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Δ−Convergence Theorems for Multivalued Non-expansive Mappings in Hyperbolic Spaces

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Abstract

The purpose of this paper is that iteration scheme of multivalued non-expansive mappings in Banach spaces is extended to hyperbolic spaces and to prove some Δ−convergence theorems of the mixed type iteration process to approximating a common fixed point for two multivalued non-expansive mappings and two non-expansive mappings in hyperbolic spaces. The results presented in the paper extend and improve some recent results announced in the current literature.

Keywords: Δ−convergence theorems; Multivalued non-expansive mapping; Common fixed point; Hyperbolic space

Introduction

The study for fixed point problem involve that multivalued contractions and non-expansive mappings used the Hausdorff metric was initiated by Markin [1,2] later, different iterative processes have used to approximate the fixed points of multivalued non-expansive mappings in Banach space, many scholars have made extensive research in [1-17]. An interesting and rich fixed point theory for such mappings was developed which has applications in control theory, convex optimization, differential inclusion, and economics [3]. But the hyperbolic space has no set up the theory of multivalued non-expansive mappings fixed point. In order to define the concept of multivalued non-expansive mapping in the general setup of Banach spaces, we first collect some basic concepts.

Let E be a real Banach space. A subset K is called proximinal if for each x ∈ E, there exists an element k ∈ K such that

$$d(x, k) = inf\left\{\|x - y\| : y \in K\right\} = d(x, K)$$

It is known that weakly compact convex subsets of a Banach space and closed convex subset of a uniformly convex Banach space are proximinal sub- set of K by P(K). Consistent with [1], let CB(K) be the class of all nonempty bounded and closed subset of K. Let H be a Hausdorff metric induced by the metric d of E, that is

$$H(A,B) = max\{\sup_{x\in A} d(x,B), \sup_{y\in B} d(y,A)\},$$

for every A,B∈CB(E) . A multivalued mapping T: K → P (K) is said to be a contraction if there exists a constant k ∈ [0, 1) such that for any x, y ∈ K ,

$$H(Tx,Ty) \le k\|x-y\|$$

Definition 1.1 [15] A multivalued mapping T: K → P (K) is said to be non-expansive, if

$$H(Tx,Ty) \le \|x-y\|,\ \forall n\ge 1, x,y\in K \tag{1.1}$$

Lemma 1.2 [12] Let T: K → P (K) be a multivalued mapping and $P_T(x)=\{y\in Tx : \|x-y\|=d(x,Tx)\}$. Then the following are equivalent.

(1) $x\in F(T)$;

(2) $P_T(x)=\{x\}$;

(3) $x\in F(P_T)$.

Moreover, F (T)=F (P_T).

Throughout this paper, we work in the setting of hyperbolic spaces introduced by Kohlenbach [18], defined below, which is restrictive than the hyperbolic type introduced in [19] and more general than the concept of hyperbolic space in [20].

A hyperbolic space is a metric space (X, d) together with a mapping $W: X^2\times[0,1]\to X$ satisfying

(i) $d(u,W(x,y,\alpha)) \le \alpha d(u,x)+(1-\alpha)d(u,y)$;

(ii) $d(W(x,y,\alpha),W(x,y,\beta)) = |\alpha-\beta|\, d(x,y)$;

(iii) $W(x,y,\alpha)=W(y,x,(1-\alpha))$;

(iv) $d(W(x,z,\alpha),W(y,w,\alpha)) \le (1-\alpha)d(x,y)+\alpha d(z,w)$.

for all x, y, z, w ∈ X and α, β ∈ [0, 1]. A nonempty subset K of a hyperbolic space X is convex if W (x, y, α) ∈ K for all x, y ∈ K and α ∈ [0, 1]. The class of hyperbolic spaces contains normed spaces and convex subsets thereof, the Hilbert ball equipped with the hyperbolic metric [21], Hadamard manifolds as well as CAT(0) spaces in the sense of Gromov [22].

