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Comparison of Repetitive Control Schemes for a DC Servo Motor

Jacob S Glower*

Department of Electrical Engineering, North Dakota State University, Fargo, North Dakota, USA

Abstract

In this paper, four types of repetitive controllers are compared on a DC servo motor. It is shown that a Fourier-type repetitive controller is superior in terms of the adaptation rate and, for tracking a sine wave and a half-rectified sine wave.

Keywords: DC servo motor; PIC microcontroller; Repetitive control schemes

Introduction

Since repetitive controllers were first proposed [1], several variations have been proposed [2]. The four of which the author is aware include the following:

Repetitive Control

Each cycle is broken down into N time-slots. The ideal input (defined as the input required to force the desired motion) is then approximated as a constant during each time slot. The adaptive controller estimates these N constants.

Bilinear Repetitive Control

Instead of approximating the ideal input as a constant during each time slot, it is approximated as a line:

$$U_{ideal}=c_0+c_1\tau \quad (1)$$

where τ measures the time into the present time slot going from 0 to 1. By using a more accurate approximation, better tracking should result at the cost of estimating more parameters.

Polynomial Repetitive Control

Still further improvements in tracking accuracy can be obtained by using higher-order polynomials during each time slot:

$$U_{ideal}=c_0+c_1+c_2\tau^2 \quad (2)$$

Fourier Repetitive Control

Alternatively, since the ideal input is periodic, it can be expressed in terms of its Fourier series:

$$U_{ideal}=b_0+\Sigma((a_n\sin(n\omega t)+b_n\cos(n\omega t)) \quad (3)$$

where ω is the fundamental frequency of the periodic motion.

With four different schemes being proposed, one may wonder which scheme is superior. In terms of their long-term performance using an infinite number of terms, each is identical. As proven, asymptotic perfect model following can be assured using adaptive control techniques so long as the ideal input can be expressed as a known function matrix or basis function, W(t), times a constant but unknown matrix [3-11], K:

$$Uideal=W(t)K \quad (4)$$

Each of the proposed repetitive control schemes falls into this category. For example, W(t) for a repetitive controller consists of N delta functions: one per time slot. Similarly, W(t) for a Fourier repetitive controller consists of the harmonics of the repetition frequency.

Since each of the proposed repetitive control schemes can be described, each assures asymptotic perfect tracking. Likewise, any comparison must be based upon the performance in finite time and/or using a finite dimensional W(t) and K. Such a comparison is made in this paper.

This paper is organized as follows:

- In Section II, the DC servo motor is presented along with the repetitive control law with a generic basis function is derived for a DC servo motor.
- In Section III, the effect of using a finite number of terms on the tracking accuracy is presented. Unfortunately, this accuracy depends upon the particular motor and the particular path being followed. For the sake of comparison, therefore, two waveforms are selected: a 1 Hz sine wave and a 1 Hz half-wave rectified sine wave.
- In Section IV, results and conclusions are presented.

Hardware and Repetitive Control

Hardware set up

Clifton DC Servo Motor AMC 30A8T power amplifier PIC microcontroller.

Set Point

0.5 Hz Sine Wave (best case)

0.5 Hz Half Rectified Sine Wave (impulse in 2nd derivative).

Repetitive control

The ideal input is estimated using hyperstability-based MRAC [12]. For a DC servo motor, the dynamics are approximately:

$$\theta \approx \left(\frac{100}{s(s+8)}\right)V_a$$

***Corresponding author:** Jacob S Glower, Department of Electrical Engineering, North Dakota State University, Fargo, North Dakota, USA, Tel: 7012318068; E-mail: jacob.glower@ndsu.edu

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If this plant is used with a PI-type parameter estimation, a type-2 system will be created in the parameter estimates, resulting in a highly oscillatory system [13]. To prevent this, unity feedback is placed around the motor:

$V_a=(Ref-\theta)$

where Ref is the set point (reference angle), resulting in

$$\theta \approx \left(\frac{100}{s^2+8s+100}\right) Ref$$

Next, to track a given waveform, the input (Ref) is defined as

$R=k1b1(t)+k2b2(t)+$

where k_i are unknown constants and $b_i(t)$ are periodic functions that serve as the basis for the input required to force the angle to track some desired path, θR .

