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Derivations on Trellises

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Abstract

In this paper, we introduce the notion of derivations for a trellis and investigate some related properties of this subject. We give some equivalent conditions under which a derivation is isotone for trellises. Also, we study fixed points and define f-derivation on T and Cartesian derivation on $T_1 \times T_2$.

Keywords: Trellis; Derivation; F-derivation; Isotone; Pseudo-order; Associative; Distributive; Fixed point; Ideal; Cartesian; Modular 2000 Mathematics Subject Classification: Primary 06B05, 06B35, 06B99

Introduction

In 1971, Skala introduced the notions of pseudo-ordered sets and trellises. Trellises are generalization of lattices by considering sets with a reflexive and antisymmetric, but not necessarily transitive. They are also an extension of lattices by postulating the existence of least upper bounds and greatest lower bounds on pseudo-ordered sets similarly as for partially ordered sets [1]. Any reflexive and antisymmetric binary relation $\trianglelefteq$ on a nonempty set T is called a pseudo – ordered on T and $(T, \trianglelefteq)$ is called a pseudo – order set or posset [2-4]. Clearly, each partial order is a pseudo-order. A natural example of a pseudo-order on the set of real numbers is obtained be setting $x \trianglelefteq y$ if and only if $0y \trianglelefteq xa$ for a fixed positive number a. Two elements x, y are comparable if $x \trianglelefteq y$ or $y \trianglelefteq x$. For a subset L of T, the notions of a lower bound, and upper bound, the greatest lower bound (g.l.b), the least upper bound (l.u.b) are defined analogously to the corresponding notions in a posets [5-8].

Generally the notion of a derivation introduced in algebraic systems such as rings, near-rings, specially in lattice theory. Some properties of a derivation such as isotones of a derivation, the set of fixed points of a derivation and relation of derivations with meet-translation as studied in derivations on trellises already defined by Rai and Bhatta [2] with extra conditions that is made derivations isotone. Many authors investigate other properties of derivations on trellises and other algebraic systems [1-3,5]. Here we introduced the notion of a derivation on trellises with weak conditions. The remainder of this paper is organized as follows. We review the definitions and important theorems of the trellis. An equivalent condition is given for a trellis in terms of isotone derivation. Also, we study fixed points and define f-derivation on T and Cartesian derivation on $T_1 \times T_2$.

Preliminaries

Definition

Let T be a nonempty set. A trellis is a psoset $\langle T, \trianglelefteq \rangle$ where any two of whose elements have a (g.l.b) and a (l.u.b). Any psoset can be regarded as a diagram(possibly infinite) in which for any pair of distinct points u and v either there is no directed line between u and v, or if there is a directed line from u to v, there is no directed line from v to u.

Definition

A trellis T is associative if the following conditions hold for all x,y,z $\in$ T;

$(x \wedge y) \wedge z = x \wedge (y \wedge z)$ or $(x \vee y) \vee z = x \vee (y \vee z)$.

Example 1.1

The psoset A={0,a,b,c,1} with $0 \trianglelefteq a \trianglelefteq b \trianglelefteq c \trianglelefteq 1, 0 \trianglelefteq x \trianglelefteq 1$ for every x $\in$ {a,b,c} and $0 \trianglelefteq 1$ while a and c are non-comparable. Then A is a trellis.

Example 1.2

Let A be a set {0,1,a,b,c,d} with the following pseudo-order: $a \trianglelefteq c \trianglelefteq d$, $b \trianglelefteq d$, $b \trianglelefteq c \trianglelefteq d$, $0 \trianglelefteq x \trianglelefteq 1$ for every x $\in$ {a,b,c,d} and $0 \trianglelefteq 1$. Then A is a trellis but not lattice and associative since, $(a \vee b) \vee d = d$ but $a \vee (b \vee d) = 1$.

Some properties on lattices hold in trellises as following:

p_1) $x \wedge y = y \wedge x$, $x \vee y = y \vee x$;(commutatively)

p_2) $(x \wedge y) \vee x = x$, $(x \vee y) \wedge x = x$;(absorption)

p_3) $x \vee ((x \wedge y) \vee (x \wedge z)) = x \wedge ((x \vee y) \wedge (x \vee z))$.(part–preservation).

