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DETERMINATION OF LOT SIZE IN THE CONSTRUCTION OF SIX SIGMA BASED DOUBLE SAMPLING PLANS

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Abstract
Six Sigma is one of the most popular quality methods. It utilizes a statistical unit of measurement to measure the capability of the process, then achieve defect free performance, and ultimately increase the bottom-line and customer satisfaction. The concept of Six Sigma can be applied in the process of quality control in general and acceptance sampling in particular. Many sampling plans have been constructed using six sigma quality levels, under the assumption that the lot size is too large or infinite. It is unconvincing to say that the lot is accepted or rejected based on a fixed sample size irrespective of the huge lot size. So, a sampling plan is required which depends on the lot size. The major objective of this paper is to determine the size of the lot of a six sigma based on double sampling plan with Poisson distribution. A table is also constructed for the easy selection of the plan.

Keywords: Six sigma, Double Sampling Plan (DSP), size of first and second sample, Lot size, Lot quality, Average Outgoing Quality (AOQ)

1. Introduction:

Six Sigma methodologies were developed in the middle of 1980s by Motorola for application of Japanese quality concepts and control systems in processes. Companies implementing Six Sigma measure quality of their processes by an index called sigma level. An increase in the sigma level indicates a decrease in defects. Nowadays, companies are able to reach high sigma levels by using Six Sigma not only in solutions of present production problems, but also in designs of products and systems. The fundamental goal in the application of Design for Six Sigma (DFSS) is to understand the customer requirements correctly and completely, and designing the product with optimum process and service to maximize customer satisfaction and optimize the usage of resources. The target is to design processes that will function at the six sigma level as suggested by Tümer Aritürk Gülser Köksal [1].

Dodge and Romig [2] have considered Double Sampling Plan as an extension of Single Sampling Plan (SSP). A detailed comparison of various attribute sampling plans and the merits of the double sampling plan can be seen in Schilling [3] and Duncan [4]. Hald [5] has constructed tables for double sampling plan with the fixed 5% producer's and 10% consumer's risks. Guenther [6] developed trial and error procedure for constructing double sampling plan for a specified producer's and consumer's risk. Schilling and Johnson [7] have developed a table for the construction and evaluation of matched sets of double sampling plans. Soundararajan and Arumainayagam [8] have provided tables for easy selection of double sampling plan indexed through Acceptable Quality Level (AQL) and Limiting Quality Level (LQL). Devaraj Arumainayagam and Soundararajan [9] have constructed Quick Switching System (QSS) indexed through various parameters with Double Sampling Plan as a reference plan. Radhakrishnan [10] contributed to the construction of Double and Continuous Sampling Plans indexed through Maximum Allowable Average Outgoing Quality (MAAOQ). Radhakrishnan and Sampath kumar [11] have constructed mixed sampling plans indexed through MAPD and AQL with Double Sampling Plan as the attribute plan. Sekkizhar [12] constructed double sampling plan indexed through MAPD and MAAOQ using Intervened Random effect Poisson Distribution (IRPD) as the basic distribution. Radhakrishnan and Sivakumaran [13] constructed six sigma sampling plans indexed through Six Sigma Quality Level (SSQL) with Double Sampling Plans as reference plans.

Radhakrishnan and Vasanthamani [14, 15, 16] determined the lot sizes for single sampling plans, double sampling plans and also double sampling plans of the type DSP (0,1). Further Radhakrishnan and Vasanthamani [17, 18] determined the lot size for single and double sampling plans of the type DSP (0,1) based on six sigma quality levels

2. Glossary of symbols

The symbols used in this paper are as follows:

p - Proportion defective / lot quality.

N - Lot size
AOQ - Average outgoing quality.
n_1 - First sample size
n_2 - Second sample size
c_1 - First sample acceptance number
c_2 - Second sample acceptance number
d_1 - Number of non-conformities counted in the first sample
d_2 - Number of non-conformities counted in the second sample

3. Conditions for application:

- The sample units are selected from a finite lot and production is continuous.
- Production is steady, so that the results of past, present and future lots are broadly indicative of a continuous process.
- Lots are submitted sequentially in the order of their production.
- Inspection is by attributes, with the lot quality defined as the proportion defective.
- Human involvement should be minimum in the process/system.

