

Application of Grey Theory to the Field of Economic Forecasting

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Abstract

The Grey model is characterized by basic mathematics and a need for few raw data, but it also lacks the flexibility to adjust the model to increase the precision for the forecasting model. This article shows two types of grey forecasting models, GM(1,1) and NGBM. The GM(1,1) shows its fitting ability to the linear raw data and NGBM is superior in fitting nonlinear raw data. In this article, two practical cases are shown to demonstrate the calculation procedures of both models. The results show they are feasible in forecasting.

Keywords: GM(1,1); Nonlinear grey Bernoulli model; Number of nurses, Economic growth rate

Introduction

Grey theory was proposed by Prof. Deng [1] for more than 30 years. It has been successfully applied in many academic fields, such as electrical engineering, education, mechanical engineering, agriculture, high-tech sectors and so on [2-8]. Grey theory is famous for easy calculation and satisfactory result. Comparing to other methods, grey theory need only as few as four data could be modeled to forecast. There is no normality test needed beforehand and there is no level of significance test afterward. This is because that it is impossible to find any regularity based only on these four raw data. The criteria for testing the availability of modelling is relative percentage error (RPE) and average relative percentage error (ARPE) which are most commonly adopted. Therefore, the grey theory is suitable for short term forecasting or temporary reference for making decision or strategy when only limited data in hands.

Due to the need of forecasting precision, there are many improved or hybrid grey models proposed recently [9] used wavelet transform and grey model improved by PSO algorithm to forecast short term electric load [10] used Grey-Markov prediction method to enhance the electric arc furnace reactive power compensation. Sun [11] used the Hybrid Approach by Integrating Brain Storm Optimization Algorithm with Grey Neural Network for Stock Index Forecasting [12] adopted nonlinear grey prediction model with convolution integral NGMC(1,n) and its application to the forecasting of china's industrial SO2 emissions [13] used an optimized Nash nonlinear grey Bernoulli model based on particle swarm optimization and its application in prediction for the incidence of Hepatitis B in Xinjiang, China [14] predicted water consumption in hospitals based on a modified grey GM (0, 1|sin) model of oscillation sequence: the example of Wuhan city. Viewing the advancement of grey theory, it has been widely applied and accepted. Based on above reasons, the authors would like to introduce the grey theory to reader of Journal of Global Economics and the field of economic forecasting. In this article, the basic GM(1,1) and NGBM models are introduced for reader's reference.

Mathematical Methodology

The grey forecasting model is based on the formulation of differential equation. The solution of differential equation is the form of exponential function which could fit observed data with more flexibility. Unlike linear regression, the GM(1,1) and NGBM could provide better forecasting precision.

Traditional grey model GM (1,1)

This section provides the derivation of traditional grey forecasting in detail.

Step 1: Assume that the original series of data with m entries is:

$$X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(k), \dots, x^{(0)}(m)\}, \quad (1)$$

Where raw matrix $X^{(0)}$ stands for the non -negative original historical time series data.

Step 2: Construct $X^{(1)}$ by one time accumulated generating operation (1-AGO), Which is

$$X^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(k), \dots, x^{(1)}(m)\}, \quad (2)$$

where

$$x^{(1)}(k) = \sum_{i=1}^k x^{(0)}(i), k = 1, 2, \dots, m \quad (3)$$

Step 3: The result of 1-AGO is monotonic increase sequence which is similar to the solution curve of first order linear ordinary differential equation. Therefore, the solution curve of following differential equation represents the approximation of 1-AGO data.

$$\frac{d \hat{X}^{(1)}}{dt} + a \hat{X}^{(1)} = b, \quad (4)$$

where $\hat{X}^{(1)}$ represents Grey forecast value. The a and b are model parameters and a could not equal to zero [15]. $x(1) \hat{X}^{(1)}(1) = X^{(0)}(1)$ is the corresponding initial condition.

