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Approximate Analytic Solution of the Heat Equation in the Cast-Mould Heterogeneous Domain by Differential Transformation Method

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Abstract

In this article, a new application of Differential Transformation Method (DTM) is presented to find exact and approximate solutions of the heat equation in the cast-mould heterogeneous domain. It is indicated that the solutions obtained by the two dimensional (DTM) are reliable, useful and effective method for decouple partial differential equations. Exact solutions can also be obtained from the known forms of the series solutions.

Keywords: Differential transformation method; Approximate analytic solutions; The heat equation in the cast-mould heterogeneous domain

Introduction

In the current paper, we consider the problem of determining distribution of temperature in the heat equation in the cast-mould heterogeneous domain. Let's start by formulating the first mathematical model of the problem. D_1 and D_2 regions are given below (Figure 1):

$$D_1 = \{(x,t): x \in [x_1, 0], t \in [0, t^*)\} \quad \text{and} \quad D_2 = \{(x,t): x \in [0, x_2], t \in [0, t^*)\} \tag{1}$$

Five-component boundaries of this region are dispatched:

$$\begin{aligned} \Gamma_1 &= \{(x,0): x \in [x_1, 0]\}, \\ \Gamma_2 &= \{(x,0): x \in [0, x_2]\}, \\ \Gamma_3 &= \{(x_1,t): x \in [0, t^*)\}, \\ \Gamma_4 &= \{(0,t): t \in [0, t^*)\}, \\ \Gamma_5 &= \{(x_2,t): t \in [0, t^*)\}, \end{aligned} \tag{2}$$

In the cast (domain D_1) and mould (domain D_2) we consider the heat transfer equations:

$$\frac{\partial u(x,t)}{\partial t} = a_1 \frac{\partial^2 u(x,t)}{\partial x^2} \quad \text{in } D_1 \tag{3}$$

$$\frac{\partial v(x,t)}{\partial t} = a_2 \frac{\partial^2 v(x,t)}{\partial x^2} \quad \text{in } D_2 \tag{4}$$

Where $a_i, i = 1,2,$ are the thermal diffusivity, u, v denote the temperature, and t and x indicate the time and spatial location. At the boundaries of the above functions satisfy the initial and boundary conditions:

$$\begin{aligned} u(x,0) &= \varphi_1(x), \quad x \in \Gamma_1, \\ v(x,0) &= \varphi_2(x), \quad x \in \Gamma_2, \\ u(x_1,t) &= \psi(t), \quad t \in \Gamma_3, \\ v(x_2,t) &= q(t), \quad t \in \Gamma_5, \\ u(0,t) &= v(0,t), \quad t \in \Gamma_4, \\ -\lambda_1 \frac{\partial u(0,t)}{\partial x} &= -\lambda_2 \frac{\partial v(0,t)}{\partial x}, \quad t \in \Gamma_4, \end{aligned} \tag{5}$$

where the $\lambda_i, i = 1,2$, denote the thermal conductivity.

The investigation of the exact and numerical solution to nonlinear equations plays an important role in the study of nonlinear physical phenomena. The modified KdV types of equations have been an important class of non-linear evolution equations with numerous applications in physical sciences and engineering fields. For example, in geophysical fluid dynamics, they describe a long wave in shallow seas and deep oceans [1-3]. In plasma physics these equations give rise to the ion acoustic solutions [4-6]. However, the physical situations in which the KdV equations arise tend to be highly idealized due to the assumption of constant coefficients. In the recent years, many authors mainly had paid attention to study solutions of coupled equations by using various methods. Among these are Trigonometric function transform method [7], the homogeneous balance method [8], the F-expansion transform method [9], the He's variational iteration method [10-13], Homotopy perturbation method [14,15], Adomian decomposition method [16,17] and other [18-20].

In this paper, the differential transformation method (DTM) [21-28] is used to solve the coupled-mKdV equations.

