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Bounded Coordination Control of Second-order Dynamic Agents

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Abstract

This paper presents a constructive design of distributed and bounded coordination controllers that force mobile agents with second-order dynamics to track desired trajectories and to avoid collision between them. The control design is based on the new bounded control design technique for second-order systems, and new pairwise collision avoidance functions. The pair wise collision functions are functions of both the relative position and velocity of the agents instead of only the relative position as in the literature. Desired features of the proposed control design include:

1) Boundedness of the control inputs by a predefined bound despite collision avoidance between the agents considered,

2) No collision between any agents,

3) Asymptotical stability of desired equilibrium set, and

4) Instability of all other undesired critical sets of the closed loop system. The proposed control design is then applied to design a coordination control system for a group of vertical take-off and landing (VTOL) aircraft.

Keywords: Second-order agents; Bounded coordination control; Collision avoidance; Potential functions; VTOL aircraft

Introduction

Coordination control of multiple agents has received a lot of attention from researchers in the control community due to its various applications to search, rescue, coverage, surveillance, reconnaissance and cooperative transportation. Therefore, a number of approaches have been available for coordination control of networked agents. Here, three common approaches are briefly mentioned. The leader-follower approach (e.g., [1-4] uses several agents as leaders and others as followers. This approach is easy to understxand and ensures coordination maintenance if the leaders are disturbed. However, the desired coordination shape cannot be maintained if followers are perturbed unless a feedback is implemented [5]. The behavioral approach (e.g., [6,7]), where each agent locally reacts to actions of its neighbors, is suitable for decentralized control but is difficult in control design and stability analysis since group behavior cannot explicitly be defined. The virtual structure approach (e.g., [8-17]) treats all agents as a single entity. The virtual structure approach is amenable to mathematical analysis but has difficulties in controlling critical points, especially when collision avoidance between the agents is a must. The control coordination design in this paper belongs to the virtual structure approach. In the literature, cooperative control of multiple agents with second-order dynamics was also addressed, see for example [12,18-30]. However, the problem of bounded control has not been addressed for the case where collision avoidance between the agents must be considered.

Let us discuss the reasons why it is difficult to design a bounded coordination controller for multiple agents with second-order dynamics when collision avoidance between the agents must be considered using the virtual structure approach. In this approach, the control design is usually based on a nontrivial potential function and the direct Lyapunov method. The potential function essentially consists of two parts. The first part is usually referred to as the goal part. This part is designed such that it puts penalty on the tracking errors of the agents, and is equal to zero when all the agents perfectly track their desired trajectory. The second part is usually referred to as the collision avoidance part, which is chosen such that it is equal to infinity whenever any two agents come in contact, i.e., a collision occurs, and attains the minimum value when the agents perfectly track their desired trajectories. Since the collision avoidance part of the potential function is designed to be equal to infinity when there is a collision between any two agents, the control signals become extremely large when any two agents are close to each other. To overcome this drawback, a method was proposed in [10] to design bounded controllers for the agents with the first-order dynamics. However, the method in [10] cannot be extended to the agents with second-order dynamics by applying the back stepping technique in [31]. To see where the problem lies, let us consider a group of N agents with the dynamics $\dot{q}_i = u_i,\ i \in N$, where q_1 and u_1 denote the position and control input vectors of the agent *i*, respectively, and N is the set containing all the agents in the group. Next, let us denote the collision avoidance part of the potential function be β, which is equal to infinity when there is a collision between any two agents. Then the function β must be a summation or a product of all pairwise collision avoidance functions β_{ij} of the agents i and j, i.e., $\beta = \sum_{(i,j)\in N, j\neq i} \beta_{ij}$ or $\beta = \prod_{(i,j)\in N, j\neq i} \beta_{ij}$. Moreover, the function β_{ij} must be a positive definite function of $\frac{1}{\|q_{ij}\|}$, where $q_{ij}=q_i-q_j$, i.e., the relative position vector between the agents i and j, As a result, the control u_i must be a function of $\beta_{ij} = \frac{\partial \beta_{ij}}{\partial q_{ij}}$. Since β_{ij} is a positive definite function of $\frac{1}{\|q_{ij}\|}$, β_{ij} tends to infinity faster than β_{ij} when $\|q_{ij}\|$ approaches zero, i.e., when a collision between the agents i and j tends to occur. Consequently, it is not possible to extend the bounded control design in [10] using the

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back stepping technique to design a bounded coordination controller for the agents with second-order dynamics. Using the same analysis as above shows that it is not possible (or at least not clear how) to extend the bounded controllers for connectivity preservation of a network of the agents with the first order dynamics in [32] and [33] to design a bounded controller for the agents with second-order dynamics.

Motivated by the above discussion and the fact that many mechanical systems in practice are of second-order dynamics and require bounded control inputs with a predefined bound, this paper contributes a new method to design bounded coordination controllers for mobile agents with second-order dynamics. The method is based on a new technique to design a bounded controller for second-order systems, and a construction of new pairwise collision avoidance functions, which are functions of both the relative position and velocity of the agents. The proposed bounded coordination controllers guarantee no collision between any agents, asymptotical stability of desired equilibrium set, and instability of all other undesired critical sets of the closed loop system. The applicability of the proposed method is illustrated through an example of designing a coordination control system for a group of VTOL aircraft.

The rest of the paper is organized as follows. In the next section, the control objective is stated. Section 3 gives essential preliminary results including saturation functions, a technique for designing bounded controllers for a second-order system, a non-zero convergent lemma for a differential inequality, smooth step functions, pair wise collision avoidance functions, and Barbalat-like lemma. Proofs of preliminary results are given in [34,10] and Appendices A and B. These results are to be used in the control design in Section 4 and stability analysis in Appendix C. An application of the proposed coordination control design to a group of VTOL aircraft is presented in Section 5. Stability analysis of coordination control of VTOL aircraft is briefly given in Appendix D. Finally, conclusions are given in Section 6.

Problem Statement

Agent dynamics

We consider a group of mobile agents, of which each agent has the following dynamics

$$\begin{aligned} \dot{q}_i &= p_i, \\ \dot{p}_i &= u_i,\ i = 1, \ldots, N, \end{aligned} \tag{1}$$

Where $q_i \in R^n$ and $u_i \in D \subset R^n$ are the state and control input of the agent i. We assume that n>1 and N>1.

Coordination control objective

In order to design a coordination control system, it is necessary to specify a common goal for the group and initial positions and velocities of the agents. We therefore impose the following assumption on the reference trajectories and initial conditions between the agents.