Hyperstability-based MRAC is used to estimate these constants as:

$$\left(\frac{100}{s^2+8s+100}\right)\left(\frac{s+1}{s}\right)$$

where the PI compensator gains were selected to keep the root locus of $\left(\frac{100}{s^2+8s+100}\right)\left(\frac{s+1}{s}\right)$ negative real, assuring stability of the MRAC system [13].

This algorithm is implemented on a PIC18F4620 microcontroller with a sampling rate of 50 Hz.

The set point to be tracked is a 1 Hz sine wave and a 1 Hz half-rectified sine wave to give a flavor for how each scheme behaves in the best case and in a fairly typical case.

The tracking error of the feedback control system with Ref=θR is show in Figures 1 and 2. Note that for each case, a significant delay exists, as is expected for an error driven device. In addition, for the half-wave rectified sine wave, the high-frequency terms have poor tracking.

Tracking with MRAC and a Finite Number of Terms

In theory, given any basis, $bi(t)$, which spans the ideal input (defined as the input required to track the set point exactly) will result in perfect tracking. With a finite number of terms, however, the tracking will be

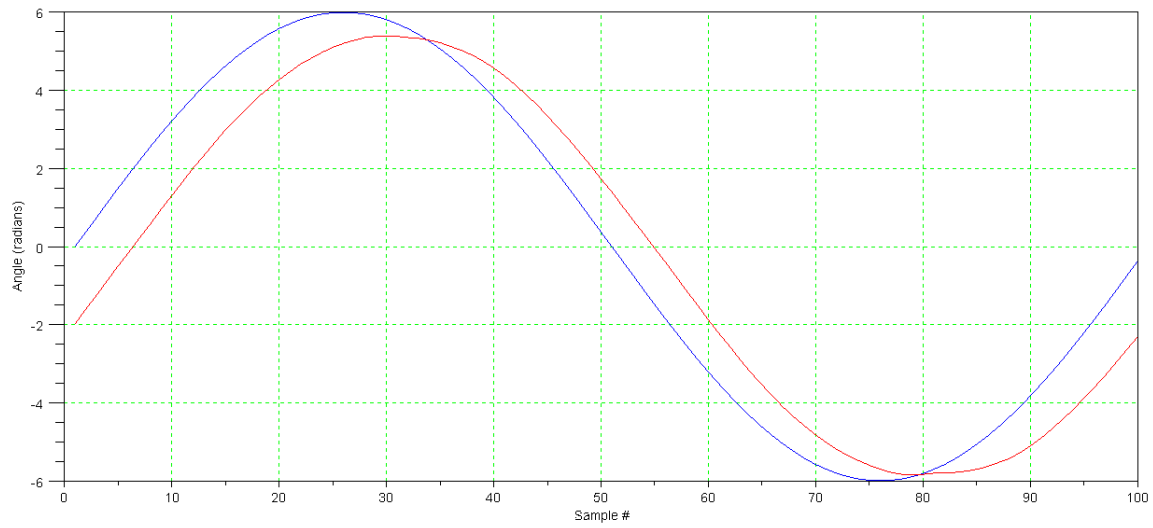

Figure 1: rms error = 0.12466 radians.

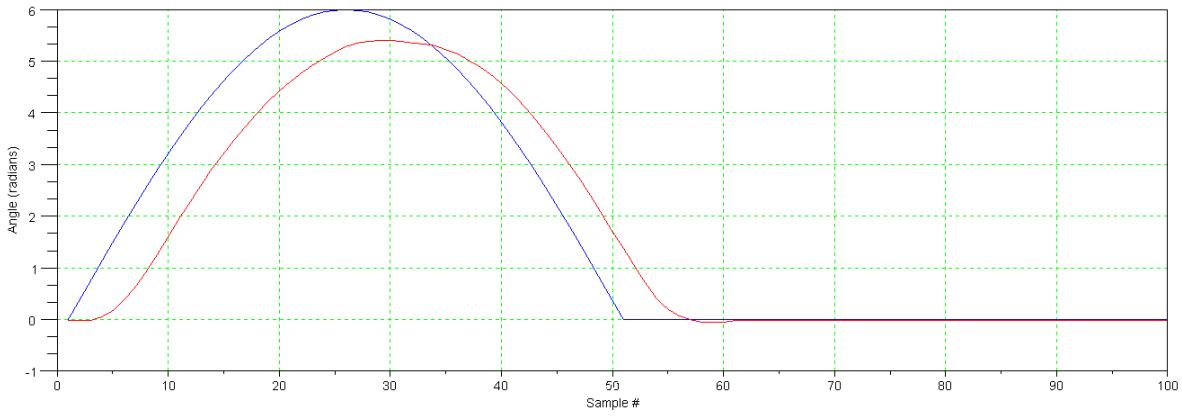

Figure 2: rms error = 0.0776009.