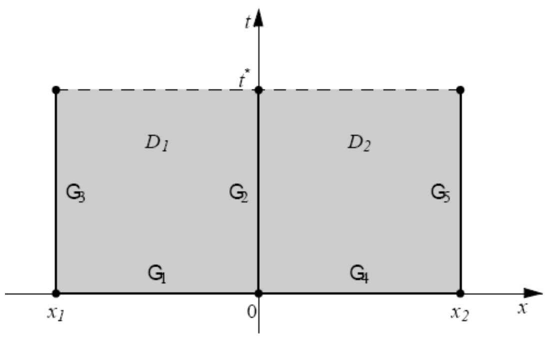

Figure 1: Domain of discussed problem.

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This paper is organized as follows:

In section 2, we describe DTM briefly. To show in efficiency of this method, we give the implementation of the DTM for the coupled-mKdV equation and numerical results in Section 3. The conclusions are then given in the final section 4.

Differential Transformation Method

The basic definitions and fundamental operations of the two dimensional differential transform function of the function are expressed as follows [21,22,25]. Two dimensional differential transform of $u(x,y)$ is the following form

$$U(k,h)=\frac{1}{k!h!}\left(\frac{\partial^{k+h}u(x,y)}{\partial x^k\partial y^h}\right)_{(x_0,y_0)} \tag{6}$$

where $u(x,y)$ is the original function and $U(k,h)$ is the transformed function. The inverse differential transform of $U(k,h)$ is defined as

$$u(x,y)=\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}U(k,h)(x-x_0)^k(y-y_0)^k \tag{7}$$

when (x_0,y_0) are taken as $(0,0)$, the function $u(x,y)$, Eq.(7), is showed as the following

$$u(x,y)=\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}\frac{1}{k!h!}\left[\frac{\partial^{k+h}u(x,y)}{\partial x^k\partial y^h}\right]_{(0,0)}x^k y^k. \tag{8}$$

From the above definitions, it can be found that the concept of two-dimensional differential transform is derived from two-dimensional differential transform is obtained from two-dimensional Taylor series expansion.

The DTM Applied to the Heat Equation in the Cast-Mould Heterogeneous Domain

In this section, we will investigate the solution of the heat equation in the cast-mould heterogeneous domain, which have been widely examined in the literature. We described the implementation of the DTM the heat equation in the cast-mould heterogeneous domain in detail. Application of the presented procedure will be examined with the help of parameters values, in which $x_1=-1,\ x_2=1,\ a_1=\frac{1}{4},\ a_2=1,\ \lambda_1=1$ and $\lambda_2=2$ To solve Eqs (3)-(5), according to DTM, Eqs. (5) with initial condition,

$$u(x,0)=e^{2x},v(x,0)=e^{x} \tag{9}$$

the boundary conditions,

$$u(-1,t)=e^{t-2},v(1,t)=e^{t+1}, \tag{10}$$

applying the differential transform of (3), (4), (9) and(10) , then

$$(h+1)U(k,h+1)=a_1(k+1)(k+2)U(k+2,h) \tag{11}$$

$$(h+1)V(k,h+1)=a_2(k+1)(k+2)V(k+2,h) \tag{12}$$

$$U(k,0)=\frac{2^k}{k!},V(k,0)=\frac{1}{k!} \tag{13}$$

$$U(-1,k)=\frac{e^{-2}}{k!},V(1,k)=\frac{e}{k!}, \tag{14}$$

Substituting Eq. (11)-(12) into Eq. (13)-(14), we obtain the closed form solution as

$$\begin{aligned}u(x,t)=\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}U(k,h)x^kt^h&=1+t+\frac{t^2}{2}+2x+2xt+xt^2\\&+2x^2+2x^2t+x^2t^2+\frac{4}{3}x^3+\frac{4}{3}x^3t+\frac{2}{3}x^3t^2+\ldots=e^{t+2x}\end{aligned} \tag{15}$$

$$\begin{aligned}v(x,t)=\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}V(k,h)x^kt^h&=1+t+\frac{1}{2}t^2+x+xt+\frac{1}{2}xt^2+\frac{1}{2}x^2\\&+\frac{1}{2}x^2t+\frac{1}{4}x^2t^2+\frac{1}{6}x^3+\frac{1}{6}x^3t+\frac{1}{12}x^3t^2+\ldots=e^{t+x}\end{aligned} \tag{16}$$

which are the exact solutions [6]. The graph of exact and DTM solutions belonging to examples examined the above are shown at Figures 1-2 for $\lambda_1=1$ and $\lambda_2=2$. it can be deduced that DTM solution corresponds to the exact solutions (Figure 2 and 3).