A hyperbolic space is uniformly convex [23] if for any r>0 and ε ∈ (0, 2] there exists a δ ∈ (0, 1] such that for all u, x, y ∈ X , we have

$$d(W(x,y,\tfrac{1}{2}),u) \le (1-\delta)r$$

provided $d(x,u)\le r, d(y,u)\le r$ and $d(x,y)\ge \varepsilon r$

A map $\eta:(0,\infty)\times(0,2]\to(0,1]$ which provides such a $\delta=\eta(r,\varepsilon)$ for For given $r>0$ and $\varepsilon\in(0,2]$,is known as a modulus of uniform convexity of X. We call η monotone if it decreases with r (for a fixed ε), i.e.,

$$\forall\varepsilon>0, \forall r_2\ge r_1>0, (\eta(r_2,\varepsilon)\le\eta(r_1,\varepsilon)).$$

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In the sequel, let (X, d) be a metric space and let K be a nonempty subset of X. We shall denote the fixed point set of a mapping T by $F(T)=\{x\in K: Tx=x\}$.

A mapping T: K → K is said to be non-expansive, if $d(Tx,Ty)\le d(x,y),\ \forall x,y\in K$.

A mapping T: K → K is said to be uniformly L-Lipschitzian, if there exists a constant L>0 such that $d(T^n x,T^n y)\le Ld(x,y),\ \forall x,y\in K$.

The purpose of this paper is that iteration scheme of multivalued non-expansive mappings in Banach spaces is extended to hyperbolic spaces and to prove some Δ−convergence theorems of the mixed type iteration process for approximating a common fixed point of two multivalued non-expansive mappings and other two non-expansive mappings in hyperbolic spaces. The results presented in the paper extend and improve some recent results given in [6-9,11-13,15,16,19,21,24-26]. In order to define the concept of Δ−convergence in the general setup of hyperbolic spaces, we also collect some basic concepts and Lemmas.

Lemma 1.3: [26] Let (X, d, W) be a complete uniformly convex hyperbolic space with monotone modulus of uniform convexity. Then every bounded sequence $\{x_n\}$ in X has a unique asymptotic center with respect to any nonempty closed convex subset K of X.

Recall that a sequence $\{x_n\}$ in X is said to Δ−converge to x ∈ X if x is the unique asymptotic center of $\{u_n\}$ for every subsequence $\{u_n\}$ of $\{x_n\}$. In this case, we write $\Delta-lim_{n\to\infty}x_n=x$ and call x the Δ-limit of $\{x_n\}$.

A mapping T: K → K is semi-compact if every bounded sequence $\{x_n\}\subset K$ satisfying $d(x_n,Tx_n)\to 0$, has a convergent subsequence.

Lemma 1.4: [27] Let $\{a_n\}$, $\{b_n\}$ and $\{\delta_n\}$ be sequences of nonnegative real numbers satisfying

$$a_{n+1}\le(1+\delta_n)a_n+b_n,\forall n\ge 1 \tag{1.3}$$

If $\sum_{n=1}^{\infty}\delta_n<\infty$ and $\sum_{n=1}^{\infty}b_n<\infty$, then the limit $\lim_{n\to\infty}a_n$ exists. If there exists a subsequence $\{a_{n_i}\}\subset\{a_n\}$ such that $a_{n_i}\to 0$ then $\lim_{n\to\infty}a_n=0$.

Lemma 1.5: [28] Let (X, d, W) be a uniformly convex hyperbolic space with monotone modulus of uniform convexity η. Let x ∈ X and $\{\alpha_n\}$ be a sequence in [a, b] for some a, b ∈ (0, 1). If $\{x_n\}$ and $\{y_n\}$ are sequences in X such that

$$\limsup_{n\to\infty} d(x_n,x)\le c,\ \limsup_{n\to\infty} d(y_n,x)\le c$$

$$\lim_{n\to\infty} d(W(x_n,y_n,\alpha_n),x)=c$$

for some c ≥ 0. Then $\lim_{n\to\infty} d(x_n,y_n)=0$.

Lemma 1.6: [28] Let K be a nonempty closed convex subset of uniformly convex hyperbolic space and $\{x_n\}$ a bounded sequence in K such that $A(\{x_n\})=\{y\}$ and $r(\{x_n\})=\zeta$. If $\{y_m\}$ is another sequence in K such that $\lim_{m\to\infty} r(y_m,\{x_n\})=\zeta$, then $\lim_{m\to\infty} y_m=y$.

