

Combinatorial Optimization: Quantum, AI, Metaheuristics

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Introduction

Combinatorial optimization problems are central to many fields, requiring innovative algorithmic approaches ranging from exact methods to heuristics and quantum-inspired techniques. The solution landscape is constantly evolving, with new computational paradigms like quantum computing and advanced Artificial Intelligence (AI) models. Understanding the strengths of different methodologies is crucial for tackling increasingly complex real-world scenarios.

One paper explores the Quantum Approximate Optimization Algorithm (QAOA) for solving Quadratic Unconstrained Binary Optimization (QUBO) problems. It discusses how QAOA can be applied to find approximate solutions for complex combinatorial problems by leveraging quantum principles, offering a promising avenue for tackling NP-hard challenges with potential quantum advantage in the future. The work details the algorithmic structure and its application to specific instances of optimization problems [1].

Another comprehensive survey provides an overview of various metaheuristic algorithms used to solve combinatorial optimization problems. It categorizes and analyzes different metaheuristic approaches, including evolutionary algorithms, swarm intelligence, and local search methods, discussing their strengths, weaknesses, and applicability to a wide range of real-world problems. The paper highlights recent advancements and future research directions in the field [2].

A dedicated survey focuses on the application of Reinforcement Learning (RL) techniques to combinatorial optimization problems. It reviews different RL paradigms, such as value-based, policy-based, and actor-critic methods, and their adaptation for solving discrete optimization challenges like the Traveling Salesperson Problem and vehicle routing. The article critically evaluates the current state of the art, pinpointing open problems and potential future research avenues in this rapidly evolving interdisciplinary field [3].

A novel paper introduces a cut-generating plane algorithm for exactly solving Quadratic Unconstrained Binary Optimization (QUBO) problems. The method leverages cutting planes within a branch-and-cut framework to strengthen the linear programming relaxation, leading to efficient exact solutions for challenging instances of QUBO. The authors demonstrate its computational effectiveness on various benchmark problems, showcasing its potential for solving large-scale discrete optimization tasks [4].

Further work investigates quantum-inspired computing approaches for tackling combinatorial optimization problems. It explores how principles from quantum mechanics, such as superposition and entanglement, can be simulated on classical computers to develop heuristic algorithms that outperform traditional methods for certain problem classes. The paper demonstrates the application of these tech-

niques to specific optimization challenges, highlighting their potential for improved solution quality and computational efficiency [5].

A review delves into the significant advancements in applying Deep Reinforcement Learning (DRL) techniques to combinatorial optimization problems. It systematically explores how DRL models, leveraging neural networks, learn to make sequential decisions for finding optimal solutions in discrete spaces. The authors discuss various DRL architectures, their successful applications across different problem domains, and highlight critical challenges and promising future research directions in this interdisciplinary field [6].

Another review meticulously examines the advancements in using Deep Reinforcement Learning (DRL) to tackle combinatorial optimization problems. It discusses how deep learning architectures are integrated with reinforcement learning frameworks to learn effective heuristics for complex problems, often outperforming traditional algorithms. The authors categorize various DRL models and their applications across different optimization domains, offering insights into their mechanisms and performance characteristics, alongside future research challenges [7].

A paper provides an overview of variational quantum algorithms (VQAs) applied to combinatorial optimization. It explains how hybrid quantum-classical approaches, such as the Variational Quantum Eigensolver (VQE) and the Quantum Approximate Optimization Algorithm (QAOA), are designed to tackle complex optimization problems by leveraging quantum hardware. The authors discuss the theoretical foundations, implementation challenges, and potential future impact of VQAs in addressing problems currently intractable for classical computers [8].

A thorough examination reviews recent advancements in metaheuristic algorithms specifically tailored for solving facility location problems, a class of combinatorial optimization challenges. It surveys various metaheuristic approaches, including nature-inspired algorithms and local search techniques, highlighting their effectiveness in determining optimal or near-optimal locations for facilities under different constraints and objectives. The authors discuss the methodologies, comparative performance, and future research directions within this critical operational research domain [9].

A paper presents a multi-objective evolutionary algorithm designed to address robust combinatorial optimization problems, where solutions need to perform well under uncertainty. The algorithm aims to find a set of Pareto-optimal solutions that are not only efficient but also resilient to variations in problem parameters. The authors detail the algorithmic structure, its robustness-enhancing mechanisms, and demonstrate its performance on benchmark problems, showing its capability to handle uncertainty effectively in complex optimization scenarios [10].