Assumption 4.1:

i) The reference position vector $q_{id}(t)$ for the agent i to track satisfies the condition:

$$\begin{aligned} &\| q_{ijd}(t) \| \geq \varepsilon_{1ijd}, \\ &\dot{q}_{id}(t) = \dot{q}_{od}(t),\ \ddot{q}_{id}(t) = \ddot{q}_{od}(t), \\ &\| \dot{q}_{od}(t) \| \leq \varepsilon_{2od},\quad \| \ddot{q}_{od}(t) \| \leq \varepsilon_{3od}, \end{aligned} \tag{2}$$

for all (i, j) € N, j ≠ i and $t \geq t_0$ where $q_{ijd}=q_{id}(t)-q_{jd}(t)$, ε_{1ijd} is a strictly positive constant, ε_{2od} and ε_{3od} are nonnegative constants, and $q_{od}(t)$ is referred to as the common reference trajectory.

ii) Let us define

$$\begin{aligned} q_{ij} &= q_i - q_j, \\ p_{ie} &= p_i - \dot{q}_{id}, \\ \chi_{ij} &= q_{ij}^T [K q_{ij} + (I_n + \Delta(p_{ie})) p_{ie} - (I_n + \Delta(p_{je})) p_{je}], \end{aligned} \tag{3}$$

for all (i, j) € N, j ≠ i, with N the set of all the agents, k is a symmetric positive definite matrix, I_n is an n×n dimensional identity matrix, and $\Delta(x)$ with $x \in R^n$ is an n×n dimensional diagonal and nonnegative definite matrix, whose elements are bounded if x is bounded. In addition, the matrix $\Delta(x)$ possesses the following properties:

$$\begin{aligned} &1)\ \dot{\Delta}(x) x = L \Delta(x) \dot{x},\ \forall\ (x, \dot{x}) \in R^{2n}, \\ &2)\ \| (I_n + (I_n + L)\Delta(x))^{-1} x \| \leq \varrho,\ \forall\ x \in R^n, \\ &3)\ \Delta(x_1)x_1 - \Delta(x_2)x_2 = A(x_1, x_2)(x_1 - x_2),\ \forall\ (x_1, x_2) \in R^{2n}, \end{aligned} \tag{4}$$

Where L is a diagonal positive definite matrix, ϱ is a nonnegative constant, and $A(x_1,x_2)$ is a diagonal and nonnegative definite matrix for all $(x_1,x_2) \in R^{2n}$. There are many matrices that satisfy the above properties. An example is $\Delta(x)= \mathrm{diag}\,(x_1^2,\ldots, x_i^2,\ldots x_n^2)$ where $x_1,\ldots, x_i,\ldots,x_n$ are elements of x, i.e., $x=[x_1,\ldots, x_i,\ldots x_n]^T$. At the initial time $t_0 \geq 0$, each agent starts at a different location and all the agents do not approach each other at high relative velocities. Specifically, there exist strictly positive constants ϱ_{ij0} and χ_{ij0} such that for all (i, j) € N, with i ≠ j, with, where N is the set of all the agents, the following conditions hold:

$$\begin{aligned} &\| q_{ij}(t_0) \| \geq \varrho_{ij0}, \\ &\chi_{ij}(t_0) \geq \chi_{ij0}. \end{aligned} \tag{5}$$

iii) The agents i and j can communicate with each other, i.e., the agent i can measure the states q_j and p_j of the agent j, and the agent j can measure the states q_i and p_i of the agent i, if the following condition holds:

$$\chi_{ij} \leq \chi_{ij}^M, \tag{6}$$

Where χ_{ij}^M is a strictly positive constant.

Remark 4.1:

i) Assumption 4.1.1 specifies feasible reference trajectory qid (t) for the agent i in the group to track since it has to satisfy the condition (2). A desired coordination shape can be specified by the reference trajectories $q_{id}(t)$ with i € N. Let us consider the virtual structure approach in [8-10] to generate the reference trajectories $q_{id}(t)$ the agent i to track. First, a virtual structure consisting of N vertices is designed as a desired coordination shape. Second, we let the center of the virtual structure move along the common reference trajectory $q_{od}(t)$. Third, as the virtual structure moves, its vertex i generates the reference trajectory $q_{id}(t)$ for the agent i to track. Specifically, the reference trajectory $q_{id}(t)$ can be generated as $q_{id}(t) = q_{od}(t) + l_i$ where l_i is a constant vector.

Moreover, the second equation in (2) implies that all the agents have the same desired velocity and acceleration. Since this paper focuses on designing bounded coordination controllers for the agents with second-order dynamics, we do not consider the case where the shape of the virtual structure is time-varying, i.e., the vectors l_i are time-varying, to avoid complication of the presentation. When l_i are time-varying, the technique proposed in [35] can be used in conjunction with the coordination control design in this paper.

ii) If at the initial time t_0 the agents approached each other at high relative velocities, a bounded controller would not be able to prevent the agents from colliding with each other. Therefore, it is reasonable to impose Assumption 1.2 for the design of bounded coordination controllers, which guarantee collision avoidance between the agents.

Assumption 4.1.3 implies that we need to design a distributed coordination control system since the condition (6) specifies that when $\chi_{ij} > \chi_{ij}^M$ the agents and do not communicate with each other. The constant $\sqrt{\chi_{ij}^M}$ can be considered as the generalized communication range between the agents and . This generalized communication range is different from usual communication ranges used in the literature, e.g. [12, 10,19] in the sense that the generalized communication range relates to both the relative position and the relative velocity between the agents while the usual communication range relates only to the relative position. Dependence of $\sqrt{\chi_{ij}^M}$ on the relative velocity between the agents is necessary because we are to design bounded controllers.

Coordination control objective 4.1: Under Assumption 2.1, for each agent i design the bounded and distributed control u_i such that the position vector q_i of the agent i tracks its reference position vector q_{id} while avoiding collision with all other agents in the group. Specifically, we will design u_i such that

$$\begin{aligned} &\| u_i(t) \| \le \delta, \\ &\lim_{t\to\infty}(q_i(t) - q_{id}(t)) = 0, \\ &\| q_i(t) - q_j(t) \| \ge \varrho_{ij}, \end{aligned} \tag{7}$$

For all (i, j) € N, i ≠ j and, $t \ge t_0 \ge 0$ where δ and ϱ_{ij} are strictly positive constants. The constant δ is strictly larger than ε_{3od}, which is defined in (2). Moreover, the control u_i should depend only on its own states and the states of other agents j if the condition (6) between the agent i and the agents j holds.

Preliminaries

This section presents saturation functions, a technique for designing bounded controllers for a second-order system, and non-zero convergent lemma for a differential inequality, smooth step functions, pairwise collision avoidance functions, and Barbalat-like lemma. These preliminary results will be used in the control design and stability analysis later.

Saturation functions

This subsection defines saturation functions that will be used in the control design later.

Definition 5.1: The function σ(x) is said to be a smooth saturation function if it possesses the following properties:

$$\begin{aligned} &1)\ \sigma(x) = 0 \text{ if } x = 0,\ \sigma(x)x > 0 \text{ if } x \ne 0, \\ &2)\ \sigma(-x) = -\sigma(x),\ (x-y)[\sigma(x) - \sigma(y)] \ge 0, \\ &3)\ |\sigma(x)| \le 1,\ |\frac{\sigma(x)}{x}| \le 1,\ |\frac{d\sigma(x)}{dx}| \le 1, \end{aligned} \tag{8}$$

for all (x, y) €R2. Some functions satisfying the above properties include σ(x) = than (x) and $\sigma(x) = \frac{x}{\sqrt{1+x^2}}$. For the vector $x=[x_1,\ldots,x_i,\ldots,x_n]^T$, we use the notation $\sigma x=[\sigma(x_1),..,\sigma(x_i)\ldots,\sigma(x_n)]^T$ to denote the smooth saturation function vector of x.