Theorem 1.3

Let $(T, \trianglelefteq)$ be a trellis. Then by taking $x \wedge y$=g.l.b{x,y} and $x \vee y$=l.u.b{x,y} the binary operation $\vee$ and $\wedge$ satisfy in p_1, p_2, p_3.

From now on, by trellis $(T, \vee, \wedge)$ we mean $(T, \trianglelefteq)$ that x $x \trianglelefteq y$ y is defined by $x \wedge y = x$ or $x \vee y = y$.

Remark 1.4

It is trivial that every associative trellis is a lattice.

Theorem 1.5

A set T with two commutative, absorption and part-preserving operations " $\vee$ ", " $\wedge$ " is a trellis if a E b is defined as $a \wedge b = a$ or $a \vee b = b$.

Proof

Refer to [3, page on 219] .

Definition

(i) A subtrellis S of a trellis $(T, \vee, \wedge)$ is a nonempty subset of T such that a,b $\in$ S implise $a \wedge b$ and $a \vee b$ belong to S.

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(ii) An ideal I of a trellis T is a subtrellis of T such that i ∈ I and a ∈ T imply that a ∧ i ∈ I or equivalently for any i ∈ I and a ∈ T, $a \trianglelefteq i$ implies a ∈ I. Moreover, an ideal I of a trellis T is called a prime ideal if x ∧ y ∈ T implies x ∈ T or y ∈ T for all x,y ∈ T. Note that if I_1,I_2 are ideals of a trellis T, so is $I_1 \cap I_2$.

A trellis T is modular if the following condition holds for all x,y,z ∈ T; $x \trianglelefteq z \Rightarrow x \vee (y \wedge z)=(x \vee y) \wedge z$.

Theorem 1.6

In any trellis, the following statements are equivalent:

$\trianglelefteq$ is transitive,

The operation ∨ and ∧ are associative,

One of the operations ∨ or ∧ is associative.

Proof

Refer to [3, Theorem 3].

As [2, lemma 2.2] by simple argument we have the following lemma:

Lemma 1.7. Let k:T→T be a mapping on a trellis T satisfying the property

k(x ∧ y)=kx ∧ y. Then for all x,y ∈ T:

i) $x \trianglelefteq y$ implies $kx \trianglelefteq ky$,

ii) $kx \trianglelefteq x$,

iii) k(kx)=kx, i.e., k is idempotent,

iv) k(x ∧ y)=kx ∧ ky.

v) the fixed elements of k (x is said to be a fixed element of k if kx=x) form an ideal of T which will be called the fixed ideal of k, denoted by Fix k,also

Fix k=k(T).

Proposition 1.8

If A is an ideal of a trellis T and k is a mapping k:T→T satisfying k(x ∧ y)=kx ∧ y, then k(A) is an ideal of A and hence an ideal of T.

Proof

Refer to [2, proposition 2.6]

Although some trellis are not distributive, by a weaker condition we have the following:

Proposition 1.9

If a trellis T satisfies the inequality x∧(y∨z)E(x∧y)∨(x∧z), then every mapping k on T satisfying k(x ∧ y)=kx ∧ y, implies k(x ∨ y)=kx ∨ ky.

Proof

Suppose that k be a mapping on T satisfying the property above. For any x,y ∈ T,

$$\begin{aligned} k(x \vee y) &= k(x \vee y) \wedge (x \vee y) \\ &\trianglelefteq (k(x \vee y) \wedge x) \vee (k(x \vee y) \wedge y) \\ &= k((x \vee ky.y) \wedge x) \vee k((x \vee y) \wedge y). \\ &= kx \vee ky \end{aligned}$$

Since $x, y \trianglelefteq x \vee y$ we have $kx, ky \trianglelefteq k(x \vee y)$ and so $kx \vee ky \trianglelefteq k(x \vee y)$.

Therefore $k(x \vee y) \triangleleft\!= kx \vee ky$.

On Derivations of Trellises

Definition

Let ⟨T,∧T,∨⟩ be a trellis. A mapping d of a trellis T into itself is called a derivation of T if it satisfies the following condition for all x,y ∈ T :

d(x ∧ y)=(d(x) ∧ y) ∨ (x ∧ d(y))

We can often d(x) written as an abbreviation dx.

Example 2.1.

(i) Let T be a trellis with the least element 0. We define a function d by dx=0 for all x ∈ T. Then d is a derivation on T, which is called the zero derivation.