Step 4: The model parameters a and b can be determined by discrete form of Eq. (4)

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$$\frac{d\hat{X}^{(1)}}{dt} = \lim_{\Delta t \rightarrow 0} \frac{\hat{X}^{(1)}(t + \Delta t) - \hat{X}^{(1)}(t)}{\Delta t} \quad (5)$$

If the sampling time interval is unit, then let $\Delta t \rightarrow 1$

$$\frac{d\hat{X}^{(1)}}{dt} \cong x^{(1)}(k+1) - x^{(1)}(k) = x^{(0)}(k+1), k=1,2,3 \quad (6)$$

and $\hat{X}^{(1)}(t)$ is defined as

$$\hat{X}^{(1)}(t) \cong px^{(1)}(k) + (1-P)x^{(x)}(k+1), k=1,2,3, \quad (7)$$

where $z^{(1)}$ is termed background value, P is traditionally set to 0.5. The source model then can be obtained as

$$X^{(0)}(k) + az^1(k) = b, k=2,3,4 \quad (8)$$

From Eq. (8), by least square method the model parameters a and b are

$$\begin{bmatrix} a \\ b \end{bmatrix} = (B^T B)^{-1} B^T Y_N, \quad (9)$$

Where B and Y_N are defined as follows

$$B = \begin{bmatrix} -Z^{(1)}(2) & 1 \\ -Z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -Z^{(1)}(m) & 1 \end{bmatrix}, Y_N = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(m) \end{bmatrix} \quad (10)$$

or, a and b can be expressed in the following form

$$a = \frac{\sum_{k=2}^m [z^{(1)}(k)] \sum_{k=2}^m x^{(0)}(k) - (m-1) \sum_{k=2}^m x^{(0)}(k) z^{(1)}(k)}{(m-1) \sum_{k=2}^m [z^{(1)}(k)]^2 - \left(\sum_{k=2}^m z^{(1)}(k) \right)^2} \quad (11)$$

$$b = \frac{\sum_{k=2}^m x^{(0)}(k) \sum_{k=2}^m [z^{(1)}(k)]^2 - \sum_{k=2}^m x^{(0)}(k) z^{(1)}(k) \sum_{k=2}^m z^{(1)}(k)}{(m-1) \sum_{k=2}^m [z^{(1)}(k)]^2 - \left(\sum_{k=2}^m z^{(1)}(k) \right)^2} \quad (12)$$

Step 5: Solve the Eq.(4) together with initial condition, and the particular solution is

$$\hat{X}^{(1)}(K+1) = \left(x^{(0)}(1) - \frac{b}{a} \right) e^{-ak} + \frac{b}{a}, k=1,2,3,\dots \quad (13)$$

Hence, the desired forecasting output at k step can be estimate by inverse accumulated generating operation(IAGO) which is defined as

$$\hat{X}^{(0)}(K+1) = (1 - e^{-a}) \left(x^{(0)}(1) - \frac{b}{a} \right) e^{-ak}, k=1,2,3,\dots \quad (14)$$

Nonlinear Grey Bernoulli model

The procedures for deducing NGBM are as follow [16]. The step 1 and 2 are as same as traditional grey model.

Step 3: Equation (4) is a linear differential equation and the only adjustable variable is P [17]. Based on the differential equation theory, Bernoulli equation is introduced to replace the traditional grey differential equation. The Bernoulli equation has the following form

$$\frac{d\hat{X}^{(1)}}{dt} + a\hat{X}^{(1)} = b \left[\hat{X}^{(1)} \right]^n, \quad (15)$$

where n belongs to real number. Observing the above equation, when $n=0$, the solution reduces to traditional GM(1,1), when $n=2$, the solution reduces to Grey-Verhulst equation [18].

Step 4: Discrete form of Eq. (15)

$$x^{(0)}(k) + az^{(1)}(k) = b[z^{(1)}(k)]^n, k=2,3,4 \quad (16)$$

where NGBM parameters a and b are calculated by the following matrix manipulation,

$$\begin{bmatrix} a \\ b \end{bmatrix} = (B^T B)^{-1} B^T Y_N, \quad (17)$$

The parameters a and b are shown below

$$a = \frac{\sum_{k=2}^m [z^{(1)}(k)] \sum_{k=2}^m \left\{ x^{(0)}(k) [z^{(1)}(k)]^n \right\} - \sum_{k=2}^m [z^{(1)}(k)] \sum_{k=2}^m x^{(0)}(k) z^{(1)}(k)}{\sum_{k=2}^m [z^{(1)}(k)]^{2n} \sum_{k=2}^m [z^{(1)}(k)]^2 - \left(\sum_{k=2}^m [z^{(1)}(k)]^{n+1} \right)^2} \quad (18)$$

$$b = \frac{\sum_{k=2}^m \left\{ x^{(0)}(k) [z^{(1)}(k)]^n \right\} \sum_{k=2}^m [z^{(1)}(k)]^2 - \sum_{k=2}^m \left\{ x^{(0)}(k) z^{(1)}(k) \right\} \sum_{k=2}^m [z^{(1)}(k)]^{n+1}}{\sum_{k=2}^m [z^{(1)}(k)]^{2n} \sum_{k=2}^m [z^{(1)}(k)]^2 - \left(\sum_{k=2}^m [z^{(1)}(k)]^{n+1} \right)^2} \quad (19)$$

