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Covariant Functors Finite Degree and Stratifiable Spaces

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Abstract

In this article, we studied covariant functors are finite-valued at the categories Comp-compact spaces, metrizable spaces, S-stratifiable spaces, ℵ-spaces, paracompact p-spaces and its continuous maps.

Keywords: Spaces; Covariant functors; Finite-level functors; Stratifiable spaces; p-Spaces; σ-Parakompact spaces; Finite dimensional spaces; Metriziable spaces

Introduction

In this note we consider covariant functors in the categories *Comp* of compacta, metr-metrizable spaces, S-stratified spaces, ℵ-spaces, and paracompact *P*-spaces and continuous mappings into itself. It is proved that functors with finite carriers that act in these or those categories preserve finite-dimensional spaces and weakly countable-dimensional spaces. Closed functors with finite supports are defined and it is proved that the closed functors preserve the class of *S*-spaces.

- We recall the definition and some properties of the normality of the covariant functor *F*:*Comp*→*Comp* Acting in the category of compacta. We say that the functor F: Saves the empty set and the point if F(∅)= and F({1})={1} where {k}, k≥0 we denote the set of nonnegative integers {0,1…,k-1}, less than *k*. In this terminology {0}=∅;
- Monomorphism if for every (topological) embedding *f*:*A*→*X* the map *F*(*f*):*F*(*A*)*F*(*X*) is an embedding.

Epimorphic if for every map f:A→Y onto Y the map $F(f):F(A)\to F(Y)$ is also a mapping "onto"; It preserves intersections if for any family{Aα:α∈A} of closed subsets of the compact space X and the identity embeddings iα:Aα→X, the map $F(i_\alpha):\cap\{F(A_\alpha):\alpha\in A\}\to X$, defined by the equality $F(i)(\alpha)=F(i_A)(\alpha)$, is an embedding for everyα∈A

Saves the preimages if for every map *f*:X→Y and every closed set A⊂Y, the mapping $F(f\big|_{f^{-1}(A)})(f^{-1}(A))\to F(A)$ is a homeomorphism;

Preserves weight if ω(F(X))= ω(X) for an infinite bicompactum X;

It is continuous if for any inverse spectrum $S=\{X_\alpha;\pi_\beta^\alpha:\alpha\in A\}$ of abacompact sets, the homeomorphism is display, $f:F(\lim S)\to\lim F(S)$ which is the limit of the maps $F(\pi_\alpha)$if $\pi_\alpha:\lim S\to X_\alpha$ are the through projections of the spectrum S.

In what follows we assume that all the functors under consideration are monomorphic and preserve intersections. We also assume that all functors preserve non-empty spaces. This restriction is irrelevant, since by this we exclude from consideration only the empty functor, i.e. the functor F, which takes every space into an empty set. In fact, let F(X)= ∅ for some non-empty bicompactum X.

Then F(X)=F(1)=∅ by the monomorphism of F. Now let Y be an arbitrary non-empty bicompactum. Consider a constant map f:Y→1 Then $F(f)(F(f))\subset F(1)=\varnothing$. Consequently, the space *F*(*Y*) is empty, since it is mapped to an empty set. Thus, we have proved that there exists a unique monomorphic functor preserving non-empty sets.

Let *F*:*Comp*→*Comp* be a functor. We denote by C(*X*,*Y*) the space of continuous mappings from X and Y in a bicompact-open topology. In particular, *C*({*k*},*Y*) is naturally homeomorphic to the k^{th} power of Y^k in the space Y.

The point $\xi:\{k\}\to Y$ is mapped to a point $(\xi(0),\ldots\xi(k-1))\in Y^k$.

For the functor F, the bicompactum *X* of the natural number *k* the map $\pi_{F,X,k}:C(\{k\},X)\times F(\{k\})\to F(X)$ by the equality $\pi_{F,X,k}(\xi,a)=F(\xi)(a)$ where $\xi\in C(\{k\},X),\alpha\in F(\{k\})$.