4. Objective:

The objective of the paper is to determine the lot size of the Double Sampling Plan subject to a specified AOQ and sample size by assuming the basic distribution as Poisson distribution, using six sigma quality level

5. Operating Procedure of DSP Plan

In Double sampling plan by attributes the lot acceptance procedure is characterized by the parameters N, n_1, n_2, c_1 and c_2. The operating procedure for double sampling plan is given below

1. Select a random sample of size 'n_1' from a lot of size 'N'
2. Inspect all the items in the sample. Let 'd_1' be the number of non-conformities in the sample.
3. If $d_1 \leq c_1$, accept the lot.
4. If $d_1 > c_2$, reject the lot.
5. If $c1+1 \leq d_1 \leq c_2$, take a second sample of size 'n_2' from the remaining lot and find the number of non-conformities 'd_2'.
6. If $d_1 + d_2 \leq c_2$, accept the lot.
7. If $d_1 + d_2 > c_2$, reject the lot.

6. Operating Characteristic function

Under Poisson model, the OC function of the Double sampling plan is given by Dodge [2] in (1)

$$P_a(p) = \sum_{r=0}^{c_1} \frac{e^{-n_1 p}(n_1 p)^r}{r!} + \left[\sum_{k=c_1+1}^{c_2} \frac{e^{-n_1 p}(n_1 p)^k}{k!} \left\{ \sum_{r=0}^{c_2-k} \frac{e^{-n_2 p}(n_2 p)^r}{r!} \right\} \right] \quad (1)$$

6. Determination of the Lot Size

By fixing sample sizes n_1 and n_2, the acceptance numbers c_1 & c_2 and AOQ with the probability of acceptance as 0.999997, the lot size are determined using the formula (2, 3, 4):

$$AOQ = \left(\frac{p}{N}\right)\left[(N-n_1)P_{a1} + (N-n_1-n_2)P_{a2}\right] \quad (2)$$

$$\text{where } P_{a1} = \sum_{r=0}^{c_1} \frac{e^{-n_1 p}(n_1 p)^r}{r!}, \quad (3)$$

$$P_{a2} = \sum_{k=c_1+1}^{c_2} \frac{e^{-n_1 p}(n_1 p)^k}{k!} \left\{ \sum_{r=0}^{c_2-k} \frac{e^{-n_2 p}(n_2 p)^r}{r!} \right\} \quad (4)$$

Where P_{a1} is the probability of acceptance in the first stage and P_{a2} is the probability of acceptance in the second stage, provided by Duncan [4] and the results are presented in Table1 (for various values of n_1, n_2, $c_1 = 1$ and $c_2 = 3$) using excel packages.

Example: For a given AOQ = 0.0436%, n_1 = 100, n_2 = 150, c_1 = 1, c_2 = 3 and probability of acceptance 0.999997, the value of the lot size can be obtained from Table1 as N = 11016 (this value can be converted to the nearest tens or hundreds). The selected Double Sampling Plan is N = 11016, n_1 = 100, n_2 = 150, c_1 = 1 and c_2 = 3 with the corresponding sigma level (3.6, 3.9). The OC curve for the suggested plan is presented in figure 1.

7. Practical Application:

In a manufacturing dish washers company which practices six sigma initiatives, the quality characteristic is selected as "the total number of scraps among selected piece's monthly production". Take a sample of 100 pieces from a lot of 11000 pieces produced in a month and count the number of non-conformities (d_1). If $d_1 \leq 1$ accept the lot and if $d_1 > 3$ reject the whole lot and inform the management for corrective action. If $1 < d_1 \leq 2$, take another sample of size 150 from the same lot and count the number of non-conformities (d_2). If $d_1 + d_2 \leq 3$, accept the lot and if $d_1 + d_2 > 3$, reject the lot and inform the management.

