

## Application of an Eigenvalue Search Method for Flow Instabilities in Jet Flow

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One of the most notable areas in the study of fluid dynamics involves the phenomenon of flow instabilities. The reason for much interest in this topic is its association with flow turbulence. Although still many questions abound with regards to the nexus between flow instability and turbulence, it has been experimentally established that the former generally presages the occurrence of the latter [1]. Flow turbulence increases skin friction on aircraft leading to significant drag increments as well as is responsible for inducing acoustical noise in aircraft engines [2]. However, studies have shown that flow instabilities in the boundary layer of aircraft wings under some circumstances delay the onset of boundary-layer separation, which provides favorable increments in aerodynamic lift [3,4].

In a previous study [5], I developed a technique to search for the eigenvalues of a matrix which lead to flow instabilities for the case of a semi-infinite shear flow modeled as a hyperbolic-tangent function over a flat boundary. The governing equation in the study was the fourth-order Orr-Sommerfeld (O-S) differential equation. The (O-S) equation was reconfigured in matrix form as a system of first order differential equations. Two parameters in the coefficient matrix included the complex wavenumber  $\alpha$  and real wave speed  $c$  of a disturbance. The search algorithm I developed looked for complex values of the eigenvalue  $\alpha$  such that the O-S equation was satisfied and where the real part was positive and the imaginary part was negative. These conditions are conducive to flow instabilities [6]. The algorithm is predicated on the fact that instability is associated with elevated levels of turbulent kinetic energy in the flow so that the technique was trained to seek values of  $\alpha$  which produce higher values of flow turbulent kinetic energy [7].

I am proposing a new study that will examine flow instabilities in a jet flow. The jet flow will be represented as the square of a hyperbolic secant function and the same technique that I had developed in the previous study using the O-S equation will be applied. The solution to the O-S equation is a stream function  $\psi$  from which the velocity  $u$  of the disturbance can be determined so that the turbulent kinetic

energy per unit mass can be calculated. As before, complex values of the wavenumber  $\alpha$  will be sought that are in quadrant IV in the complex domain that produce the highest levels of turbulent kinetic energy.

The existing computer program is coded in MATLAB and produces plots of turbulent kinetic energy as a function of real and imaginary values of the complex wavenumber  $\alpha$  for a particular wave speed  $c$ . A typical plot of the calculated turbulent kinetic energy per unit mass is shown in the figure below and was taken from my earlier work. The data were calculated for a specific Reynolds number  $R$ , which is a similarity parameter and a normalized wave speed  $c$ .

Similar plots will be produced for the study presently under consideration. The results from the proposed study will be compared with prior computational and experimental studies. If the comparisons are favorable, then added validity will be attributed to the eigenvalue search technique.

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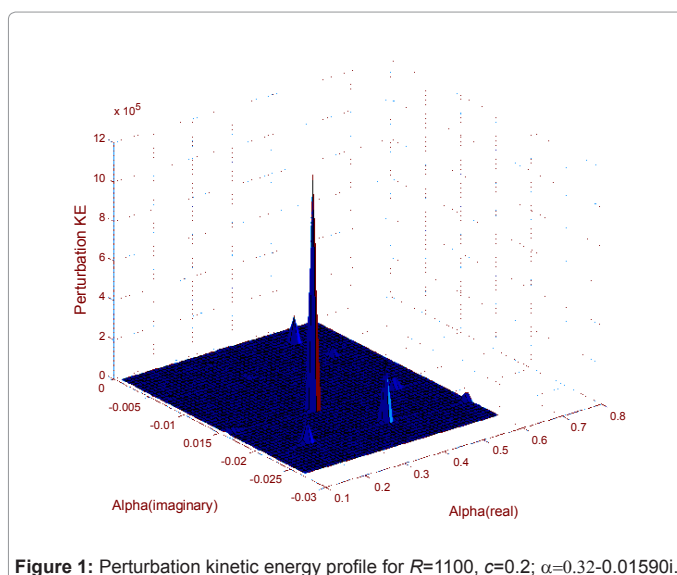


Figure 1: Perturbation kinetic energy profile for  $R=1100$ ,  $c=0.2$ ;  $\alpha=0.32-0.015901i$ .

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Received July 27, 2012; Accepted July 28, 2012; Published July 31, 2012

Citation: Walker IJ (2012) Application of an Eigenvalue Search Method for Flow Instabilities in Jet Flow. J Applied Computat Mathemat 1:e118. doi:10.4172/2168-9679.1000e118

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