Step 5: The Corresponding particular solution of Eq.(16) together with initial condition

$$x^{(1)}(1) = x^{(0)}(1) \text{ is } \hat{X}(K+1) = \left[\left(x^{(0)}(1)^{(1-n)} - \frac{b}{a} \right) e^{-a(1-n)k} + \frac{b}{a} \right]^{\frac{1}{1-n}} \quad (20)$$

Error analysis

Error analysis is needed for examining the precision of forecasted results. Relative percentage error (RPE) compares the real and forecast values to evaluate the precision at specific time k . RPE is defined as

$$RPE = \varepsilon(k) = \frac{x^{(0)}(k) - \hat{x}(k)}{x^{(0)}(k)} \times 100\% \quad (21)$$

where $x^{(0)}(k)$ is the actual value and $\hat{X}^{(0)}(k)$ is the forecasted value by Eq. (20). The total model precision can be defined by average relative percentage error (ARPE) as follows

$$ARPE = \varepsilon(\text{avg}) = \frac{1}{m-1} \sum_{k=2}^m \varepsilon(k), k=2,3,4,\dots,m. \quad (22)$$

Forecasting by GM(1,1) and NGBM

In this section, two practical cases are calculated to demonstrate how grey theory is applied. Normally, GM(1,1) is a linear model which is suitable for slow changing raw data. NGBM is nonlinear model which is applicable to drastically change raw data. In the Table 1, according to the statistic data of Ministry of Health and Welfare of Taiwan, the number of nurses grows steadily year by year. Examining the raw data, it shows linear characteristics as the increment is small. Modelling the data from year 2009 to 2012, the point errors are all greatly less than 1%. The average relative percentage error is only 0.074%. The modelling

Year	2009	2010	2011	2012	2013	
raw data	87,361	91,724	95,529	99,801	103,277	
predicted data	87,361	91,659	95,613	99,737	104,040	a=-0.042
point error	0%	0.037%	-0.007%	0.012%	0.738%	b=86046.9
average error		0.074%				

Table 1: Number of nurses in Taiwan from 2009 to 2013.

Model	year	2010	2011	2012	2013	
	raw data	10.760%	4.190%	1.480%	2.090%	
	predicted data	10.760%	3.838%	2.327%	1.411%	a=0.543,
GM (1,1)	point error	0%	8.39%	-57.243%	32.488%	b=10.26
	average error		32.71%			
	predicted data	10.76%	3.893%	1.696%	1.866%	a=-0.108
NGBM	point error	0%	7.081%	-14.604%	10.721%	b=1.633*10 ³¹
	average error		10.802%			n=-27.72

Table 2: Economic Growth Rate of Taiwan from 2010 to 2013.

precision is more than 99.9%. The data of year 2013 is served as the criteria for checking the forecasting precision. The forecasted value is 104,040 which is close to the real data 103,277.

Source: Statistic data of Ministry of Health and Welfare of Taiwan

As the previous case is linear, the forecasting result is satisfactory. If the nonlinear case is encountered, the forecasted result will be unacceptable if GM(1,1) is applied. The case 2 is forecasting of Taiwan economic growth rate from year 2010 to 2013. From Table 2, the modelling error of GM(1,1) is 32.71% which is high. This is because there is a drastic drop from year 2010 to 2011, which is from 10.76% to 4.19%. The linear GM(1,1) obviously fails in this case. By applying nonlinear grey forecasting, NGBM, the modelling error is greatly reduced from 32.71% to 10.8% which demonstrates the power of nonlinear model.

Conclusions

The grey theory is easy in its mathematics and the forecasting result is satisfactory. Classified by raw data distribution, GM(1,1) is suitable for linear data and NGBM is suitable for nonlinear data. The derivation and calculation procedure are shown in this article. Also, two practical cases are demonstrated to show how GM(1,1) and NGBM are adopted to model the raw data. The results show that NGBM is superior than GM(1,1) if nonlinear data pattern is considered.

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