8. Comparison of the plans:

In table 2, the lot sizes of the sampling plan with different sample sizes for a specified AOQ are given. The engineer working in the floor of assembly can decide about the lot size to be fixed for production/inspection based on the AOQ and size of the sample selected for inspection. This will help the engineer and manager in optimizing the production run.

9. Conclusion:

This paper provides the size of the lot (N) of six sigma based double sampling plan which has the probability of acceptance Pa (p) of the lot as $1-3.4*10^{-6}$ using Poisson distribution as a base line distribution and suitable table is also provided for specified values of AOQ. This will help the floor engineers working in companies which are practicing six sigma initiatives in their organization, to suggest on the size of the lot to be submitted for inspection. This procedure can be extended to other pairs of values of sample sizes (n_1 and n_2) and acceptance numbers (c_1 and c_2). This work can also be extended to plans such as skip-lot, chain and mixed sampling plans.

Table 1: Values of the Lot size of the six sigma based double sample plans ($c_1 = 1, c_2 = 3$)

AOQ%	Sample sizes n_1=150, n_2=300	n_1=150, n_2=200	n_1=100, n_2=300	n_1=150, n_2=150	n_1=100, n_2=250	n_1=100, n_2=150
0.0215	868					
0.0023	1302					
0.0250	3906					
0.0255	7813	1001				
0.0257	13023	1048				
0.0259	39088	1099				
0.0292		5633				
0.0294		7511				
0.0296		11267				
0.0298		22539	1033			
0.0299		45099	1066			
0.0324			5510	2020		
0.0326			8265	2188		
0.0328			16534	2387		
0.0340				5251	1235	
0.0346				13130	1544	
0.0348				26267	1685	
0.0356					2647	
0.0358					3089	
0.0360					3707	
0.0362					4633	
0.0364					6178	
0.0366					9268	
0.0368					18541	
0.0400						1101
0.0420						2203
0.0432						5507
0.0436						11016
0.0438						22040

Table 2: Comparison of Lot Sizes

AOQ%	Sample Sizes				
	n1=150, n2=300	n1=150, n2=200	n1=100, n2=300	n1=150, n2=150	n1=100, n2=250
0.0257	13023	1048	533		
0.0258	19536	1073	542		
0.0259	39088	1099	551		
0.0282		2503	689	772	
0.0285		3004	735	808	

0.0289		4097	806	861	
0.0291		5007	848	890	
0.0296		11267	972	972	501
0.0297		15024	1002	991	508
0.0298		22539	1033	1010	515
0.0299		45099	1066	1030	522
0.0324			5510	2020	806
0.0326			8265	2188	842
0.0327			11021	2283	862
0.0328			16534	2387	882
0.0329			33085	2500	904
0.0342				6564	1324
0.0343				7502	1373
0.0344				8753	1425
0.0345				10504	1483
0.0346				13130	1544
0.0347				17508	1611
0.0348				26267	1685

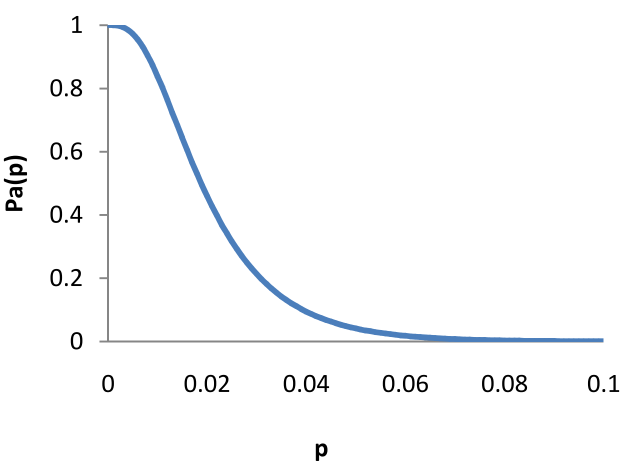

Figure 1: OC curve of six sigma based double sampling plan ($n_1 = 100$, $n_2 = 150$, $c_1 = 1$ and $c_2 = 3$)

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