Bounded control design for second-order systems

This subsection presents the idea of a bounded control design technique through a simple example. This technique will be used to design bounded cooperative controllers later. As such, consider the following second-order system:

$$\begin{aligned} &\dot{x}_1 = x_2, \\ &\dot{x}_2 = u, \end{aligned} \tag{9}$$

were x_1 and x_2 are the states, and is the control input. Let us address a control problem of designing the control input u to asymptotically stabilize (9) at the origin for any initial values$(x_1(t_0),x_2(t_0))$€R^2 at the initial time $t_0 \ge 0$ such that $|u(t)| \le \varepsilon$ for all $t \ge t_0$. A solution to the above control problem is given in the following lemma.

Lemma 5.1: Let the positive constants K and C be chosen such that 0.5k + c ≤ ε, and let $\sigma(\bullet)$ be a smooth saturation function of • defined i n Definition 3.1. The bounded control law

$$u = \frac{}{1+1.5x_2^2}(-kx_2 - c\sigma(kx_1 + (1+0.5x_2^2)x_2)) \tag{10}$$

Globally asymptotically stabilizes the system (9) at the origin.

Proof: Refer to Appendix A.

Remark 5.1: There are other methods (e.g., [36,37]) to design bounded control laws for a chain of integrators inspired by the work in [38]. However, these methods are not suitable for designing a bounded coordination controller for multiple agents in this paper.

Non-zero convergent lemma

This subsection presents a non-zero convergent result for a first order system. This result will be used to construct pairwise collision avoidance functions in Subsection 3.5.

Lemma 5.2: Assume the continuous vector x (t) € R^n and its derivative x, which is also continuous, satisfy the following conditions:

$$\begin{aligned} &\| x(t_0) \| \ge a_0, \\ &x^T(Bx + (I_n + Q(t))\dot{x}) \ge a,\ \forall t \ge t_0 \ge 0, \end{aligned} \tag{11}$$

where $t_0 \ge 0$ is the initial time, B is a symmetric positive definite matrix, I_n is an n×n dimensional identity matrix, $Q(t)$ is a diagonal and nonnegative definite matrix whose elements are bounded for all, t ≥ t0 ≥ 0, a_0 and a are strictly positive constants. Then

$$\| x(t) \| \ge \min(a_0, \sqrt{\frac{a}{\lambda_M(B)}}),\ \forall\ t \ge t_0 \ge 0, \tag{12}$$

where $\lambda_M(B)$ is the maximum eigenvalue of the matrix .

Proof: Refer to Appendix B.

Smooth step function

This subsection gives a definition of the smooth step function followed by a construction of this function. The smooth step function is to be embedded in a pairwise collision avoidance function to avoid discontinuities in the control law in solving the collision avoidance problem.

Definition 5.2: A scalar function h(x, a, b) is said to be a smooth step function if it possesses the following properties

$$\begin{aligned} &1)\ h(x,a,b) = 0,\ \forall\, x \in (-\infty, a], \\ &2)\ h(x,a,b) = 1,\ \forall\, x \in [b, \infty), \\ &3)\ 0 < h(x,a,b) < 1,\ \forall\, x \in (a,b), \\ &4)\ h(x,a,b) \text{ is smooth}, \\ &5)\ h'(x,a,b) > 0,\ \forall\, x \in (a,b), \end{aligned} \tag{13}$$

where $h'(x,a,b) = \frac{\partial h(x,a,b)}{\partial x}$, and a and b are constants such that a<b.

Lemma 5.3: Let the scalar function h(x,a,b) be defined as

$$h(x,a,b) = \frac{f(\tau)}{f(\tau)+f(1-\tau)} \text{ with } \tau = \frac{x-a}{b-a}, \tag{14}$$

where

$$f(\tau) = 0 \text{ if } \tau \le 0, \text{ and } f(\tau) = e^{-\frac{1}{\tau}} \text{ if } \tau > 0, \tag{15}$$

with a and b being constants such that a<b. Then the function (x,a,b) is a smooth step function.

Proof: See [34]. An alternative smooth step function is available in [39] but it requires a numerical integration.

Pair wise collision avoidance functions

This subsection defines and constructs pair wise collision avoidance functions. In constructing these functions, we utilize Lemma 3.1, Lemma 3.2, Definition 3.2, and Lemma 3.3. The pair wise collision avoidance functions will be embedded in a potential function for the coordination control design in the next section.

Definition 5.3: Let β_{ij} with (i, j) € N and i ≠ j be a scalar function of χ_{ij}, which is defined in (3). The function β_{ij} is said to be a pair wise collision avoidance function if it possesses the following properties:

$$\begin{aligned}
&1)\ \beta_{ij} = 0,\ \beta_{ij} = 0,\ \beta_{ij} = 0,\ \forall\, \chi_{ij} \in [\chi_{ij}^*, \infty),\\
&2)\ \beta_{ij} > 0,\ \forall\, \chi_{ij} \in (0, \chi_{ij}^*),\ \beta_{ij} \le 0,\ \forall\, \chi_{ij} \in R,\\
&3)\ \lim_{\chi_{ij}\to 0} \beta_{ij} = \infty,\ \lim_{\chi_{ij}\to 0} \beta_{ij} = -\infty,\ \lim_{\chi_{ij}\to 0} \beta_{ij} = -\infty,\\
&4)\ \beta_{ij} \text{ is smooth, } \forall\, \chi_{ij} \in (0, \infty),
\end{aligned} \tag{16}$$

where $\beta_{ij} = \frac{\partial \beta_{ij}}{\partial \chi_{ij}}$, $\beta_{ij} = \frac{\partial^2 \beta_{ij}}{\partial \chi_{ij}^2}$, and χ_{ij}^* is a strictly positive constant satisfying the following condition:

$$\chi_{ij}^* \le \min(\chi_{ijd}, \chi_{ij}^M), \tag{17}$$

where $\chi_{ijd} = \chi_{ij}\,|_{q_{ij}=q_{ijd},\, p_{ie}=0,\, p_{je}=0} = q_{ijd}^T K q_{ijd}$, and the constant χ_{ij}^M is defined in (6).

Remark 5.2: Property 1) implies that the function β_{ij} is zero when the agents i and j are at their desired locations or are sufficiently faraway from each other since the constant χ_{ij}* satisfies the condition (17). Property 2) implies that the function β_{ij} is positive definite when the agents i and j are sufficiently close to each other. Property 3) means that the function β_{ij} is equal to infinity when a collision between the agents i and j occurs. Property 4) allows us to use control design and stability analysis methods found in [40] for continuous systems instead of techniques for switched and discontinuous systems found in [41] to handle the collision avoidance problem.

Using the smooth step function given in Definition 3.2, we can find many functions that satisfy all the properties listed in (16). An example is

$$\beta_{ij} = \ell_{ij} \frac{1 - h(\chi_{ij}, a_{ij}, b_{ij})}{\chi_{ij}}, \tag{18}$$

where l_{ij} is a positive constant, and the positive constants a_{ij} and b_{ij} satisfy the following condition:

$$0 < a_{ij} < b_{ij} \le \chi_{ij}^*. \tag{19}$$

The function $h(\chi_{ij}, a_{ij}, b_{ij})$ is a smooth step function defined in Definition 3.2. It can be directly verified that the function β_{ij} given in (18) possesses all the properties listed in (16). In the rest of the paper, the function β_{ij} defined in (18) will be used.

