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Applications of Lie Point Symmetries

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Introduction

Lie point symmetry is a concept in advanced mathematics. Towards the end of the nineteenth century, Sophus Lie introduced the notion of Lie group in order to study the solutions of ordinary differential equations (ODEs). He showed the following main property: the order of an ordinary differential equation can be reduced by one if it is invariant under one-parameter Lie group of point transformations. This observation unified and extended the available integration techniques. Lie devoted the remainder of his mathematical career to developing these continuous groups that have now an impact on many areas of mathematically based sciences. The applications of Lie groups to differential systems were mainly established by Lie and Emmy Noether, and then advocated by Élie Cartan. Roughly speaking, a Lie point symmetry of a system is a local group of transformations that maps every solution of the system to another solution of the same system. In other words, it maps the solution set of the system to itself. Elementary examples of Lie groups are translations, rotations and scalings. For example, brain gliomas in particular have been extensively modelled using reaction-diffusion equations, as have biological invasions. These models have more recently been employed to represent and clarify a variety of nonlinear physical, chemical, and biological phenomena. It can be helpful to characterise the dynamics of glioma by finding analytical solutions that represent the passage through white and grey matter in the brain. At its interface, we thus discover analytical answers for a tumour growth model.

Description

The subject of fractional calculus is as old as the calculus of differentiation and integration and dates back to the time when Leibniz, Newton, and Gauss developed this type of calculation. Fractional calculus is the generalisation of ordinary differentiation and integration to noninteger (arbitrary) order. Due to its applications in modelling physical processes related to their historical states (nonlocal property), which can be effectively described by using the theory of derivatives and integrals of fractional order, it is also regarded as one of the most interesting topics in a variety of fields, particularly mathematics and physics. This is because models described by fractional order are more realistic than those described by integer order. A point transformation in the space of variables known as Lie point symmetry of an ordinary differential equation (ODE) maintains the set of solutions to the ODE. The set of solution curves can be preserved by a Lie point symmetry, which is equivalently thought of when considering these solutions as curves in the space of variables [1]. The Lie point symmetries of the geodesic equations in any Riemannian (affine) space are the automorphisms that preserve the set of these curves, according to our application of this finding to the geodesic curves in a Riemannian (affine) space. However, it is understood from differential geometry that the set of geodesics is preserved by the point transformations of a Riemannian.

The process of figuring out the Lie point symmetries of a particular system of ODEs entails two steps: (a) figuring out the requirements that the elements of the Lie symmetry vectors must meet, and (b) solving the system of those requirements. The second step is crucial, because the solution might be complicated in higher dimensions if there are many simultaneous equations. The Lie symmetry method is the most crucial method for creating analytical solutions to nonlinear PDEs, it might be argued. In addition, the conservation laws (CLs) can be created by employing the symmetries of the differential equations. It is based on the study of the invariance of differential equations (DEs) under a one-parameter group of transformations that converts one solution into another new solution [2]. The Lie group approach became valid for FDEs as a result of this work, and many studies are now devoted to it. Recently, the Lie symmetry analysis was also employed for FDEs; Gazizov et al. demonstrated how the prolongation formulae for fractional derivatives are formulated [3].

The use of systematic methods leading to the integration by quadrature (or at least lowering the order) of ordinary differential equations, the identification of invariant solutions to initial and boundary value problems [4], the development of conservation laws, and the construction of links between various differential equations that turn out to be equivalent is made possible by the Lie symmetry analysis of differential equations [5]. The combination of symmetry and simplicity has been, is, and most likely will always be a beautiful and practical tool in the creation and application of natural laws. The demand for symmetry explains the rules' regularities, which are unaffected by some inessential conditions [6]. For example, the repeatability of experiments depends on the principles of nature's invariance under space translation and rotation (homogeneity and isotropy of space), as well as time translation (homogeneity of time).

Conclusion

The development of systematic methods leading to the integration by quadrature (or at least lowering the order) of ordinary

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differential equations, the identification of invariant solutions to initial and boundary value problems, the derivation of conserved quantities, or the construction of relations between various differential equations that turn out to be equivalent all rely on Lie's theory, which is strong, adaptable, and fundamental. Although the application of Lie's theory to differential equations is entirely algorithmic, it frequently necessitates lengthy and laborious calculations. For instance, it is fairly uncommon to have to manage hundreds of equations in order to locate a single solution when searching for symmetries of a system of partial differential equations.

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