When it is clear which functor and which bicompactum Y we are talking about, we denote the mapping $\pi_{F,X,k}$ by $\pi_{X,k}$ or π_k. The necessary facts pertaining to covariant functors and their properties can be found [1,2].

The Main Part

Lemma 1

Let *F*:*Tych*→*Tych* be a monomorphic preserving intersection, inverse images of mappings, continuous supports of the functor of degree ≤*n*.Then for any $i=\overline{0,n}$ the set $F_{i-i}(i)$ is open-closed in F(*i*) if *F*(*comp*)⊂*comp.*

Proof: The map $supp_F$:F(i)→exp(*i*) is continuous by condition. The set $exp_{i-1}(i)$ is open-closed in $exp_{i-1}(i)$ since the space $exp_{i-1}(i)$ is discrete. Therefore, the set $exp_{i-1}(i)$-$supp_F$(expi-1(i)) is open-closed in F(*i*) Lemma 1 is proved [3]. If the functor satisfies the conditions of Lemma 1, then by Theorem 5. 1 we have.

Theorem 1

If the functor F satisfies the conditions of Lemma 1, then any Tikhonov space X and any $i=\overline{0,n}$ mapping $\pi_{F,X,i}:X^i\times F(i)\to F_i(X)$ factor.

Theorem 2

Let *F*:*Tych*→*Tych* be a monomorphic preserving intersection, inverse images of the functor of degree ≤*n*, the set $F_{n-1}(\tilde{n})$ is open

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in $F(\tilde{n})$, then the functor F with continuous supports, if the map $\pi_{FXi}: X^n \times F(\tilde{n}) \to F_n(X)$ is closed.

Evidence: Let X be a Tikhonov space. It is necessary to show that the map $\mathrm{supp}_F: F(X) \to \exp_n X$ to each point by the carrier of this point is continuous. In this case we have the following diagram:

$$\begin{array}{ccc} X^n \times F(\tilde{n}) & \xrightarrow{\pi_{Fxn}} & F_n(X) \\ \downarrow \pi_{X^n} & & \downarrow \mathrm{supp}_F \\ X^n & \xrightarrow[\pi_n]{} & \exp_n X \end{array}$$

The mapping $\pi_n: X^n \to \exp_n X$ is determined in a natural way by the formula: for the point $x=(x_1,...,x_n) \in X^n$ put $\pi_n(x)=\{x_1,x_2,...,x_n\}$. It is known that the mapping \$_n$ is continuous.

Let $A \subset \exp_n X$ be closed. Set $B=\pi_n^{-1}(A) \subset X^n$. The set B is closed in X^n, since the mapping π_n is continuous. The following equality holds; $\pi_{Fxn}(B \times F^*(n)) = \mathrm{supp}_F^{-1}(A)$ where $F^*(\tilde{n}) = F(\tilde{n}) \setminus F(\tilde{n}-1)$. Then the set $B \times F^*(n)$ is closed in $Xn \times F(n)$. From the closedness of the mapping π_{FXn} the set $\mathrm{supp}_F^{-1}(A) = \pi_{Fxn}(B \times F^*(n))$ is closed, i.e. the mapping suppF(a) is continuous. Theorem 2 is proved.

Definition

A continuous functor F:*Tych*→*Tych* with finite support ≤*n* is said to be closed if the map $\pi_{Fxn}: X^n \times F(\tilde{n}) \to F(X)$ is closed.

T_1 - the space X is called stratified (lacy, meekly S -space) if to each open set $U \subset X$ we can associate the sequence $\{U_n : n \in N\}$ of open subsets in such a way that the following conditions are satisfied [4];

a) $\overline{U_n} \subset U$ for all $n \in N$;

b) $\cup\{Un : n \in N\} = U$

c) if $U \subset V$, then $U_n \subset V_n$ for all n.