Barbalat-like lemma

The following Barbalat-like lemma is to be used in the stability analysis of the closed loop system.

Lemma 5.4: Assume that a nonnegative scalar differentiable function f(t) satisfies the following conditions

$$1) \left|\frac{d}{dt} f(t)\right| \le k_1 f(t), \forall t \ge 0,\ \ 2) \int_0^\infty f(t)dt \le k_2 \tag{20}$$

where k_1 and k_2 are positive constants, then $\lim_{t\to\infty} f(t) = 0$.

Proof: See [10]. Lemma 3.4 differs from Barbalat's lemma found in [40]. While Barbalat's lemma assumes that f(t) is uniformly continuous, Lemma 3.4 assumes that $|\frac{d}{dt} f(t)|$ is bounded by $k_1 f(t)$. Lemma 4 is useful in proving convergence of f(t) when it is difficult to prove uniform continuity of f(t).

Coordination Control Design

To design the control u_i for the agent i that achieves Coordination Control Objective 1, we will construct a potential function ϕ. This potential function puts penalty on all the tracking errors for all the agents in the group, and includes all the pairwise collision avoidance functions β_{ij} for all i ≠ j. As such, the potential function ϕ is chosen as follows:

$$\phi = \frac{1}{2}\Big(\sum_{i=1}^N \| 2Kq_{ie} + (I_n + \Delta(p_{ie}))p_{ie} \|^2 + \frac{1}{2}\sum_{i=1}^N \sum_{j\in N_i} \beta_{ij}\Big), \tag{21}$$

where the matrices K and $\Delta(\bullet)$, and the pairwise collision avoidance function β_{ij} are given in Definition 5.3, and is the set of all the agents except for the agent *i*. Differentiating both sides of (21) along the solutions of (1) and using the first property of $\Delta(\bullet)$ in (4), we have

$$\begin{aligned}
\dot\phi = &\sum_{i=1}^N \Omega_i^T [2Kp_{ie} + (I_n + (I_n + L)\Delta(p_{ie}))\dot p_{ie}]\\
&+ \frac{1}{2}\sum_{i=1}^N \sum_{j\in N_i} \beta_{ij} p_{ij}^T [(I_n + \Delta(p_{ie}))p_{ie} - (I_n + \Delta(p_{je}))p_{je}],
\end{aligned} \tag{22}$$

Where

$$\begin{aligned}
&p_{ij} = p_i - p_j,\\
&\Omega_i = 2Kq_{ie} + (I_n + \Delta(p_{ie}))p_{ie} + \sum_{j\in N_i} \beta_{ij} q_{ij}.
\end{aligned} \tag{23}$$

It is noted that in deriving (22), we have used $p_{ij}=p_{ie}-p_{je}$ since $\dot q_{id} = \dot q_{jd} = \dot q_{od}$, see (2). Now using Property 3) of $\Delta(\bullet)$ in (4), we can write the term $[(I_n + \Delta(p_{ie}))p_{ie} - (I_n + \Delta(p_{je}))p_{je}]$ in (22) as follows:

$$[(I_n + \Delta(p_{ie}))p_{ie} - (I_n + \Delta(p_{je}))p_{je}] = [I_n + A(p_{ie}, p_{je})]p_{ij}, \tag{24}$$

Where we have again used $p_{ij}=p_{ie}-p_{je}$. Substituting (24) into (22) gives

$$\begin{aligned}
\dot\phi = &\sum_{i=1}^N \Omega_i^T [2Kp_{ie} + (I_n + (I_n + L)\Delta(p_{ie}))\dot p_{ie}]\\
&+ \frac{1}{2}\sum_{i=1}^N \sum_{j\in N_i} \beta_{ij} p_{ij}^T [I_n + A(p_{ie}, p_{je})]p_{ij}.
\end{aligned} \tag{25}$$

Since $\beta_{ij} \le 0$ for all $\chi_{ij} \in R$, see Property 2 of β_{ij} in (16), and $A(p_{ie},p_{je})$ is a diagonal and nonnegative definite matrix, see Property 3) of $\Delta(\bullet)$ in (4), we have the following inequality

$$\beta_{ij} p_{ij}^T [I_n + A(p_{ie}, p_{je})]p_{ij} \le 0, \tag{26}$$

For all $q_{ij} \in R_n$, $p_{ij} \in R_n$, $p_{ie} \in R_n$, and $p_{je} \in R_n$. Substituting (26) and $\dot p_{ie} = u_i - \ddot q_{id}$ into (25) results in

$$\dot\phi \le \sum_{i=1}^N \Omega_i^T [2Kp_{ie} + (I_n + (I_n + L)\Delta(p_{ie}))(u_i - \ddot q_{id})], \tag{27}$$

which suggests that we choose the control u_i as follows:

$$u_i=[I_n+(I_n+L)\Delta(p_{ie})]^{-1}(-C\sigma(\Omega_i)-2Kp_{ie})+\ddot{q}_{id}, \quad (28)$$

Where C is a positive definite matrix.

Remark 6.1:

1. Substituting Ω_i defined in (23) into (28) results in

$$u_i=[I_n+(I_n+L)\Delta(p_{ie})]^{-1}(C\sigma(-2Kq_{ie} -(I_n+\Delta(p_{ie}))p_{ie}-\sum_{j\in N_i}\beta_{ij}q_{ij})-2Kp_{ie})+\ddot{q}_{id}. \quad (29)$$

The elements of u_i are explained as follows. The term $[I_n+(I_n+L)\Delta(p_{ie})]^{-1}$ is to make the control u_i bounded with a predefined bound since $\sigma(\bullet)$ and $[I_n+(I_n+L)\Delta(p_{ie})]^{-1}(-2Kp_{ie})$ are bounded with a predefined bound. The term $-2Kq_{ie}-(I_n+\Delta(p_{ie}))p_{ie}$, which is a part of the argument of $\sigma(\bullet)$ and the term $-2Kp_{ie}$ referred to as the attractive force plays the role of forcing the agent to track its desired trajectory. The term $-\sum_{j\in N_i}\beta_{ij}q_{ij}$, which is a part of the argument of $\sigma(\bullet)$, referred to as the repulsive force, takes care of collision avoidance for the agent i with the other agents. Moreover, the control u_i of the agent i given in (28) depends only on its own state and the states of other neighbor agents j if these agents are communicated with the agent i, i.e., when the condition (6) holds. This is because if the condition (6) does not satisfy, we have $\beta_{ij}=0$ due to Property 1) of β_{ij} listed in (16) with the constants a_{ij} and b_{ij} of the function β_{ij} chosen according to (19), and χ_{ij}^* satisfied the condition (17).