Main Results

Theorem 2.1: Let K be a nonempty closed convex subset of a complete uniformly convex hyperbolic space X with monotone modulus of uniform convexity η. Let T_i: K → P (K), i=1, 2 be a multivalued mapping and be a uniformly L_i-Lipschitzain and T_{T_i} be a non-expansive mapping, let

S_i: K → K, i=1, 2 be a uniformly L_i-Lipschitzian and be a non-expansive mapping. Assume that $F:=\bigcap_{n=1}^{2}F(T_{T_i})\cap F(S_i)\ne\Phi$, *and* for arbitrarily chosen $x_1\in K,\{x_n\}$ is defined as follows

$$\begin{cases}x_{n+1}=W(S_1x_n,T_{T_1}u_n,\alpha_n)\\ y_n=W(S_2x_n,T_{T_2}v_n,\beta_n)\end{cases}\forall n\ge 1 \tag{2.1}$$

where $v_n\in T_{T_i}x_n, u_n\in S_iy_n, d(v_n,u_n)\le H(T_{T_i}x_n,S_iy_n)+\tau_n$ and i=1 2, $\{\tau_n\}$, $\{\alpha_n\}$ and $\{\beta_n\}$ satisfy the following conditions:

1) $lim_{n\to\infty}\tau_n=0,\ \sum_{n=1}^{\infty}\tau_n<\infty;$

2) There exist constants $a,b\in(0,1)$ with $0<b(1-a)\le\frac{1}{2}$ such that $\{\alpha_n\}\subset[a,b]$ *and* $\{\beta_n\}\subset[a,b]$.

3) $\|x_n-p\|=d(x_n,p),\|y_n-p\|=d(y_n,p);$

4) $d(x,T_{T_i}y)\le d(S_ix,T_{T_i}y)$ for all $x,y\in K$ and i=1,2.

Then the sequence $\{x_n\}$ defined by (2.1) Δ−converges to a common fixed point of $F:=\bigcap_{i=1}^{2}F(T_{T_i})\cap F(S_i)$.

Proof: The proof of Theorem 2.1 is divided into three steps:

Step: First we prove that $\lim_{n\to\infty}d(x_n,p)$ exists for each $p\in F$.

For any given p∈F, since $T_{T_i}, i=1,2$, is a multivalued non-expansivemapping, S_i, i=1,2 is a non-expansive mapping, by condition (2) and (2.1), we have

$$\begin{aligned}d(x_{n+1},p)&=d(W(S_1x_n,T_{T_1}u_n,\alpha_n),p)\\&\le(1-\alpha_n)d(S_1x_n,p)+\alpha_nd(T_{T_2}u_n,p)\\&=(1-\alpha_n)d(S_1x_n,S_1p)+\alpha_nd(T_{T_1}u_n,T_{T_1}p)\\&\le(1-\alpha_n)d(x_n,p)+\alpha_nd(u_n,p)\\&\le(1-\alpha_n)d(x_n,p)+\alpha_nH(S_1y_n,S_1p)+\alpha_n\tau_n\\&\le(1-\alpha_n)d(x_n,p)+\alpha_n\|y_n-p\|+\alpha_n\tau_n\\&=(1-\alpha_n)d(x_n,p)+\alpha_nd(y_n,p)+\alpha_n\tau_n\end{aligned} \tag{2.2}$$

Where

$$\begin{aligned}d(y_n,p)&=d(W(S_2x_n,T_{T_2}v_n,\beta_n),p)\\&\le(1-\beta_n)d(S_2x_n,p)+\beta_n d(T_{T_2}v_n,p)\\&=(1-\beta_n)d(S_2x_n,S_2p)+\beta_nd(T_{T_2}u_n,T_{T_2}p)\\&\le(1-\beta_n)d(x_n,p)+\beta_nd(v_n,p)\\&\le(1-\beta_n)d(x_n,p)\beta_n H\left|(T_{T_2}x_n,T_{T_2})+\beta_n\tau_n\right.\\&\le(1-\beta_n)d(x_n,p)\beta_n\|x_n-p\|+\beta_n\tau_n,\\&=(1-\beta_n)d(x_n,p)+\beta_n d(x_n,p)+\beta_n\tau_n,\\&=d(x_n,p)+\beta_n\tau_n\end{aligned} \tag{2.3}$$

Substituting (2.3) into (2.2) and simplifying it, we have

$$d(x_{n+1},p)\le d(x_n,p)+(1+\beta_n)\alpha_n\tau_n, \tag{2.4}$$

where $\delta_n=0,b_n=(1+\beta_n)\alpha_n\tau_n$. Since $\sum_{n=1}^{\infty}\tau_n<\infty$ and condition it follows from Lemma 1.4 that $\lim_{n\to\infty}d(x_n,p)$ exist for $p\in$ F.