We note [5] that S -spaces are completely normal and paracompact, and that the finite union and countable product of an S -space again is an S -space. It was shown that every S-space is a σ-space. Hence an S -space is a paracompact σ-space [5].

If the functor F:*Tych*→*Tych* is closed, then the space $F(X)$ is an S-space if and only if X is an S-space and $F(\tilde{n})$ is also an S-space. Since the space $X^n \times F(\tilde{n})$ is an S-space [4-6]. The S-space is preserved with closed mappings [3].

Hence it is the case.

Theorem 3

Normal closed functors F:*Tych*→*Tych* with finite support ≤n, preserve the category of S-spaces.

Definition: A Hausdorff space X- is called a $\aleph$-space if it can be mapped onto some S-space S by means of a perfect mapping [6].

Let F:*Tych*→*Tych* be a normal or semi-normal functor preserving S-spaces and perfect mappings. $F(St) \subset S$ и $F(f)$ is a perfect mapping if f is a perfect mapping in this case.

Theorem 4

Let F:*Tych*→*Tych* be a seminormal functor preserving S-spaces and perfect mappings. Then the functor F- preserves $\aleph$-spaces.

Since the closed functors preserve the category of S -spaces and perfect mappings, from this we have:

Theorem 5

Closed functors F:S→S preserve $\aleph$-spaces.For a seminormal functor F:*Comp*→*Comp* and a Tikhonov space X, we set $F_\beta(X) = \{a \in F(\beta X) : \mathrm{supp} a \subset X\}$, where βX-The Stuno-Chekhov expansion of the space X.

Lemma 2

If f:X→Y is a perfect map between Tikhonov spaces and A⊂Y then the equality $a \subseteq F(f^{-1}(A))$.

Proof: Let X,Y be Tikhonov spaces, and f:X→Y be completely continuous. $f^1(y)$ is bicompact for any $y \in Y$. Let $a \subseteq F(f^{-1}(A))$ from here $f(\mathrm{supp}_F a) \subset f^{-1}(A)$. From the fact that F preserves the inverse images of mappings, we have $f(\mathrm{supp}_F a) \subset F(A)$ or the same $\beta f(\mathrm{supp}_F(a)) \subset A$ By the seminormality of the functor F:*Comp*→*Comp*, $(\beta f)(\mathrm{supp}_F a) = \mathrm{supp}_F F(\beta f)(a)$. By the definition of the functor F in the categories of Tixonov spaces, we have $a \in F(f)^{-1} F(A)$ i. $a \in F(f)^{-1} F(A)$

Suppose that $a \in F(f)^{-1} F(A)$ .Therefore, $F(f)a \in F(f) F(f)^{-1} F(A) = F(f) a \in F(A)$ i. $\mathrm{supp} F(\beta f)(a) \subset A$. The inequality $(\beta f)\mathrm{supp} a = F(\beta f) a$ has $\beta(f)\mathrm{supp} a \subset A$. But for the perfect mapping f:X→Y and an arbitrary $A \subset Y$, the equality $(\beta f)^{-1}(A) = f^{-1} A$ Thus, for the seminormal functor F:*Comp*→*Comp* of the perfect mapping f:X→Y and for an arbitrary subset A⊂Y, the following equality:

(1) $(\beta f)^{-1}(A) = f^{-1} A$

(2) $(\beta f)(\mathrm{supp}_F a) = \mathrm{supp}_F F(\beta f)(a)$.

Therefore, $\mathrm{supp}_F(a) \subset f^{-1}(A)$. i. $a \in F(f)^{-1} F(A)$ Lemma 2 is proved.

Lemma 3

If f:X→Y is a perfect map between Tikhonov spaces X and Y then the map $F_\beta(f): F_\beta(X) \to F_\beta(Y)$ is also perfect.