2. Since $\Delta(p_{ie})$ satisfies Property 2) in (4) and $\Delta(p_{ie})$ is a nonnegative definite matrix, L is a positive definite matrix, a calculation shows that the control u_i is bounded by

$$\|u_i(t)\|\le\lambda_M(C)+2\lambda_M(K)\varrho+\varepsilon_{3od},\ \forall\ t\ge t_0\ge 0, \quad (30)$$

as long as the closed loop system (32) is forward complete. Forward completeness of the closed loop system (32) is to be proved in Appendix C. In (30), $a_M(\bullet)$ is the maximum eigenvalue of •, and ϱ and ε_{3od} are defined in (4) and (2), respectively. Therefore, the upper-bound δ of the control u_i can be specified as

$$\delta\ge\lambda_M(C)+2\lambda_M(K)\varrho+\varepsilon_{3od}. \quad (31)$$

It is seen from (31) that the upper-bound δ of the control u_i can be predetermined as long as the pre-specified value of δ is strictly larger than ε_{3od}, which is the suprenum value of the reference trajectory acceleration, $\ddot{q}_{id}$. This is because when δ is strictly larger than ε_{3od} we can always appropriately choose the matrices and , and the constant ϱ, which comes from a proper choice of the matrix $\Delta(\bullet)$, see (4).

Substituting the control u_i given in (28) into (1) gives the closed loop system

$$\dot{q}_i=p_i, \\ \dot{p}_i=(I_n+(I_n+L)\Delta(p_{ie}))^{-1}(-C\sigma(\Omega_i)-2Kp_{ie})+\ddot{q}_{id}. \quad (32)$$

On the other hand, substituting the control u_i into (27) results in

$$\dot{\phi}\le-\sum_{i=1}^{N}\vartheta_i, \quad (33)$$

Where

$$\vartheta_i=\Omega_i^T C\sigma(\Omega_i). \quad (34)$$

The coordination control design has been completed. We summarize the main results in the following theorem.

Theorem 6.1: Under Assumption 4.1, the smooth bounded control vector u_i given in (28) with a predetermined bound δ defined in (31) for the agent i solves the coordination control objective. In particular, no collisions between any agents can occur for all $t \ge t_0 \ge 0$ the closed loop system (32) is forward complete, and the trajectory $q_i(t)$ of the agent i asymptotically track its reference trajectory $q_{id}(t)$, for all i € N.

Proof: Refer to Appendix C.

Coordination Control of VTOL Aircraft

In this section, we present an application of the proposed coordination control design in the previous section to design a coordination control system for a group of VTOL aircraft.

Related work

Control of a single VTOL aircraft has been considered by many researchers since it is under actuated and non minimum phase. An approximate input-output linearization approach was used in [42-46] to develop controllers for stabilization and output tracking/regulation of a VTOL aircraft. In [47], by noting that the output at a fixed point with respect to the aircraft body (Huygens center of oscillation) can be used, an interesting approach was introduced to design a local output tracking controller. Simple approaches were developed in [48,49] to provide global controllers for the stabilization and tracking control of a VTOL aircraft. A dynamic high-gain approach was used in [50] to design a controller to force the VTOL aircraft to globally practically track a reference trajectory.

Since the VTOL aircraft are underactuated it is necessary to address the bounded control problem for the agents with second-order dynamics in order to design a coordination control system for a group of VTOL aircraft, see discussion in Subsection 7.4. Due to the mentioned difficulties, only few results on cooperative control of multiple aircraft are available. In [51], direct coordination (using inertial measurements) and nearest-neighbor coordination (using relative measurements) were addressed. In [52], a sliding mode formation controller was proposed using the leader-follower approach. In [53,54], several formation controllers were designed to force a group of the quadrotor aircraft to track a desired reference linear velocity and to maintain a desired formation. In the above works, collision avoidance between the aircraft is not considered. The above review motivates an inclusion of this section to present an application of the coordination control design for agents with second-order dynamics to a group of VTOL aircraft.

Mathematical model and coordinate transformations

A scaled mathematical model of the i^{th} VTOL aircraft in the group can be described [43]:

$$\ddot{x}_i=-\overline{F}_i\sin(\theta_i)+\xi_i\tau_i\cos(\theta_i), \\ \ddot{z}_i=\overline{F}_i\cos(\theta_i)+\xi_i\tau_i\sin(\theta_i)-g, \\ \ddot{\theta}_i=\tau_i, \quad (35)$$

where x_i, z_i and θ_i denote position of the aircraft center of mass and roll angle of the i^{th} aircraft, respectively, $\overline{F}_i$ and τ_i are the vertical control force and rotational moment, g>0 is the gravitational acceleration, and ξ_i is the constant coupling between the roll moment and the lateral force, see Figure 1.

Since the VTOL dynamics (35) is non-minimum phase if its position (x_i,z_i) is controlled directly, we introduce the following coordinate transformations [49].

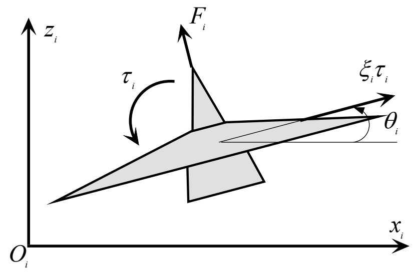

Figure 1: The VTOL aircraft.

$$\begin{aligned} \bar{x}_i &= x_i - \xi_i \sin(\theta_i), \\ \bar{z}_i &= z_i + \xi_i \cos(\theta_i), \end{aligned} \tag{36}$$

With the coordinate transformations (36), we can write (35) as

$$\begin{aligned} \ddot{\bar{x}}_i &= -\bar{F}_i \sin(\theta_i), \\ \ddot{\bar{z}}_i &= \bar{F}_i \cos(\theta_i) - g, \\ \ddot{\theta}_i &= \tau_i, \end{aligned} \tag{37}$$

where

$$\bar{F}_i = F_i - \xi_i \dot{\theta}_i^2. \tag{38}$$

If we consider $(\bar{x}_i, \bar{z}_i)$ as the new position of the aircraft to be controlled, it is seen that (37) is of a triangular form. It is noted $(\bar{x}_i, \bar{z}_i)$ coincides with the aircraft's center of oscillation [47]. Let us define

$$q_i = \begin{bmatrix} \bar{x}_i \\ \bar{z}_i \end{bmatrix}, \; p_i = \begin{bmatrix} \dot{\bar{x}}_i \\ \dot{\bar{z}}_i \end{bmatrix}, \; u_i = \begin{bmatrix} -\bar{F}_i \sin(\theta_i) \\ \bar{F}_i \cos(\theta_i) - g \end{bmatrix}. \tag{39}$$

With the definition of q_1, p_1 and u_i in (39), we can write (37) as

$$A) \begin{cases} \dot{q}_i = p_i \\ \dot{p}_i = u_i \end{cases}, \quad B) \; \{\ddot{\theta}_i = \tau_i. \tag{40}$$

This is a convenient form that will be used for the coordination control design in Subsection 5.4.

Coordination control objective

Roughly speaking, we will consider a group of N aircraft with a coordination control objective of designing the controls $\bar{F}_i$ and τ_i for the aircraft such that its position $q_i = [\bar{x}_i \; \bar{z}_i]^T$ tracks its reference trajectory $q_{id}(t) = [\bar{x}_{id} \; \bar{z}_{id}]^T$ and avoids collision with all other aircraft in the group. We will state the coordination control objective precisely after the following assumption stated.