imperfect. In this section, the result of limiting the basis to 10 or fewer terms is investigated.

MRAC: Model-reference adaptive control is used to control the motor. The idea here is as follows:

The 'ideal' input is the input required to force the output to track the set point perfectly. Since the 'ideal' input is not known a-priori, express it in terms of a basis function:

$Uideal=\Sigma kiBi(t)$

where ki is a set of unknown constants and $Bi(t)$ is a set of basis functions.

Hyperstability-based model reference adaptive schemes tend to use a type of PID control with the estimation of the constants, ki. When trying to control a type-1 system, this in essence creates a type-2 system for the parameter estimates, creating stability issues. [10]. To eliminate this second integrator, a feedback loop around motor angle is created:

$U=k(Ref-\theta)$

where R is the set point or reference angle. Now, the 'ideal' input is the set point, R, which causes the motor to track the set point, θR.

For the following MRAC schemes, this technique was used as follows. The error between the set point and the actual angle is defined as:

$E=\theta R-\theta$

The reference angle is found using a set of basic functions:

$Ref=\Sigma ki\ Bi$

where ki are unknown constants estimated using the following PI scheme:

$$k_i=\left(\frac{s+1}{s}\right)E\cdot B_i$$

In theory, any basis function which spans all periodic inputs can work. In practice, some are probably better than others. The tracking ability for several basis functions follow [14-16]:

Repetitive control

$$B_i=\begin{cases}1 & \text{in time slot i}\\ 0 & \text{otherwise}\end{cases}.$$

Bilinear repetitive control

MRAC sets the midpoint in each time slot. Use linear interpolation between time slots (Figures 3-6).

Problem: This isn't guaranteed stable. The basis function is a constant for each time slot (the midpoint). The actual basis interpolates between adjacent time slots.

Legendre polynomials

A Legendre polynomial is an orthonormal power series (1, t, t^2, t^3,

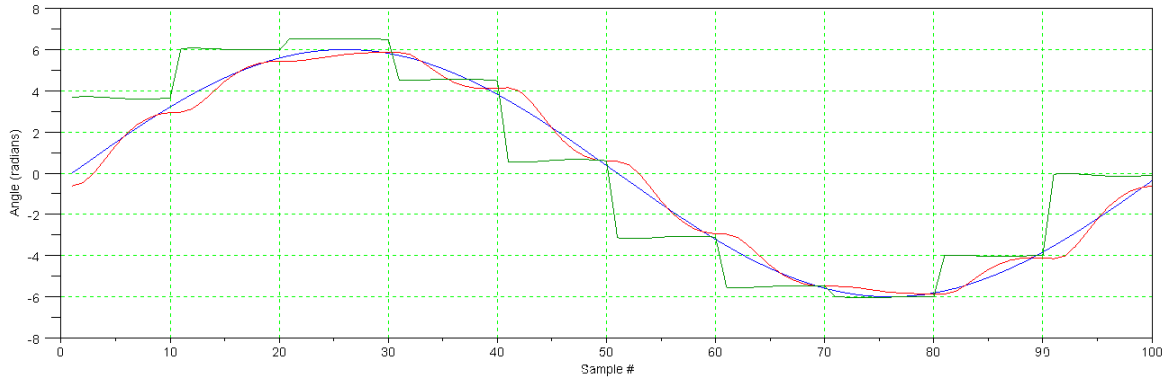

Figure 3: Ability to track a sinusoidal set point: Blue = reference angle, red = actual angle, green = input from MRAC algorithm. RMS error = 0.0360065 radians.

