

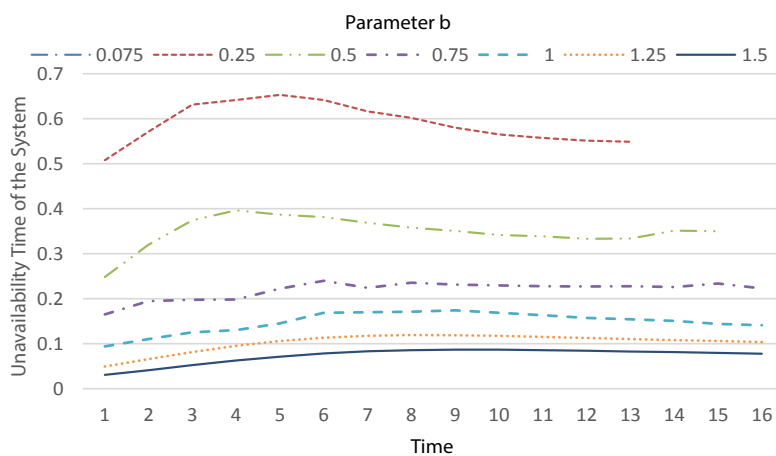


Figure 16: Sensitivity of the model to parameter b of the defect occurrence rate.

## Conclusion

In this paper, maintenance of the equipment, which have been got degraded gradually over time and will be failed ultimately, is optimized through determination of the appropriate inspection times using delay time concept and semi-Markov process. Inspection policies are considered with the aim of reducing or removing likely failures or decreasing their effects in terms of the system availability. The proposed model in this paper presents a probabilistic method based on semi-Markov process to calculating the amount of system availability. The model could be applied for determining the optimum inspection times in the system degradation phase. Compared to existing models have been established based on Markov process, the semi-Markov based model has the following advantages:

- (i) Obtaining transition probabilities between states in different time intervals given the degradation phase.
- (ii) Obtaining the sojourn time in any state in different time intervals based on the amount of degradation and number of failures.

In this paper, firstly, transition probabilities matrix was defined for the model, and these probabilities were calculated using the defect occurrence rate function and delay time function. In the next phase, calculation of the limit probabilities of semi-Markov process was eased thanks to obtaining the sojourn times based on the existing states. Afterwards, the optimum inspection intervals were obtained using MATLAB software with the aim of increasing availability.

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