Journal of Physical Mathematics
ISSN: 2090-0902
Keilman, J Phys Math 2018, 9:2
DOI: 10.4172/2090-0902.1000271
Commentary Open Access

Deformation-less Acceleration of a Stick in Special Relativity

Keilman Y*
University of Oslo Oslo, Oslo, USA

Introduction

Can we accelerate a stick in SR so that it remains not deformed during acceleration? The infinitely rigid stick will require that. It was long time that this question was not answered and it was thought that infinitely rigid stick does not exist in SR. It appears that we can accelerate without deformation in SR (no matter if the stick is soft or hard).

Explanation

A resting clock usually described in SR by the line parallel to time axis and displaying time T on its dial which coincides with the local coordinate time t. But if the same clock moves with a constant velocity V in x direction then the local time differs from the dial time: $t=T/\sqrt{1-V^2}$. (I assume c=1). It is so called "time dilation". There is no actual change of the dial time T (the coordinate time t gets dilated). The dial time T is invariant and called proper time.

A resting stick of the physical length L usually described in SR by 2 lines parallel to the time axis and cutting **x**-coordinate length l=L on the x-axis. But if the same stick moves with a constant velocity V in x direction then the lines (the tracks of the end points of the stick) make an angle with the time axis and they cut the length $l=L\sqrt{1-V^2}$ on x-axis. It is so called "length contraction". There is no actual deformation (contraction) of the physical length of the stick (called proper length) because in both cases we have inertial movement.

In his article Einstein [1] put stress on coordinate time and x-coordinate length that change with frame. But for the practical reasons (like measurements or calculating deformation) we need to put stress on proper time and proper length which are invariant. All physical clocks and meter sticks are invariant and can be used in any coordinate system. We do not need any additional "normalization" like proposed by Winkler [2] Now let us turn to acceleration. Physically we can imagine deformation-less movement with acceleration in a case of gravitational forces, or in a case of absolutely rigid stick.

Let us prescribe the trajectory of the first end of the stick in the parametric form:

$$x_1 = x_1(p);\ t_1 = p \tag{1}$$

In particular, in the case of acceleration it can be:

$$x_1(p) = Vp + \frac{ap^2}{2} \tag{2}$$

We want to find the trajectory of the 2nd world line (another end of a stick). Let us take the arbitrary point P1(t1,x1) on the line (1). Let us build a normal to the 1st world line at the point P1. Suppose the point P(x,t) is located on this normal. The invariant distance between P1 and P is:

$$S = \sqrt{(x - x_1(p))^2 - (t-p)^2} \tag{3}$$

This distance has to be maximum with respect to a variation of p (we keep the point P fixed while P1 is moving along the 1st world line by the variation of p). We have:

$$\frac{\partial S}{\partial p} = 0;\ x = x_1(p) + \frac{t-p}{V(p)};\ V(p) = k(p) \tag{4}$$

where V is just a notation. With p fixed the last equation indicates that we can move the point P only along the line. This line represents the normal to the 1st world line in the point P1. Substituting this back to eqn. (3) we get:

$$t = p \pm \frac{LV}{\sqrt{1-V^2}};\ \ x = x_1(p) \pm \frac{L}{\sqrt{1-V^2}} \tag{5}$$

Where L is the value of S at maximum. Changing L we will move the point P(t,x) along the normal. This normal will intersect the 2nd world line in some point P2(t2,x2). The t2 and x2 will satisfy to the equation of normal (5):

$$t_2 = p \pm \frac{LV}{\sqrt{1-V^2}};\ \ x_2 = x_1(p) \pm \frac{L}{\sqrt{1-V^2}} \tag{6}$$

Given s and changing parameter p we can find the world line of the second end of a stick in the parametric form with the same parameter p : $t_2(p);x_2(p)$. Considering that the invariant length of the stick L is constant and changing p (moving the point P1 along the 1st world line) we actually find the 2nd world line in a parametric form (eqn. (6)). Namely the time of the reciprocal point (we consider the points P1 and P2 are reciprocal) on the world line of the first end of the stick, so it uses the same parameter as the 1st world line [3].

By direct differentiation of eqn. (6) we can find:

$$\frac{\dot{x}_2(p)}{\dot{t}_2(p)} = V(p) \tag{7}$$

That means that the reciprocal points have the same velocity.

Let us build a normal to the 2nd world line in the point P2. Suppose the point P(x,t) is located on this normal. The invariant distance between P2 and P is:

$$S = \sqrt{(x - x_2(p))^2 - (t - t_2(p))^2} \tag{8}$$

This distance has to be maximum with respect to a small variation of p:

$$\frac{\partial S}{\partial p} = 0;\ \ x = x_2(p) + (t - t_2(p))\frac{\dot{t}_2(p)}{\dot{x}_2(p)} \tag{9}$$

where the last equation represents another normal. This normal will intersect the 1st world line at some point P1'(t1',x1') (t1' and x1' can be found as a joint solution of both eqns. (1) and (8)). It is easy to see from eqns. (4) and (8) that as a result of eqn. (7) P1' coincides with P1. That means that the points P1 and P2 are "reciprocal" points. Given

***Corresponding author:** Keilman Y, University of Oslo Oslo, Oslo, United State, Tel: +47 22 85 50 50; E-mail: altsci1@gmail.com

Received April 22, 2018; **Accepted** June 11, 2018; **Published** June 18, 2018

Citation: Keilman Y (2018) Deformation-less Acceleration of a Stick in Special Relativity. J Phys Math 9: 271. doi: 10.4172/2090-0902.1000271

the 1st world line we found the way to build the reciprocal 2nd world line. These 2 lines represent the world lines of the ends of moving and accelerating stick if it is not deformed. This is the proof that existence of absolute hard stick does not contradict to SR. The objection to the existence of a hard stick can come only from dynamics of the stick. This dynamics will also put limit on the velocity of transfer information along a stick.

References

1. Einstein A (1905) Zur Electrodynamik bewegter Korper. Annalen der Physik 16: 895-896.
2. Winkler FG (2005) The Normalization Problem in Special Relativity.
3. https://arxiv.org/pdf/physics/0604214