Proof: It is known that for perfect mappings we have the equality $X=(\beta f)^{-1}Y$

In this case, by Lemma 2, we have the equality: $F_\beta(X) = F(\beta f)^{-1}(F_\beta(Y))$ Hence the image of $F\beta(f)$ is perfect. As the restriction of the perfect map of $F\beta(f)$ to the full preimage $F_\beta(\beta f)^{-1}(F(Y)) = F_\beta(X)$. Lemma 3 is proved. By Lemma 3 and Theorem 4-5, we obtain Theorem 6.

Theorem 6

Let $F\beta$: S→S be a seminormal functor. Then the functor $F\beta$ preserves $\aleph$ spaces. It is known that paracompact p-spaces are perfect preimages of metrizable spaces. In this case, from Lemma 2-3 in particular, we obtain

Theorem 7

Let F_β:Metr→Metr be a seminormal functor. Then the functor F preserves paracompact p-spaces.

Theorem 8

Let F:Metr→Metr be a seminormal functor with finite support ≤n, and $\dim F(\tilde{n}) < \infty$.

Then F_β preserves finite-dimensional metrizable spaces. Moreover, it is true: $\dim F(X) \le n \dim X + \dim F(\tilde{n})$.

Evidence: Let X be a metrizable space. In this case, the space $F\beta(X)$ is metrizable and $\dim F_n(X) = \dim \beta F_n(X)$. $\dim \beta F_n(X) \le \dim F_n(\beta X)$. By the Basmann's theorem, we have $\dim F_n(\beta X) \le n \dim \beta X + \dim F(\tilde{n})$. Hence the space $F_n(X)$ is finite-dimensional [7,8]. Moreover, the following inequality holds: $\dim F_n(X) \le n \dim X + \dim F_n(n)$. Theorem 8 is proved.

Using Theorem 4 similarly, as Theorem 8 we prove the following

Theorem 9

Let F:Metr→Metr be a seminormal functor with finite ≤n and $\dim F_n(n) < \infty$. Then *F* preserves finite-dimensional paracompact *p*-spaces. Moreover, it is true: $\dim F_n(X) \le n \dim X + \dim F_n(\tilde{n})$.

Applying the corresponding results [8] and Theorem 8, we prove the following.

Theorem 10

Let *F*: *S*→*S* be a seminormal functor with finite support ≤n and $\dim F_n(\tilde{n}) < \infty$. Then *F* preserves finite-dimensional *S*-spaces. Moreover, it is true: $\dim F_n(X) \le n \dim X + \dim F_n(\tilde{n})$.

Theorem 11

Let F:ℵ→ℵ be a seminormal functor with finite supports ≤n and $\dim F_n(\tilde{n}) < \infty$. Then the functor *F* preserves finite-dimensional ℵ-spaces. Moreover, it is true: $\dim F_n(X) \le n \dim X + \dim F_n(\tilde{n})$.

Now let *X* be a weakly-countable-dimensional space; *X* is the countable union of finite-dimensional subspaces X_i where $i \in N$. Hence $X = \bigcup_{i=1}^{\infty} X_i$ is closed in *X* and $\dim X_i < \infty$. We can assume that $X_i \subseteq X_{i+1}$, then the functor $F: Tych \to Tych$ holds $F = (\bigcup_{i=1}^{\infty} X_i) = \bigcup_{i=1}^{\infty} F(X_i) = F(X)$ If the functor F satisfies the conditions of Theorems 7-11, then the spaces *F*(*Xi*) are also finite-dimensional. Consequently, the space *F*(*X*) is weakly countable. Hence the versions of the assertions of Theorem 7-10 are true, and in the case of weakly-dimensionality of the spaces *X* i.e. if the functor *F* satisfies the conditions of Theorem 7-11, then *F* preserves weakly countable-dimensional spaces, respectively.

Conclusion

In this note we study covariant functors that are finite in the categories of *Comp* -compacta, *Metr*-metrizable, *S*-stratified, ℵ-spaces and paracompact *p*-spaces and continuous mappings into itself resulted in new search results of Finite Degree anyumaed stratifiable spaces.

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