Step 2: We show that

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$$\lim_{n\to\infty} d(x_n, T_{T_i} x_n) = 0, \lim_{n\to\infty} d(x_n, S_i x_n) = 0, \ i = 1,2 \tag{2.5}$$

For each $p \in F$, from the proof of Step 1, we know that $lim_{n\to\infty} d(x_n, p)$ exists.

We may assume that $lim_{n\to\infty} d(x_n, p) = c \ge 0$. If c=0, then the conclusion is trivial. Next, we deal with the case c>0. From (2.3), we have $d(y_n, p) \le d(x_n, p) + \beta_n \tau_n$ (2.6)

Taking limsup on both sides in (2.6), we have

$$\limsup_{n\to\infty} d(y_n, p) \le c \tag{2.7}$$

In addition, since $d(T_{T_1} y_n, p) = d(T_{T_1} y_n, T_{T_1} p) \le d(y_n, p)$, and $d(S_1 x_n, p) = d(S_1 x_n, S_1 p) \le d(x_n, p)$ then we have

$$\limsup_{n\to\infty} d(T_{T_1} y_n, p) \le c \tag{2.8}$$

and

$$\limsup_{n\to\infty} d(S_1 x_n, p) \le c \tag{2.9}$$

Since $\limsup_{n\to\infty} d(x_{n+1}, p) = c$, it is easy prove that

$$\lim_{n\to\infty} d(W(S_1 x_n, T_{T_1} y_n, \alpha_n), p) = c \tag{2.10}$$

It follows from (2.8)-(2.10)and Lemma 1, 5 that

$$\lim_{n\to\infty} d(S_1 x_n, T_{T_1} y_n) = 0 \tag{2.11}$$

By the same method, we can also prove that

$$\lim_{n\to\infty} d(S_2 x_n, T_{T_2} x_n) = 0 \tag{2.12}$$

By virtue of the condition (4), it follows from (2.11) and (2.12) that

$$\lim_{n\to\infty} d(x_n, T_{T_1} y_n) \le \lim_{n\to\infty} d(S_1 x_n, T_{T_1} y_n) = 0 \tag{2.13}$$

and

$$\lim_{n\to\infty} d(x_n, T_{T_2} x_n) \le \lim_{n\to\infty} d(S_2 x_n, T_{T_2} x_n) = 0 \tag{2.14}$$

From (2.1) and (2.12) we have

$$\begin{aligned} d(y_n, S_2 x_n) &= d(W(S_2 x_n, T_{T_2} x_n), S_2 x_n) \\ &\le \beta_n d(T_{T_2} x_n, S_2 x_n) \ \to 0 \ (\text{as } n \to \infty) \end{aligned} \tag{2.15}$$

and

$$\begin{aligned} d(y_n, S_1 x_n) &= d(W(S_1 x_n, T_{T_1} x_n, \beta_n), S_1 x_n) \\ &\le \beta_n d(T_{T_1} x_n, S_1 x_n) \ \to 0 \ (\text{as } n \to \infty) \end{aligned} \tag{2.16}$$

Observe that $d(x_n, y_n) = d(x_n, T_{T_2} x_n) + d(T_{T_2} x_n, S_2 x_n) + d(S_2 x_n, y_n)$

It follows from (2.14) and (2.15) that

$$\lim_{n\to\infty} d(x_n, y_n) = 0 \tag{2.17}$$

This together with (2.13) implies that

$$\begin{aligned} d(x_n, T_{T_1} x_n) &\le d(x_n, T_{T_1} y_n) + d(T_{T_1} y_n, T_{T_1} x_n) \\ &\le d(x_n, T_{T_1} y_n) + d(y_n, x_n) \to 0 \ (n \to \infty) \end{aligned} \tag{2.18}$$