Description

Research into combinatorial optimization often branches into fundamentally different computational paradigms. Quantum computing presents a particularly promising frontier. The Quantum Approximate Optimization Algorithm (QAOA) is actively investigated for solving Quadratic Unconstrained Binary Optimization (QUBO) problems [1]. This approach leverages quantum principles to find approximate solutions for complex NP-hard challenges, hinting at a quantum advantage down the line. The foundational structure of QAOA and its specific applications to various optimization problems illustrate this path. Extending this, variational quantum algorithms (VQAs) offer another perspective for combinatorial optimization [8]. These are hybrid quantum-classical strategies, encompassing methods like the Variational Quantum Eigensolver (VQE) and QAOA itself. They leverage quantum hardware to approach complex problems. Discussions around their theoretical underpinnings, implementation hurdles, and future impact are critical for pushing the boundaries of what is currently considered intractable for classical systems. Quantum-inspired computing on classical machines also plays a part [5]. It simulates quantum mechanical principles such as superposition and entanglement to create heuristic algorithms that can sometimes outperform traditional methods for specific problem types, showing potential for better solution quality and computational efficiency in various optimization challenges.

Traditional and modern metaheuristic algorithms continue to be central to solving combinatorial optimization problems. A comprehensive survey categorizes and analyzes diverse metaheuristic approaches [2]. This includes evolutionary algorithms, swarm intelligence, and local search methods, detailing their strengths, weaknesses, and how they apply to many real-world problems. This work also highlights where the field is heading. Specifically, for challenges like facility location problems, metaheuristic algorithms have seen considerable advancements [9]. These techniques, which include nature-inspired and local search methods, are effective at finding optimal or near-optimal facility locations under varying constraints. Understanding their methodologies and comparative performance provides crucial insights for operational research. Beyond single-objective problems, a multi-objective evolutionary algorithm has been developed to tackle robust combinatorial optimization problems [10]. Here, solutions must perform well even when faced with uncertainty. This algorithm seeks Pareto-optimal solutions that are both efficient and resilient to parameter variations, showcasing its ability to handle complex scenarios effectively.

Artificial Intelligence, particularly Reinforcement Learning (RL), has emerged as a powerful tool for combinatorial optimization. A key survey focuses on RL techniques [3], reviewing different paradigms like value-based, policy-based, and actor-critic methods. It shows how these methods adapt to solve discrete optimization problems such as the Traveling Salesperson Problem and vehicle routing. This critical evaluation also points out open problems and future research directions in this interdisciplinary area. Taking this a step further, Deep Reinforcement Learning (DRL) has made significant strides [6]. DRL models utilize neural networks to learn sequential decision-making for finding optimal solutions in discrete spaces. Various DRL architectures and their successful applications across different problem domains are discussed, along with critical challenges and promising research paths. Another detailed review underscores these advancements [7], showing how deep learning architectures integrate with RL frameworks. This integration allows for learning effective heuristics that often outperform conventional algorithms. The categorization of DRL models and their applications, mechanisms, and performance characteristics, alongside future challenges, paints a clear picture of this evolving field.

While heuristics and approximate methods are widely used, the pursuit of exact solutions for complex problems remains a vital area. For Quadratic Unconstrained

Binary Optimization (QUBO) problems, a novel cut-generating plane algorithm offers an exact solution [4]. This method enhances linear programming relaxation through cutting planes within a branch-and-cut framework. This leads to efficient and precise solutions for even very challenging QUBO instances. The algorithm's effectiveness has been demonstrated on various benchmark problems, proving its potential for large-scale discrete optimization tasks. The ongoing development of both exact and approximate methods underscores the complexity and importance of combinatorial optimization, with researchers continually exploring new avenues to improve solution quality and computational efficiency across a broad spectrum of problem types.

Conclusion

Research into combinatorial optimization explores a wide range of algorithmic solutions, from quantum computing paradigms to classical metaheuristics and advanced Artificial Intelligence methods. For instance, the Quantum Approximate Optimization Algorithm (QAOA) is actively investigated for solving Quadratic Unconstrained Binary Optimization (QUBO) problems. This approach leverages quantum principles to find approximate solutions for complex NP-hard challenges, presenting a promising path for future quantum advantage. Concurrently, comprehensive surveys provide overviews of various metaheuristic algorithms, including evolutionary algorithms, swarm intelligence, and local search methods, discussing their strengths and applicability to real-world problems. Reinforcement Learning (RL) techniques are also gaining prominence in this field. Surveys focus on adapting different RL paradigms, such as value-based, policy-based, and actor-critic methods, for discrete optimization challenges like the Traveling Salesperson Problem. For exact solutions, a novel cut-generating plane algorithm has been introduced for QUBO problems, employing cutting planes within a branch-and-cut framework to achieve efficient exact solutions. Quantum-inspired computing explores simulating quantum mechanics principles on classical computers to develop heuristic algorithms, aiming for improved solution quality and computational efficiency. Deep Reinforcement Learning (DRL) shows significant advancements, with models leveraging neural networks to make sequential decisions for optimal solutions in discrete spaces. Variational Quantum Algorithms (VQAs), hybrid quantum-classical approaches like VQE and QAOA, are also being developed to tackle complex optimization problems using quantum hardware. Furthermore, multi-objective evolutionary algorithms address robust combinatorial optimization problems, focusing on finding efficient and resilient Pareto-optimal solutions under uncertainty.

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Conflict of Interest

None.

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