(ii) Let d be the identity function on a trellis T. Then d is a derivation on T, which is called the identity derivation.

Example 2.2

Let T={0,1,a,b} }be a trellis with the following pseudo-order: $a \trianglelefteq b, 0 \trianglelefteq x \trianglelefteq 1$ for every x∈{a,b} and $0 \trianglelefteq 1$. We define two functions d_1, d_2

$$d_1x=\begin{cases} x, & x=0,1 \\ b & x=a \\ a & x=b \end{cases} \qquad d_2x=\begin{cases} 0, & x=0,1 \\ a & x=a \\ a & x=b \end{cases}$$

d_2 is a derivation on T but d1 is not a derivation, because d_1(a∧a)=(d_1a∧a) implies b≠a.

Example 2.3.

Let T={0,1,a,b,c,d} be a trellis with the following pseudo-order : $a \trianglelefteq c \trianglelefteq d$, $b \trianglelefteq d$, $b \trianglelefteq c \trianglelefteq d$, $0 \trianglelefteq x \trianglelefteq 1$ for every x∈{a,b,c,d} and $0 \trianglelefteq 1$.

Define d: T→T by:

$$dx=\begin{cases} 0, & x=0,1 \\ a & x=a \\ b & x=b,c,d \end{cases}$$

d is a derivation on T.

Example 2.4.

Every mapping k on trellis T satisfying k(x∧y)=kx ∧ y is a derivation on T. The above example is a derivation on T that it does not satisfy in this property.

Remark 2.5

It should be noted that principle derivation on lattices [7] is not a derivation on a trellis. Because it does not have associative property, necessarily.

Proposition 2.6

Let T be a trellis and d be a derivation on T. Then the following statements hold:

dx$\trianglelefteq x$

If I is a ideal of T, then dI ⊆ I

If T has a greatest element 1 and d is a derivation on T, then dx=(x ∧ d1) ∨ dx for all x ∈ T.

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iv) If T has a least element 0 and a greatest element 1, then d0=0 and $d1 \wedge x \trianglelefteq dx$.

Proof

(i) If x ∈ T, then dx=d(x ∧ x)=(dx ∧ x) ∨ (x ∧ dx)=x ∧ dx.

(ii) If I is an ideal of T, then for any x ∈ T, $dx \trianglelefteq x$. implies that dx ∈ I, thus dI⊆ I.

(iii) Note that dx=d(x ∧ 1)=(dx ∧ 1) ∨ (x ∧ d1)=dx ∨ (x ∧ d1).

(iv) It is trivial that d0=0. If x ∈ T, we have,

dx=d(x ∧ 1)=(dx ∧ 1) ∨ (x ∧ d1) implies dx=dx ∨ (x ∧ d1)

then, d1 ∧ x E dx.

By applying proposition 3.6 (iii), in the cases $d1 \trianglelefteq x$ and $x \trianglelefteq d1$ we have the following:

Corollary 2.7

If T has a greatest element 1 and d is a derivation on T, then for all x ∈ T we have,

$d1 \trianglelefteq x$ Implies $d1 \trianglelefteq dx$.

$x \trianglelefteq d1$ Implies dx=x.

Corollary 2.8

Let T be a trellis with a greatest element 1 and d be a derivation on T. Then d1=1 if and only if d is the identity derivation.

Remark 2.9

As derivation on trellises for a derivation d satisfing the dual formula of d(x ∧ y)=(dx ∧ y) ∨ (x ∧ dy),i.e. d(x ∨ y)=(dx ∨ y) ∧ (x ∨ dy), implies that d is a identity derivation.

Proposition 3.1

Let T be a trellis and d be a derivation on T. Then the following conditions are equivalent:

(i) d is the identity derivation;

(ii) d(x ∨ y)=(dx ∨ y) ∧ (x ∨ dy);

Proof

The implication (i) ⇒ (ii) is trivial.

Taking x=y with together contraction property of d implies (ii) ⇒ (i).

Definition

Let T be a trellis and d be a derivation on T. If $x \trianglelefteq y$ implies $dx \trianglelefteq d\,y$, we call d is an isotone derivation.

Example 3.2

The example of 3.2, d_2 is an isotone derivation but in 3.3, d is not an isotone derivation since, $a \trianglelefteq c$ then da=a, dc=b that ab.