Conclusions

In this article, to find the numerical solution of the heat equation in the cast-mould heterogeneous domain with a number of initial and boundary values, the two dimensional differential transformation method (DTM) was has been performed successfully application. The results obtained from DTM show that in full compliance with exact solution. The solution obtained by differential transformation method shown as an expression of the form of a series of the exact solution.

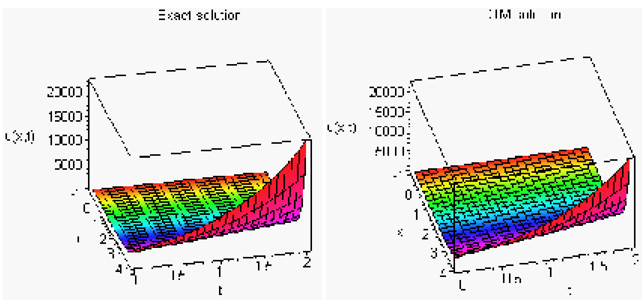

Figure 2: Exact solution and DTM solutions for u(x,t) when for $\lambda_1=1$ and $\lambda_2=2$.

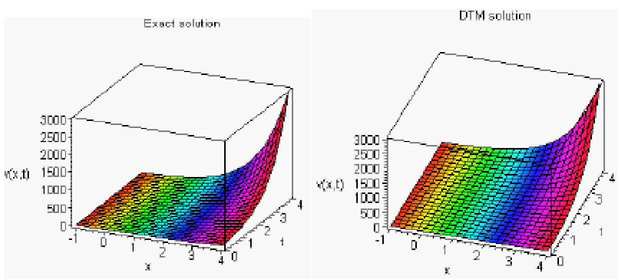

Figure 3: Exact solution and DTM solutions for $u(x,t)$ when for $\lambda_1=1$ and $\lambda_2=2$.

Original function	Transformed function
$u(x,y)=f(x,y)\mp g(x,y)$	$U(k,h)=F(k,h)\mp G(k,h)$
$u(x,y)=\xi f(x,y)$	$U(k,h)=\xi F(k,h)$
$u(x,y)=\frac{\partial f(x,y)}{\partial x}$	$U(k,h)=(k+1)F(k+1,h)$
$u(x,y)=\frac{\partial^{m+n} f(x,y)}{\partial x^m \partial y^n}$	$U(k,h)=(k+1)\ldots(k+m)(h+1)\ldots(h+n)F(k+m,h+n)$
$u(x,y)=x^r y^s$	$U(k,h)=\delta(k-r,h-s)=\begin{cases}1, & k=r,h=s\\ 0, & otherwise\end{cases}$
$u(x,y)=f(x,y)g(x,y)$	$U(k,h)=\sum_{m=0}^{k}\sum_{n=0}^{h}F(m,h-n)G(k-m,n)$
$u(x,y)=f(x,y)g(x,y)h(x,y)$	$U(k,h)=\sum_{k_4=0}^{k}\sum_{k_3=0}^{k-k_4}\sum_{k_2=0}^{h}\sum_{k_1=0}^{h-k_2}F(k_4,h-k_2-k_1)$ $G(k_3,k_2)H(k-k_4-k_3,k_1)$

Table 1: Operations of the two dimensional differential transform.

DTM can be applied to many complicated linear and strongly nonlinear partial differential equations and does not require linearization, discretization or perturbation. The results of the present method show that this method is useful and effective for solving in the cast-mould heterogeneous domain.

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