Assumption 7.1:

1. Assumption 2.1 holds. In addition, the reference trajectories q_{id} satisfy the following conditions:

$$\begin{aligned} & q_{id}(t) = q_{od}(t), \; q_{id}(t) = q_{od}(t), \\ & \| q_{od}(t) \| \le \varepsilon_{4od}, \quad \| q_{od}(t) \| \le \varepsilon_{5od}, \end{aligned} \tag{41}$$

for all $t \ge t_0 \ge 0$ and i € N, where ε_{4od} and ε_{5od} are nonnegative constants.

2. $\varepsilon_{3od} \le g - \mu$, where ε_{3od} is specified in (2), and μ is a strictly positive constant.

Assumption 7.1 covers both the stabilization and trajectory tracking of the VTOL aircraft and implies that the aircraft is not allowed to land faster than it freely falls under the gravitational force.

Coordination Control Objective 7.1: Under Assumption 7.1, for each aircraft i design the smooth control inputs $\bar{F}_i$ and τ_i such that the position vector q_i of the aircraft i tracks its reference position vector q_{id} while avoiding collision with all other agents in the group. Specifically, we will design $\bar{F}_i$ and τ_i such that

$$\begin{aligned} & \lim_{t \to \infty} (q_i(t) - q_{id}(t)) = 0, \\ & \| q_i(t) - q_j(t) \| \ge \varrho_{ij}, \\ & |\theta_i(t)| \text{ is bounded,} \end{aligned} \tag{42}$$

for all $(i, j) \in N$, $i \ne j$ and $t \ge t_0 \ge 0$, where ϱ_{ij} is a positive constant.

Coordination control design

It is seen that (40) consists of two subsystems, namely the subsystem describes linear motion of the aircraft's center of oscillation and the subsystem gives rotational motion of the aircraft. These two subsystems are connected in a lower triangular structure. The subsystem is of the form (1) with n=2. This structure suggests that we should use the back stepping technique [31] to design the controls $\bar{F}_i$ and τ_i.

Let us discuss the reason why we need to apply the smooth bounded coordination control design proposed in the previous section to the VTOL aircraft described by (40). Assume that we have already designed the smooth vector u_i and that θ_i is a control. We will need to solve the third equation of (39) for $\bar{F}_i$ and θ_i. As such, we write the third equation of (39) as

$$\begin{aligned} -\bar{F}_i \sin(\theta_i) &= u_{1i}, \\ \bar{F}_i \cos(\theta_i) - g &= u_{2i}, \end{aligned} \tag{43}$$

where u_{1i} and u_{2i} are the first and second elements of ui, i.e., $u_i = [u_{1i} \; u_{2i}]^T$. Clearly, we need a bounded u_i with $|u_{2i}| < g$ to obtain a bounded $\theta_i = \arctan(\frac{-u_{1i}}{u_{2i}+g})$ and a bounded $\bar{F}_i = -u_{1i}\sin(\theta_i) + (u_{2i} + g)\cos(\theta_i)$. Motivated by the aforementioned discussion, we will proceed the coordination control design for the group of VTOL aircraft in two stages.

Stage 1: In this stage, we consider the subsystem A defined in (40) with u_i, i.e., $\bar{F}_i$ and θ_i, as a control vector to achieve asymptotical tracking of $q_i(t)$-$q_{id}(t)$ and to guarantee no collision between any aircraft. As such, define

$$\begin{aligned} \theta_{ie} &= \theta_i - \alpha_{\theta_i}, \\ u_{ie} &= u_i - \alpha_{u_i}, \end{aligned} \tag{44}$$

where $\alpha_{\theta i}$ is a virtual control of θ_i, and α_{ui} is a virtual control of u_i defined by

$$\alpha_{u_i} = \begin{bmatrix} -\bar{F}_i \sin(\alpha_{\theta_i}) \\ \bar{F}_i \cos(\alpha_{\theta_i}) - g \end{bmatrix}. \tag{45}$$

With the third equation of (39), the second equation of (44) and (45), we can write u_{ie} as

$$u_{ie} = \bar{F}_i \begin{bmatrix} -\sin(\theta_{ie})\cos(\alpha_{\theta_i}) - (\cos(\theta_{ie}) - 1)\sin(\alpha_{\theta_i}) \\ (\cos(\theta_{ie}) - 1)\cos(\alpha_{\theta_i}) + \sin(\theta_{ie})\sin(\alpha_{\theta_i}) \end{bmatrix}. \tag{46}$$

The virtual control vector α_{ui} is designed in the same way as the one in (28), i.e.,

$$\alpha_{u_i} = [I_n + (I_n + L)\Delta(p_{ie})]^{-1}(-C\sigma(\Omega_i) - 2Kp_{ie}) + \ddot{q}_{id}, \tag{47}$$

where the matrices In ,L, C, and K, $\Delta(p_{ie})$,and the constant ϱ are defined in the previous section. It is noted that $\|\alpha_{u_i}(t)\| \le \lambda_M(C) + 2\lambda_M(K)\varrho + \varepsilon_{3od}$, see (30). Therefore, to make it possible to solve (45) for αθi, we impose the following condition on the matrices L , K and C, and the constant ϱ:

$$\lambda_M(C) + 2\lambda_M(K)\varrho + \varepsilon_{3od} < g, \tag{48}$$

There always exist the matrices L ,K and C, and the constant ϱ such that the condition (48) under Item 2) of

Assumption 7.1: Solving (45) gives

$$\alpha_{\theta_i} = \arctan\left(\frac{-\alpha_{u_{1i}}}{\alpha_{u_{2i}}+g}\right), \quad \overline{F}_i = -\alpha_{u_{1i}}\sin(\alpha_{\theta_i}) + (\alpha_{u_{2i}}+g)\cos(\alpha_{\theta_i}), \tag{49}$$

where α_{u1i} and α_{u2i} are the first and second elements of α_{ui}, i.e., $\alpha_{ui}=[\alpha_{u1i}\ \alpha_{u2i}]$ $\alpha_{u_i}=[\alpha_{u_{1i}}\ \alpha_{u_{2i}}]^T$. It is noted that $\alpha_{\theta i}$ in (49) is well defined because $g-\|\alpha_{u_i}\|>0$.

Remark 7.1:

1. The virtual control $\alpha_{\theta i}$ is a smooth function of $q_1,..,q_N$, $p_1,..,p_N$, $q_{1d},..,q_{Nd}$, $\dot{q}_{od}$ and $\ddot{q}_{od}$ because $\dot{q}_{id}=\dot{q}_{od}$ and $\ddot{q}_{id}=\ddot{q}_{od}$, see Assumption 2.1.1.

2. The virtual control α_{ui} depends only on its own state and the states of other neighbor aircraft j if these aircraft are communicated with the aircraft , i.e., when the condition (6) holds, see Remark 4.1.2 for more details. Therefore, the control $\overline{F}_i$ of the aircraft i depends only on its own state and the states of other aircraft j if these aircraft are communicated with the aircraft i.