Proposition 3.3

Let T be a trellis with a greatest element 1 and d be a derivation on T. If d is an isotone derivation Then dx=x ∧ d1.

Proof

Since d is an isotone, then $dx \trianglelefteq d1$. Note that $dx \trianglelefteq x$, we can get $dx \trianglelefteq (x \wedge d1)$, by proposition 3.6(iii), dx=dx ∨ (x ∧ d1)=x ∧ d1.

Remark 3.4

The above proposition illustrates a condition that makes isotone derivation, principle [7] .

Lemma 3.5

Let T be a trellis and d:T→T be a derivation. Then d(dx)=dx.

Proof

We can get, $dx \trianglelefteq (dx \wedge dx) \vee (d(dx) \wedge x) = d(x \wedge dx) = d(dx)$ and also by 3.6, $d(dx) \trianglelefteq dx$ thus, d(dx)=dx.

Theorem 3.6

Let T be a trellis and d:T→T be a derivation satisfying d(x ∨ y)=dx ∨ dy. Then for all x,y ∈ T:

i) d is a isotone derivation;

ii) $x \trianglelefteq y$ implies dx=x ∧ dy;

iii) dx ∧ y=dx ∧ dy;

Proof

(i) Let $x \trianglelefteq y$, then x ∨ y=y and so dx E (dx ∨ dy)=d(x ∨ y)=dy.

Let $x \trianglelefteq y$. Then by (i), $dx \trianglelefteq dy$ and $dx \trianglelefteq x$. Therefore $dx \trianglelefteq x \wedge dy$. Also $x \wedge dy \trianglelefteq dx$ since, dx=d(x ∧ y)=(dx ∧ y) ∨ (x ∧ dy).

By definition of the derivation, we have $dx \trianglelefteq x \wedge dy$ for all x,y ∈ L. Taking x=dx ∧ y and y=dx in (ii), we have d(dx ∧ y)=(dx ∧ y) ∧ d(dx)=(dx∧y)∧dx=dx∧y. Thus (dx∧y)∨(dx∧dy)=dx∧y implies $dx \wedge dy \trianglelefteq y$. Since $dx \wedge y \trianglelefteq dx$ thus $d(dx \wedge y) \trianglelefteq dy$. Then by above equality we have $dx \wedge y \trianglelefteq dy$. Also $dx \wedge y \trianglelefteq dx$. Then we can get, $dx \wedge y \trianglelefteq dx \wedge dy$. With attention to above inequalities we have dx ∧ y=dx ∧ dy.

Corollary 3.7

Let T be a trellis and d:T –→ T be a derivation satisfying d(x ∨ y)=dx ∨ dy. Then for all x,y ∈ T, d(x ∧ y)=dx ∧ y.

Proof

We have $d(x \wedge y) \trianglelefteq dx \wedge dy = dx \wedge y$, since d is isotone and if $x \wedge y \trianglelefteq x$ then $d(x \wedge y) \trianglelefteq dx \wedge dy$. By definition of the derivation, we can get the inverse relation and so d(x ∧ y)=dx ∧ y for all x,y ∈ L.

Corollary 3.8

If a trellis T satisfies the inequality $x \wedge (y \vee z) \trianglelefteq (x \wedge y) \vee (x \wedge z)$ and d be a derivation on T, Then the following conditions are equivalent:

(i) d(x ∨ y)=dx ∨ dy;

(ii) d(x ∧ y)=dx ∧ y.

Corollary 3.9

Let T be a trellis and d be a derivation on T. If d(x ∧ y)=dx ∧ y, then d(x ∧ y)=dx ∧ dy.

Proof

We have d(x∧y)=dx∧y thus, d(d(x∧y))=d((y∧dx)) then d(x∧y)=dy ∧ dx.

Corollary 4.1

Suppose k be a mapping on a trellis T satisfying k(x ∨ y)=kx ∨ ky. Then k is an isotone derivation if and only if k(x ∧ y)=kx ∧ y.

Remark 4.2

This corollary implies that inverse relation 2.7 (i) is established.

Proposition 4.3

Let T be a trellis and d:T→T be a derivation. If d(x ∧ y)=dx ∧ dy then d is an isotone derivation.

Proof

For all x,y in T. If x E y, then $dx = d(x \wedge y) = dx \wedge dy \trianglelefteq dy$.