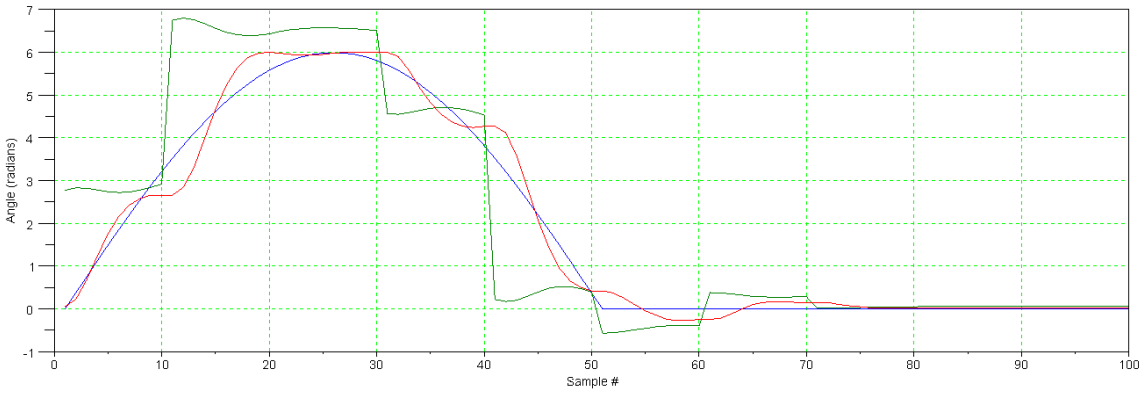

Figure 4: Ability to track a half-rectified sine wave. The RMS error is 0.0299643 radians.

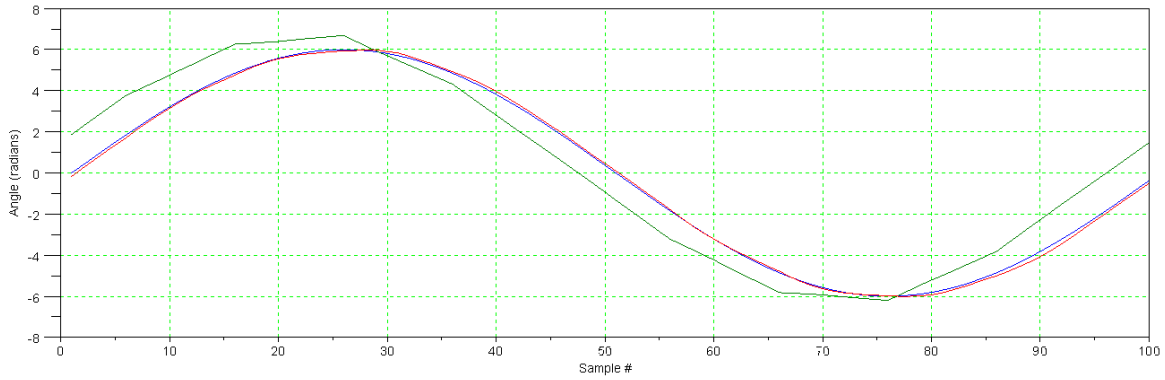

Figure 5: Tracking a sine wave using bilinear repetitive control RMS error = 0.0106850.