Due to $u_i=u_{ie}+\alpha_{ui}$, see (44), the derivative of the function ϕ defined in (21) is

$$\dot{\phi} \le -\sum_{i=1}^{N}\vartheta_i + \sum_{i=1}^{N}\Omega_i^T[I_n+(I_n+L)\Delta(p_{ie})]u_{ie}, \tag{50}$$

where, ϑ_i, Ω_i and $\Delta(p_{ie})$, are defined in (34), (23), and Assumption 1.1, respectively.

Stage 2: In this stage, we will design the control τ_i to asymptotically stabilize u_{ie} at the origin by using the back stepping technique [31]. We first note that the subsystem B defined in (40) can be written as

$$\dot{\theta}_i = \omega_i, \quad \dot{\omega}_i = \tau_i. \tag{51}$$

Now, we define

$$\omega_{ie} = \omega_i - \alpha_{\omega_i}, \tag{52}$$

where α_{wi} is a virtual control of ω_i. We now design the virtual control α_{wi} to asymptotically stabilize u_{ie} at the origin by considering the following Lyapunov function candidate

$$V_1 = \phi + \frac{1}{2}\sum_{i=1}^{N}\theta_{ie}^2, \tag{53}$$

where θ_{ie} is defined in (44). Differentiating both sides of (53) along the solutions of (50), (52), and the first equation of (51) gives

$$\dot{V}_1 \le -\sum_{i=1}^{N}\vartheta_i + \sum_{i=1}^{N}\theta_{ie}\left(\Omega_i^T[I_n+(I_n+L)\Delta(p_{ie})]\frac{u_{ie}}{\theta_{ie}} + \alpha_{\omega_i} + \omega_{ie} - \dot{\alpha}_{\theta_i}\right), \tag{54}$$

Where

$$\dot{\alpha}_{\theta_i} = \sum_{i=1}^{N}\left(\frac{\partial\alpha_{\theta_i}}{\partial q_i}p_i + \frac{\partial\alpha_{\theta_i}}{\partial p_i}u_i + \frac{\partial\alpha_{\theta_i}}{\partial q_{id}}\dot{q}_{od}\right) + \frac{\partial\alpha_{\theta_i}}{\partial\dot{q}_{od}}\ddot{q}_{od} + \frac{\partial\alpha_{\theta_i}}{\partial\ddot{q}_{od}}\dddot{q}_{od}. \tag{55}$$

It is noted that the term $\frac{u_{ie}}{\theta_{ie}}$ is well defined because $\frac{\sin(\theta_{ie})}{\theta_{ie}} = \int_0^1\cos(\theta_{ie}\eta)d\eta$ and $\frac{\cos(\theta_{ie})-1}{\theta_{ie}} = -\int_0^1\sin(\theta_{ie}\eta)d\eta$, which are smooth for all $\theta_{ie}\in R$. The equation (54) suggests that we choose the virtual control α_{wi} as follows

$$\alpha_{\omega_i} = -k_1\theta_{ie} + \dot{\alpha}_{\theta_i} - \Omega_i^T[I_n+(I_n+L)\Delta(p_{ie})]\frac{u_{ie}}{\theta_{ie}}, \tag{56}$$

where k_1 is a positive constant.

Remark 7.2: The virtual control α_{wi} is a smooth function of $q_1,..,q_N$, $p_1,..,p_N$, $q_{1d},..,q_{Nd}$, $\dot{q}_{od}$, $\ddot{q}_{od}$, $\dddot{q}_{od}$, and θ_i. Moreover, due to Remark 5.1 on $\alpha_{\theta i}$, the virtual control α_{wi} of the aircraft i depends only on its own state and the states of other neighbor aircraft j if these aircraft are communicated with the aircraft i.

Substituting (56) into (54) results in

$$\dot{V}_1 \le -\sum_{i=1}^{N}\vartheta_i - k_1\sum_{i=1}^{N}\theta_{ie}^2 + \sum_{i=1}^{N}\theta_{ie}\omega_{ie}. \tag{57}$$

We finally design the control τ_i to asymptotically stabilize ω_{ie} at the origin by considering the following Lyapunov function candidate

$$V_2 = V_1 + \frac{1}{2}\sum_{i=1}^{N}\omega_{ie}^2, \tag{58}$$

whose derivative along the solutions of (57), (52), and the last equation of (51) satisfies

$$\dot{V}_2 \le -\sum_{i=1}^{N}\vartheta_i - k_1\sum_{i=1}^{N}\theta_{ie}^2 + \sum_{i=1}^{N}\omega_{ie}(\theta_{ie}+\tau_i-\dot{\alpha}_{\omega_i}), \tag{59}$$

where

$$\dot{\alpha}_{\omega_i} = \sum_{i=1}^{N}\left(\frac{\partial\alpha_{\omega_i}}{\partial q_i}p_i + \frac{\partial\alpha_{\omega_i}}{\partial p_i}u_i + \frac{\partial\alpha_{\omega_i}}{\partial q_{id}}\dot{q}_{od}\right) + \frac{\partial\alpha_{\omega_i}}{\partial\dot{q}_{od}}\ddot{q}_{od} + \frac{\partial\alpha_{\omega_i}}{\partial\ddot{q}_{od}}\dddot{q}_{od} + \frac{\partial\alpha_{\omega_i}}{\partial\dddot{q}_{od}}\ddddot{q}_{od} + \frac{\partial\alpha_{\omega_i}}{\partial\theta_i}\omega_i. \tag{60}$$

The equation (59) suggests that we design the control τ_i as

$$\tau_i = -k_2\omega_{ie} - \theta_{ie} + \dot{\alpha}_{\omega_i}, \tag{61}$$

where K_2 is a positive constant. Due to Remark 7.2 on the virtual control α_{wi}, the control τ_i of the aircraft i depends only on its own state and the states of other neighbor aircraft j if these aircraft are communicated with the aircraft i, i.e., when the condition (6) holds. Substituting (61) into (59) gives

$$\dot{V}_2 \le -\sum_{i=1}^{N}\vartheta_i - k_1\sum_{i=1}^{N}\theta_{ie}^2 - k_2\sum_{i=1}^{N}\omega_{ie}^2. \tag{62}$$

The above control design results in the following closed loop system:

$$\begin{aligned} \dot{q}_i &= p_i, \\ \dot{p}_i &= [I_n+(I_n+L)\Delta(p_{ie})]^{-1}(-C\sigma(\Omega_i)-2Kp_{ie}) + \ddot{q}_{id} + u_{ie}, \\ \dot{\theta}_{ie} &= -k_1\theta_{ie} + \omega_{ie} - \Omega_i^T[I_n+(I_n+L)\Delta(p_{ie})]\frac{u_{ie}}{\theta_{ie}}, \\ \dot{\omega}_{ie} &= -k_2\omega_{ie} - \theta_{ie}. \end{aligned} \tag{63}$$

We summarize the results of the proposed coordination control design for VTOL aircraft in the following theorem:

Theorem 7.1: Under Assumption 5.1, the smooth controls $F_i = \overline{F}_i + \xi_i\dot{\theta}_i^2$ with $\overline{F}_i$ given in (49) and τ_i given in (61) for the aircraft i solve the coordination control objective 1 as long as the control design matrices K and C, and the constant ϱ, which comes from a proper choice of the matrix $\Delta(\bullet)$, see (4), are chosen such that the condition (48) holds. In particular, no collisions between any aircraft can occur for all $t \ge t_0 \ge 0$ the closed loop system (63) is forward complete, and the trajectory $q_i(t)$ of the aircraft i asymptotically tracks its reference trajectory $q_{id}(t)$, for all $i\in N$.