On the other hand, from (2.11) and (2.17), we have

$$\begin{aligned} d(S_1 x_n, T_{T_1} x_n) &\le d(S_1 x_n, T_{T_1} y_n) + d(T_{T_1} y_n, T_{T_1} x_n) \\ &\le d(S_1 x_n, T_{T_1} y_n) + d(y_n, x_n) \to 0 \ (n \to \infty) \end{aligned} \tag{2.19}$$

Hence from (2.18) and (2.19), we have that

$$d(S_1 x_n, x_n) \le d(S_1 x_n, T_{T_1} y_n) + d(T_{T_1} x_n, x_n) \to 0 (n \to \infty) \tag{2.20}$$

In addition, since $\begin{aligned} d(x_{n+1}, x_n) &\le d(W(S_1 x_n, T_{T_1} y_n, \alpha_n) x_n) \\ &\le (1-\alpha_n) d(S_1 x_n, x_n) + \alpha_n d(T_{T_1} y_n, x_n) \end{aligned}$

from (2.13) and (2.20), we get

$$\lim_{n\to\infty} d(x_{n+1}, x_n) = 0 \tag{2.21}$$

Finally, for all i = 1; 2, we have

$$\begin{aligned} d(x_n, T_{T_i} x_n) &\le d(x_n, y_n) + d(y_n, S_i x_n) \\ &+ d(S_i x_n, T_{T_i} y_n) + d(T_{T_i} y_n, T_{T_i} x_n) \\ &\le 2d(x_n, y_n) + d(y_n, S_i x_n) + d(S_i x_n, T_{T_i} y_n) \end{aligned}$$

it follows from (2.11),(2.12),(2.15),(2.16) and (2.17) that

$$\lim_{n\to\infty} d(x_n, T_{T_i} x_n) = 0, \ i = 1,2 \tag{2.22}$$

Since $d(x_n, S_i x_n) \le d(x_n, T_{T_i} x_n) + d(T_{T_i} x_n, S_i x_n)$

it follows from (2.12),(2.19) and (2.22) that

$$\lim_{n\to\infty} d(x_n, S_i x_n) = 0, i = 1,2 \tag{2.23}$$

Step 3: Now we prove the sequence $\{x_n\}$ Δ-converges to a common fixed point of

$F := \bigcap_{i=1}^{2} F(T_{T_i}) \bigcap F(S_i)$.

In fact, since for each $p \in F$, $\lim_{n\to\infty} d(x_n, p)$ exist.

This implies that the sequence $\{d(x_n,p)\}$ is bounded, so is the sequence $\{x_n\}$. Hence by virtue of Lemma1.3, $\{x_n\}$ has a unique asymptotic center $A_K(\{x_n\}) = \{x\}$.

Let $\{u_n\}$ be any subsequence of $\{x_n\}$ with $A_k(\{u_n\}) = \{u\}$ It follows from (2.5) that

$$\lim_{n\to\infty} d(u_n, T_{T_i} u_n) = 0$$

Now, we show that $u \in F(T_{T_i})$. For this, we define a sequence $\{z_n\}$ in K by $z_j = T_{T_i}^j u$. So we calculate

$$\begin{aligned} d(z_j, u_n) &\le d(T_{T_i}^j u, T_{T_i}^j u_n) + d(T_{T_i}^j u_n, T_{T_i}^{j-1} u_n) + \cdots + d(T_{T_i} u_n, u_n) \\ &= d(T_{T_i}^j u, T_{T_i}^j u_n) + \sum_{k=1}^{j} d(T_{T_i}^j u_n, T_{T_i}^{k-1} u_n) \end{aligned} \tag{2.25}$$