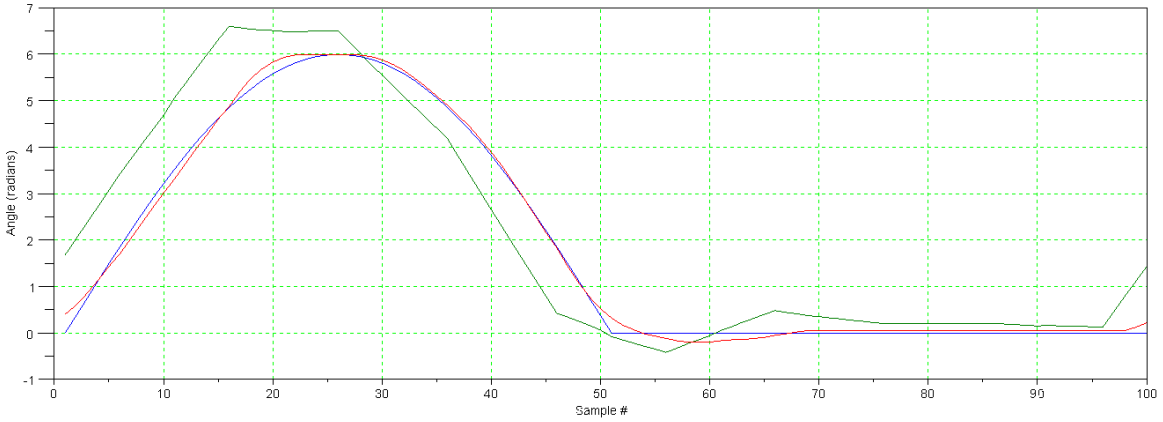

Figure 6: Tracking a half-rectified sine wave using bilinear repetitive control RMS error = 0.0119151.

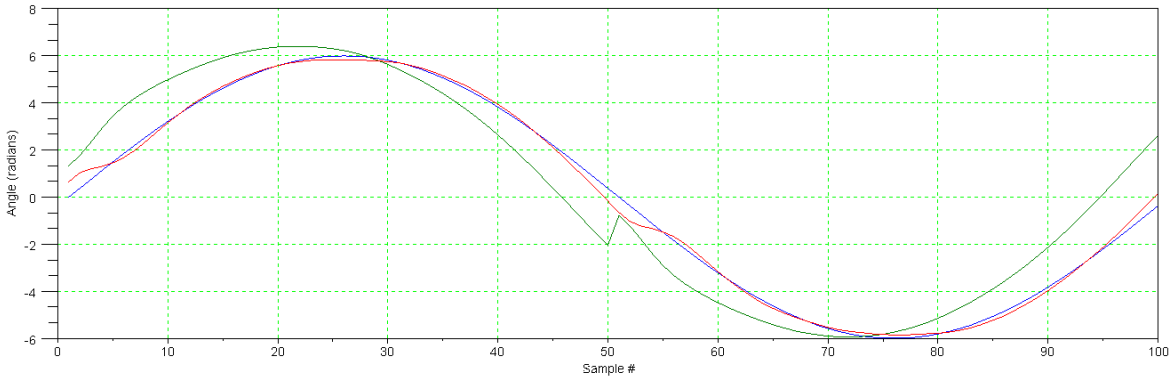

Figure 7: Tracking a sine wave using Legendre polynomials. Blue = set point, red = actual angle, green = input (Ref). Two regions are used: 0<t<T/2 and T/2<t<T. This creates the bump at T/2. RMS error = 0.0215202.

...). Defining the time slot to be $t \in [-1, 1]$ one valid set of basis functions in time are (Figures 7 and 8):

$B0=1$

$B_1=1.22t$

$B_2=1.44t^2-1$.

Fourier repetitive

Yet another valid basis comes from a Fourier series expansion. Since the 'ideal' input is periodic, it can be expressed in terms of it's

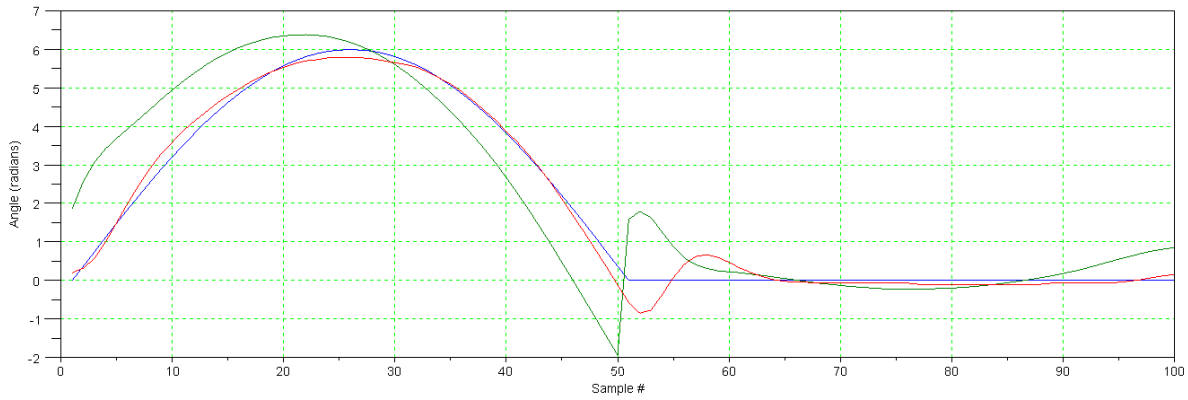

Figure 8: Tracking a half-wave rectified sine wave using Legendre polynomials. Two regions are used: 0<t<T/2 and T/2<t<T. This creates the bump at T/2. RMS error =0.0239508.