Proof: Refer to Appendix D.

Simulation results

In this subsection simulates the coordination control design proposed in the previous subsection on a group 6 aircraft. All the aircraft have the same coupling constant $\xi_i = 0.1$, i=1,….,6. The communication constant χ_{ij}^M is specified as χ_{ij}^M = 40. The initial conditions of the aircraft are chosen as follows:

$$q_i(0) = R_0[\cos(\frac{2\pi}{N}(i-1))\ \sin(\frac{2\pi}{N}(i-1))]^T,$$
$$p_i(0) = [0\ 0]^T, \tag{64}$$
$$\theta_i(0) = 0,$$

where R_0=10. This choice of the initial conditions means that the aircraft are uniformly distributed on a circle centered at the origin with a radius of 10. The reference trajectories are chosen as follows:

$$q_{id}(t) = R_d[\cos(\frac{2\pi}{N}(i-1)+\pi)\ \sin(\frac{2\pi}{N}(i-1)+\pi)]^T,\ \forall\ 0 \le t \le 20,$$
$$q_{id}(t) = R_d[\cos(\frac{2\pi}{N}(i-1)+\pi)\ \sin(\frac{2\pi}{N}(i-1)+\pi)]^T + R_d[0\ \sin(0.1(t-20))]^T,\ \forall\ 20 < t \le 80, \tag{65}$$

where R_d = 5. This choice of the reference trajectories mean that the aircraft are first to be uniformly placed on a circle centered at the origin with a radius of 5 but their positions are opposite to their initial positions, are then to track sinusoidal signals. The purpose of choosing the above reference trajectories is to illustrate both collision avoidance and reference trajectory tracking properties of the proposed coordination control. Indeed, with the above choices of the initial conditions and reference trajectories all the aircraft need to cross the origin, i.e., the center of the aforementioned circles. This is an effective illustration of the collision avoidance capacity of the proposed coordination controller.

The matrix $\Delta(p_{ie})$, is chosen as $\Delta(p_{ie}) = \text{diag}(\dot{x}_{ie}^2, \dot{z}_{ie}^2)$ where x_{ie} and z_{ie} are the first and the second elements of p_{ie}, i.e., $p_{ie} = [\dot{x}_{ie}\ \dot{z}_{ie}]^T$. With this choice of $\Delta(p_{ie})$, a simple calculation shows that the matrices L and $A(x_1,x_2)$ and the constant ϱ in (4) are L=diag(2,2), $A(x_1,x_2) = \text{diag}(x_{11}^2 + x_{11}x_{21} + x_{21}^2, x_{12}^2 + x_{12}x_{22} + x_{22}^2)$, for all $x_1 = [x_{11}\ x_{12}]^T \in R^2$ and $x_2 = [x_{21}\ x_{22}]^T \in R^2$, and ϱ=1. Moreover, the reference trajectories specified in (65) ensure that the condition (2) holds with ε_{1ijd}=5, ε_{2od}=0.5, and ε_{30d}=0.05. From the condition (48), we choose the control design matrices as C=I_2 and K=I_2. From the initial condition (64) and the chosen K, it is directly verified that ϱ_{ij0} and χ_{ij0} in (5) are $\varrho_{ij0} = 10$ and $\chi_{ij0} = \varrho_{ij0}^2$. The constants a_{ij} and b_{ij} are chosen as a_{ij}=4 and b_{ij}=16. The other control design parameters are chosen as $\sigma(\bullet) = \tanh(\bullet)$, $l_{ij}=10^2$ for all (i, j) €b N, i ≠ j,K_1=2, and K_2=4. A calculation shows that the condition (48) satisfies with this choice of the control parameters and the above choice of the reference trajectories.

Simulation results are plotted in Figures 2 and 3. Figure 2a plots the trajectories of all the agents in plane. It is seen from this figure

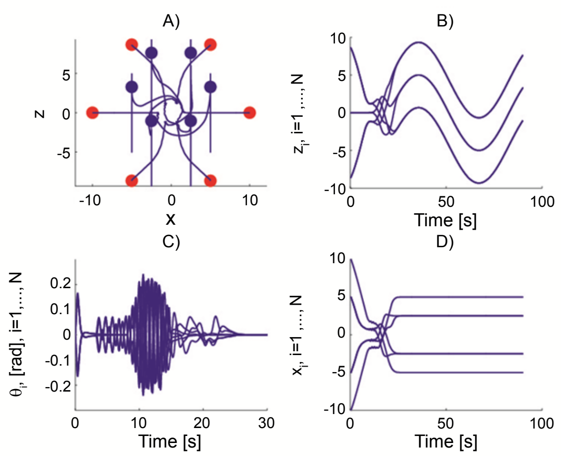

Figure 2: Trajectories of the aircraft.

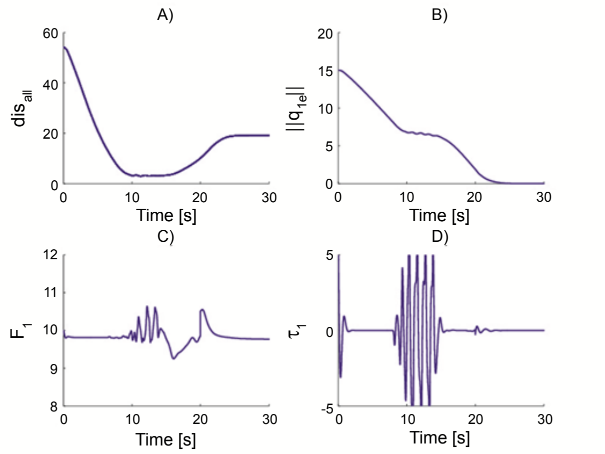

Figure 3: Scaled value of relative distances, tracking errors, and control inputs.

that the proposed coordination controller forces all the aircraft to perform collision avoidance and trajectory tracking very well, see also Figure 3a for a plot of the scaled product of all relative distance $\text{dis}_{\text{all}} = (\prod_{(i,j)\in N, i\neq j} \|q_{ij}(t)\|)^{\frac{1}{20}}$, which is always larger than zero, Figure 3b for convergence of the tracking error$||q_{1e}||$(t) of the aircraft 1 to zero. Figure 2b and Figure 2d plot the trajectories z_i and x_i of all the aircraft versus time, and Figure 2c plots θ_i of all the aircraft versus time. It is seen that $\theta_i(t)$ is bounded for all i € N and t ≥ 0Finally, the controls F_1 and τ_1 of the aircraft 1 are plotted in Figure 3c and Figure 3d, respectively.

Conclusions

A constructive method has been presented to design distributed controllers for coordination control of a group of N mobile agents. The most desired feature of the proposed coordination control design is that the controllers are bounded with a pre-specified bound. The keys to the control design include the new bounded control design technique for second-order systems, and new pair wise collision avoidance functions of both the relative position and velocity of the agents. An extension of the proposed coordination control design to agents with higher order dynamics is under consideration.

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