Theorem 4.4

Let T be a trellis and d be a derivation on T. Then the following conditions are equivalent:

i) d is an isotone derivation;

ii) $dx \vee dy \trianglelefteq d(x \vee y);$.

Proof

(i)⇒(ii). By (i), we have $dx \trianglelefteq d(x \vee y), dy \trianglelefteq d(x \vee y)$, and so $dx \vee dy \trianglelefteq d(x \vee y)$ ·

(ii)⇒(i). Assume that (ii) holds. Let $x \trianglelefteq y$.

By (ii), $dx \trianglelefteq (dx \vee dy) \trianglelefteq d(y \vee x) = dy$. Thus $dx \trianglelefteq dy$.

Remark 4.5

Despite lattices, on trellises we cannot expect the following statements for an isotone derivation d:

1) d(x ∧ y)=dx ∧ dy

2) d(x ∨ y)=dx ∨ dy.

Remark 4.6

It is trivial that every distributive trellis is a modular trellis and every distributive trellis is a associative trellis. Note that by [4, page on 224] every associative trellis is a transitive trellis, and so every distributive trellis is a lattice.

Definition

Let T be a trellis and d be a derivation on T. Define $Fix_d(T)$={x ∈ T|dx=x}. By the following proposition we can see that∈ E ∈ $Fix_d(T)$ is down-closed set, that is, x∈ $Fix_d(T)$ and $y \trianglelefteq x$ imply y∈ $Fix_d(L)$. Moreover if d is isotone, $Fix_d(T)$ is an ideal of T.

Proposition 4.7

Let T be a trellis and d be a derivation on T. If y Ex and dx=x, then dy=y.

Proof

Suppose x,y are arbitrary elements in L, $y \trianglelefteq x$, then y=x ∧ y. Thus,

Dy=d(x∧y)

=(dx∧y) ∨(x∧dy)

=(x∧y) ∨dy

=y∨dy

=y

Theorem 4.8

Let T be a trellis and d_1 and d_2 be two isotone derivations on T. Then $d_{1=}d_2$ if and only if $Fix_{d1}(T)=Fix_{d2}(T)$.

Proof

Trivially, $d_{1=}d_2$ implies $Fix_{d1}(T)=Fix_{d2}(T)$. For the converse, for all x ∈ T since $d_1x \in Fix_{d1}(T)=Fix_{d2}(T)$ we have $d_2d_1x=d_1x$. Similarly $d_1d_2x=d_2x$. On the other hand, isotonness of d_1 and d_2 implies that $d_2d_1x \trianglelefteq d_2x = d_1d_2$ and $d_2d_1x=d_1d_2x$. Also, d_1d_2x E d_2d_1x, this show that $d_2d_1x=d_1d_2x$. It follows that $d_1x=d_2d_1x=d_1d_2x=d_2x$.

Definition

Let $(A_1, \trianglelefteq_1)$ and $(A_2, \trianglelefteq_2)$ two pseudo-ordered set. By $(A_1 \times A_2, \trianglelefteq)$ we means the set A1 × A_2 with the pseudo-order $(a_1, a_2) \trianglelefteq (b_1, b_2)$ if and only if $a_1 \trianglelefteq_1 b_1$ and $a_2 \trianglelefteq_2 b_2$. If T_1 and T_2 are trellises, so is $T_1 \times T_2$.

Remark 4.9

⟨T_1,∧1,∨1⟩ , ⟨T_2,∧2,∨2⟩ are trellises. It consider that for all $a_1,b_1 \in T_1$ and $a_2,b_2 \in T_2$, $(T_1 \times T_2, \vee, \wedge)$ with $(a_1,a_2) \wedge (b_1,b_2)=(a_1 \wedge b_1, a_2 \wedge b_2)$ and $(a_1,a_2) \vee (b_1,b_2)=(a_1 \vee b_1, a_2 \vee b_2)$ is a trellis.

Definition

Suppose d_1,d_2 are arbitrary derivations on T_1,T_2 respectively. Define $d:T_1 \times T_2 \to T_1 \times T_{2:}$ $d(a,b)=(d_1a,d_2b)$ for all a ∈ T_1,b ∈ T_2. Trivially, d is a derivation and it is called a Cartesain derivation.

