

**Research Article** 

# BCR: A Simple and Efficient Method of Unidimensional Search by Elimination

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### Abstract

The algorithm proposed in this paper belongs to the methodological category of unidimensional search by elimination, and may be used, therefore, in the optimization of discontinuous functions. This new method is based on the dichotomous search algorithm and is, in many cases, superior to Fibonacci's algorithm (up to the present considered the most efficient method of elimination), with the advantage of being much simpler.

Keywords: Unidimensional search; Elimination; Dicothomy

## Introduction

Unidimensional search methods are used in many algorithms of multivariate optimization, in which successive unidimensional searches are executed in selected directions [1].

They are therefore fundamental, and the efficiency of the whole search process depends on their selection.

In the elimination methods, the initial interval of uncertainty (i.e. the interval where the optimal solution must be isolated) is progressively reduced to the desired precision.

As examples of multivariate methods that use unidimensional search, the following may be cited, among others [2,3].

1) Steepest Descent Method (gradient)

2) Powell Method

3) Davidon-Fletcher-Powell Method

4) Broyden-Fletcher-Goldfarb-Shanno Method

5) Rosen Method of Gradient Projection

The elimination methods of unidimensional search most frequently used are [2,3]:

1) Exhaustive Search

- 2) Dichotomy
- 3) Interval Halving
- 4) Golden Section

5) Fibonacci

Among these methods, the most efficient (and also the most complex) is most certainly the Fibonacci Method, which uses the famous Fibonacci sequence in the selection of samples from the function to be optimized.

In all elimination methods, the assumption of function unimodality (a unique valley in the interval in the case of minimization and a unique peak in the case of maximization) is present.

If, by chance, the function is not unimodal, the method converges for a local optimum. As in multivariate methods unidimensional searches are executed in different directions, this fact does not normally have major implications. Even if the elimination methods are, in principal, considered less efficient than the so-called interpolation methods, the former are more robust than the latter, because when the polynomial interpolator does not adequately represent the variation of the function, these methods may present very slow convergence, and may diverge, or even predict the minimum of the function outside of the initial uncertainty interval. In addition, the methods of elimination are applicable to discontinuous functions.

Furthermore, the elimination methods do not require the derivative of the function, as do many interpolation methods [2].

Normally, the optimization procedures seek out the minimum of the function, which does not restrict them in the least, as maximizing f(x) is equivalent to minimizing -f(x).

Therefore, in the following sections we discuss minimization.

## **Description of the Method**

In the elaboration of the BCR, we observed that in the dichotomous method, some pieces of information (samples of the function) were prematurely discarded. This information could be used in a more efficient way for the determination of the new interval.

In the search for dichotomy [2], sampling of the function is carried out at two points as close as possible to the center of the uncertainty interval. At each iteration the method eliminates practically half of the uncertainty interval, based on the relative values of the objective function at the sample points and at its unimodality.

Let the initial interval be  $[0, L_o]$  and the position of the two given samples be:

$$x_1 = \frac{L_0}{2} - \frac{\delta}{2}$$

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$$x_2 = \frac{L_0}{2} + \frac{\delta}{2}$$

where  $\delta$  is a very small positive constant which is chosen considering the precision of the machine, so that the two samples can produce different values.

Then, the new uncertainty interval has the length of  $(L_0+\delta)/2$ . The next pair of samples is taken from the center of the new uncertainty interval, and so on.

BCR executes the dichotomous search procedure, with the following modifications:

First, we compute f(a) and f(b) where a and b denote the limits of the interval.

Instead of always evaluating the function at the two central points of the interval, the method evaluates the function value only at the central point (*c*). If f(a) < f(c) < f(b), the new interval, considering unimodality, will be [*a*, *c*]. In the symmetrical condition f(b) < f(c) < f(a), the new interval will be [*c*, *b*]. In these conditions, half of the interval was discarded using only one function evaluation. In the other cases, the conventional dichotomy procedure is applied, taking the first point as *c* and the second point as *c*+*delta*.

BCR does not require that we establish *a priori* the total number of iterations to be completed. This is an additional advantage over the Fibonacci Method.

## Efficiency Analysis

In the best case (a monotonic function in the interval), the algorithm will always use only one central evaluation of the function, (besides the initial two), so that half of the interval is discarded at every evaluation of the function. In this case, BCR is highly superior to all other methods.

In the worst case (for example, in the case of very high derivatives and high function values, close to one of the interval limits), it becomes identical to the dichotomous algorithm. In this case, the first two function evaluations will be wasted, in contrast to the dichotomous algorithm.

However, for functions that are not so anomalous, the method has been shown to be very efficient, superseding, in many cases, the Fibonacci method, as demonstrated in the examples provided in the results section.

Table 1 illustrates the relationship between the number of function calls and the final interval length. (OBS:  $\delta = 10^{-10}$ ) [2].

## **Results and Conclusion**

For a reduction ratio of 0.5 E-9 and initial interval [-1,1], various

Method	Formula	n = 14	
Dichotomy	$L_n = \frac{L_0}{2^{n/2}} + \delta - \frac{1}{2^{n/2}}$	0 00781L <sub>0</sub> + 0992E - 10	
Fibonacci (Fn is the nth Fibonacci number)	$L_n = \frac{L_0}{F_n}$	0 00164 L <sub>o</sub>	
Golden Section	0 618 <sup>n-1</sup> L <sub>o</sub>	0 00192L <sub>0</sub>	
BCR ( best case) n > 2.	$L_n = \frac{L_0}{2^{n-2}}$	0 000244L <sub>o</sub>	
BCR (worst case)	Dichotomy (n-2)	0 0156L <sub>0</sub> + 0984E - 10	

 Table 1: Comparison of the several unidimensional search methods.

Function	<b>BCR</b> d =10-10	DICHOTOMY d =10-10	FIBONACCI
X <sup>10</sup>	31	56	42
$\cos(x) + \sin(x)$	30	56	42
sin(3x)	43	56	42
$\sqrt{\arctan(x)}$	31	56	42
exp(  x )	31	56	42
x - 0 2	44	56	42

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Table 2: Numerical tests with BCR.

functions were simulated in order to determine the lowest number of evaluations of the function that would generate such a reduction. The results are presented in the following Table 2.

#### Pseudo-Code

The BCR pseudo-code is described below:

{Given:

}

a0 and b0 (initial interval limits)

reduction\_ratio (final interval length / initial interval length) delta (small positive number - see text)

$$a = a0 \ b = b0$$

ya = f(a) yb = f(b)

 $M = reduction\_ratio^{*}(b0-a0)$  If

 $M < delta \ then \ begin$ 

Print('Your reduction ratio must be greater than delta/(b0-a0)') Halt

end

$$x = (a+b)/2$$

y = f(x)

If (y < ya) and (y < yb) then begin

{Dichotomy Procedure}

$$xa = x y1 = y$$
  

$$xb = x+delta y2 = f(xb)$$
  
If y1

$$yb=y2$$
  
end  
else begin  

$$a = xa ya = y1$$

$$a = xa ya = ya$$

end

else If y > ya then begin b = x

end

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yb = y

end

else begin

a = x y a = y

end Until (b-a) < M

*Print* ('*Minimum Point* = ', (a+b)/2

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