Since T_{T_i} be a non-expansive mapping, by

$$d(T_{T_i}^j u, T_{T_i}^j u_n) \le d(T_{T_i}^j u, T_{T_i}^{j-1} u_n) \le$$
$$\cdots \le d(u, u_n), d(T_{T_i}^j u_n, T_{T_i}^{j-1} u_n) \le d(T_{T_i}^{j-1} u_n, T_{T_i}^{j-2} u_n) \le \cdots \le d(T_{T_i} u_n, u_n)$$

from (2.25) we have:

$$d(z_j, u_n) \le d(u, u_n) + jd(T_{T_i} u_n, u_n)$$

Taking limsup on the sides of the above estimate and using (2.24), we have

$$r(z_j, \{u_n\}) = \limsup_{n\to\infty} (z_j, u_n)$$

$\le \limsup_{n\to\infty} d(u,u_n)$

$=r(u,\{u_n\})$

And so,

$\limsup_{j\to\infty} r(z_j,\{u_n\}) \le r(u,\{u_n\})$

Since $A_K(\{u_n\})=\{u\}$ by the definition of asymptotic center $A_K(\{u_n\})$ of a bounded sequence $\{u_n\}$ with respect to $K\subset X$, we have

$r(u,\{u_n\}) \le r(y,\{u_n\}),\ \forall y\in K$

This implies that

$\liminf_{j\to\infty} r(z_j,\{u_n\}) \le r(u,\{u_n\})$

Therefore we have

$\lim_{j\to\infty} r(z_j,\{u_n\}) \le r(u,\{u_n\})$

It follow from Lemma 1.6 that $\lim_{j\to\infty} T_{Ti}\, u=u$ As T_{Ti} is uniformly continuous, so that $T_{T_i}u=T_{T_i}(\lim_{j\to\infty} T_{T_i}^j u)=\lim_{n\to\infty} T_{T_i}^{j+1}u=u$. That is $u\in F(T_{T_i})$. Similarly, we also can show that $u\in F(S_i)$. Hence, u is the common fixed point of T_{T_i} and S_i. Reasoning as above by utilizing the uniqueness of asymptotic centers, we get that x=u. Since $\{u_n\}$ is an arbitrary subsequence of $\{x_n\}$, therefore $A\{u_n\}=\{u\}$ for all subsequence $\{u_n\}$ of $\{x_n\}$ This proves that $\{x_n\}$ Δ-convergence to a common fixed point of $F:=\bigcap_{i=1}^2 F(T_{T_i})\bigcap F(S_i)$. This completes the proof.

The following theorem can be obtained from Theorem 2.1 immediately.

Theorem 2.2: Let K be a nonempty closed convex subset of a complete uniformly convex hyperbolic space X with monotone modulus of uniform convexity η.

Let $T_i: K\to P(K), i=1,2$ be a multivalued mapping and be a uniformly L_i − Lipschitzian and $T_{T_i}, i=1,2$ be a non-expansive mapping. Assume that $F:=\bigcap_{i=1}^2 F(T_{T_i})\ne\phi$ for arbitrarily chosen $x_1\in K$, $\{x_n\}$ is defined as follows:

$$\begin{cases} x_{n+1}=W(x_n,T_{T_1}u_n,\alpha_n) \\ y_n=W(x_n,T_{T_2}v_n,\beta_n)\ \forall n\ge 1 \end{cases} \quad (2.26)$$

where $v_n\in T_{T_i}x_n, u_n\in Iy_n, d(v_n,u_n)\le H(T_{T_i}x_n, Iy_n)+\tau_n$ I be an identity mapping. $\{\tau_n\}$ $\{\alpha_n\}$ $\{\beta_n\}$ satisfy the following conditions:

1) $\lim_{n\to\infty}\tau_n=0, \sum_{n=1}^{\infty}\tau_n<\infty;$

2) There exist constants a, b ∈ (0, 1) with $0<b(1-a)\le\frac{1}{2}$ such that $\{\alpha_n\}\subset[a,b]$ and $\{\beta_n\}\subset[a,b]$;

3) $\|x_n-p\|=d(x_n,p), \|y_n-p\|=d(y_n,p)$.

Then the sequence $\{x_n\}$ defined by (2.26) Δ−converges to a common fixed point of $F:=\bigcap_{i=1}^2 F(T_{T_i})$.

Proof Take S_i=I, i=1,2 in Theorem 2.1. Since all conditions are satisfied. It follows from Theorem 2.1 that the sequence $\{x_n\}$ converges to a common fixed point of $F:=\bigcap_{i=1}^2 F(T_{T_i})$. This completes the proof of Theorem 2.2.

Competing Interests

The author declares that he has no competing interests.

Author's Contributions

Author contributed equally and significantly in this research work.

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