Example 5.1

Cartesian derivation of identity derivations is the identity derivation and if d_1,d_2 are isotone derivations then $d=d_1 \times d_2$ is an isotone derivation.

Remark 5.2

Nonetheless, we can product many derivations in such matter, but there is a derivation on $T_1 \times T_2$ that is not a cartesian derivation. For, let

$T_{1=}T_{2=}$T={0,1} be a trellis with $0 \trianglelefteq 1$ and define d:T × T→T × T by d(x,y)=(x,y) for all (x,y)=(1/,1) and d(1,1)=(1,0). Note that this derivation is not isotone, perhaps isotone derivation on $T_1 \times T_2$ are Cartesian derivations.

Definition

Let T be a trellis. A function d:T→T is called an f-derivation on T if there exists a function f:T→T such that

d(x ∧ y)=(d(x) ∧ f(y)) ∨ (f(x) ∧ d(y)).

for all x,y ∈ T.

Remark 5.3

It is obvious that if f is an identity function then d is a derivation on T.

Example 5.4

Let T={0,1,a,b,c} be a trellis with the following pseudo-order: $a \trianglelefteq b \trianglelefteq c$, $0 \trianglelefteq x \trianglelefteq 1$ for every x ∈ {a,b,c} and $0 \trianglelefteq 1$. Define d:T→T by:

$$dx = \begin{cases} 0, & x = 0, a \\ c & x = b, c \\ a & x = 1 \end{cases} \cdot$$

Then d is not a derivation on T since 0=d(a ∧ 1) ≠ (da ∧ 1) ∨ (a ∧ d1)=0 ∨ a=a. If we define f by:

$$fx = \begin{cases} 0, & x = 0, a \\ c & x = b, c \\ 1 & x = 1 \end{cases}$$

then d is an f-derivation on T, for all x,yT.

Proposition 5.5

Let T be a trellis and d be a f-derivation on T. Then the following identities hold for all x,y ∈ T

$dx \trianglelefteq fx$

If T has a least element 0, then f0=0 implies d0=0.

Proof

(i) For all x ∈ T, we have

dx=d(x ∧ x)=dx ∧ fx, thus dx E fx.

(ii) Since $dx \trianglelefteq fx$ for all x ∈ T, we have $0 \trianglelefteq d0 \trianglelefteq f0 = 0$.

Corollary 5.6

If T has a greatest element 1 and d is an f-derivation on T,

f1=1, then for all x ∈ T we have,

If $fx \trianglelefteq d1$, then dx=fx,

If $d1 \trianglelefteq fx$, then $d1 \trianglelefteq dx$.

Proof

(i) we have dx=d(x ∧ 1)=(dx ∧ f1) ∨ (fx ∧ d1)=dx ∨ fx, then fx E dx. From proposition 3.32 (i), we obtain dx=fx.

Since dx=d(x∧1)=(dx∧f1)∨(fx∧d1)=dx∨d1, we have $d1 \trianglelefteq dx$.

Remark 5.7

Note that if d1=1, since $d1 \trianglelefteq f1$, we have f1=1. In this case from corollary 3.33 (i), we get dx=fx.

Definition

Let T be a trellis and d be an f-derivation on T. If $x \trianglelefteq y$ implies $dx \trianglelefteq dy$, we call d is an isotone f-derivation .

Example 5.8

The example of 3.31, d is not an isotone f-derivation, since $c \trianglelefteq 1$ but it dose not follow $dc \trianglelefteq d1$, whereas f is an increasing function on T.

Corollary 5.9

Let T be a trellis and d be an f-derivation on T. Then for all x,y ∈ T we have,

If d is an isotone f-derivation, then $dx \vee dy \trianglelefteq d(x \vee y)$.

If d(x ∧ y)=dx ∧ dy, then d is an isotone f-derivation.

Proof

(i) We know that $x \trianglelefteq x \vee y$ and $y \trianglelefteq x \vee y$. Since d is isotone, $dx \trianglelefteq d(x \vee y)$ and $dy \trianglelefteq d(x \vee y)$. Hence we obtain $dx \vee dy \trianglelefteq d(x \vee y)$.

(ii) Let d(x ∧ y)=dx ∧ dy and $x \_ y$. Since dx=d(x ∧ y)=dx ∧ dy, we get $dx \trianglelefteq dy$.

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