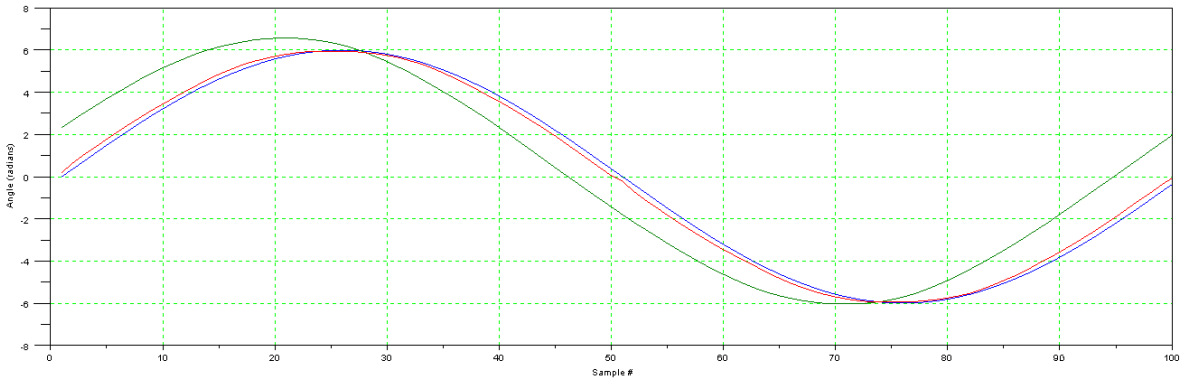

Figure 9: Tracking a sine wave using Fourier series. Blue = set point, red = actual angle, green = input (Ref). RMS error = 0.0219632.

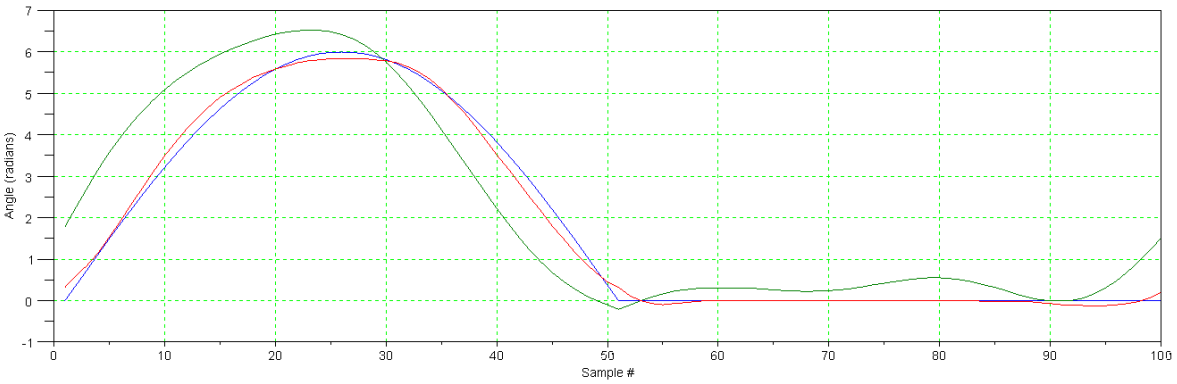

Figure 10: Tracking a half-wave rectified sine wave using a Fourier series. RMS error = 0.0158288.

Fourier series. The truncated series used here is (Figures 9 and 10):

B_i={1, sin(t), cos(t), sin(2t), cos(2t), sin(4t), cos(4t)}.

Results and Conclusions

RMS Error (radians * 10e-3).

Method	Sine Wave	1/2 Wave Rectified Sine Wave
Error Feedback	124.66	77.6009
10 Time Slots: Constant	36.0065	29.9643
10 time slots - interpolated	10.6850	11.9151
Polynomial	21.5202	23.9508
Fourier	21.9632	15.8288

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