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Announcement: Overview of Perfect Number (in 7 units)

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Abstract

A perfect number is a positive integer that is equal to the sum of its positive divisors, excluding the number itself. Currently only even perfect numbers are to be known. It is not known if any odd perfect numbers exist or not. So, is there an odd perfect number? No, there is not. (See unit 4,..., 7).

Keywords: Perfect number • Positive divisors • Odd perfect numbers

Introduction

Unit 1: Overview of perfect number

A perfect number is a positive integer that is equal to the sum of its positive divisors, excluding the number itself.

Currently only even perfect numbers are to be known. It is not known if any odd perfect numbers exist or not.

So, is there an odd perfect number? No, there is not. (See unit 4, …, 7)

Leonhard Euler (1707-1783, Swiss mathematician and physicist) Leonard Euler said: "Whether... there are any odd perfect numbers is a most difficult question."

In other hand Chris Caldwell said: "This is probably the oldest unsolved problem in all of mathematics."

The problem is originated from far ago, from Euclid, the age of ancient Greeks. And are there amateur or professional mathematicians who ignore arithmetic, omit the name of all kind of numbers, natural numbers, integers, primes, number Pi, number e,..., perfect numbers?

All mathematicians know that an odd perfect number is too difficult to exist. So why does that kind of mysterious unpredictable problem as of self-inflicted mystery trap exist? That trap is hidden in the integers.

Need any ideas of new nuance to find the proof to the problem?

My new idea is to assume that the odd perfect number is divisible by a prime number or an odd number of some forms then use the properties of integers and that of the perfect number to figure out perfect number equation model. I use some simple methods concerning the integers to address problems.

In this article, the author analyzes, argues, builds up even or odd perfect number equation model and proves that there is no odd perfect number [1].

Euclid sometimes called Euclid of Alexandria (Mid-4th century BC-Mid-3rd century BC, Greek mathematician). Euclid first devised a way to construct a set of even perfect numbers and showed that if 2n-1 is prime when n is prime and then 2n-1(2n-1) is a perfect number.

So the general formula of an even perfect number is P=2n-1(2n-1). It is formed by 2 primes: 2 and 2n-1, only when 2n-1 is a Mersenne prime.

This formula clearly shows that the pair (2n-1, 2n-1) is the key divisor pair or the final divisor pair or the last factor pair [2].

Illustration *via* explanation of several perfect numbers:

6=1+2+3=2 × 3

28=1+2+4+7+2 × 7=4 × 7

496=1+2+4+8+16+31+2 × 31+4 × 31+8 × 31=16 × 31

8128=1+2+4+8+16+32+64+127+2 × 127+4 × 127+8 × 127+16 × 127+32 × 127=64 × 127

Euler showed that an odd perfect number, if it exists, must be of the form

$N=p4\lambda+1Q2$ where p is a prime of the form 4n+1.

This formula does not show the last factor pair. It is simply a formula for an odd composite of the form $p4\lambda+1Q2$.

I think that the trap itself started here, greatly affecting mathematicians in many aspects, leading to a series of false proofs, which disturb the problem solving.

I think there should not be the so-called:

* An odd perfect number if it exists, must be of the form $N=p^{\alpha}q_1{}^{2}\beta^{1}$ … $q_r{}^{2\beta_r}$ with separate primes p, q1, …, qr and p=α=1 (mod4)
* An odd perfect number if it exists must be of the form 12k+1 or 36k+9 …
* An odd perfect number if it exists, must be larger than 10^{1500}
* An odd perfect number if it exists must have at least 101 prime factors and at least 10 distinct prime factors.
* Odd perfect numbers: A Triptych

I want to fair-play once for all to this problem.

That trap is hidden in the integers.

Perfect number as any composite,

They always have their divisors (n divisors) in a unique increasing divisor sequence and form divisor pairs (m divisor pairs, m=n/2).

There always has the two unique consecutive divisors in the middle of the divisor sequence and the product of these two divisors is always right equal to the value of the perfect number or the composite.

We call this divisor pair the key divisor pair or the final divisor pair or the last divisor pair.

I want to point out here, an odd perfect number if it exists as any odd composite, must be met 2 conditions:

1. An odd perfect number must be of the equation form $P=p_1q_1=p_2q_2=\ldots=p_iq_i=\ldots=$pmqm including the numerous products piqi. Each product is the

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product of 2 divisors in each divisor pair of P: (p_1, q_1), (p_2, q_2), (p_2, q_2),..., (p_i, q_i), ..., (p_{m-2}, q_{m-2}), (p_{m-1}, q_{m-1}), (p_m, q_m) The pair pm, qm is the last factor pair.

2. The form of an odd perfect number must match the form of the product of the two divisors in any of their divisor pairs.

This is the minimum requirement that the two end numbers of the odd perfect number of some form as well as the two end numbers of the product of the two divisors in any divisor pairs of odd perfect number are the same.

Example: Composite 819 is of the form 4k-1 has 12 divisors (n=12) in a unique increasing sequence: 1, 3, 7, 9, 13, 21, 39, 63, 91, 117, 273, 819.

It forms 6 divisor pairs (m=6):

(1, 819), (3, 273), (7, 117), (9, 91), (13, 63), (21, 39)

The divisor pair (21, 39) is the last divisor pair.

21 and 39 are the two unique consecutive divisors in the middle of the divisor sequence and the product of these two divisors 21 x 39=819 is always right equal to the value of the composite 819.

Odd number of the form 4k-1 has a property. (see unit 2)

Odd number of the form 4k-1 is always a product of an odd number of the form 4k-1 and an odd number of the form 4k-3: (4k-1)=(4k-1)(4k-3)

The form 4k-1 of odd number 819 governs the form of the product of the two divisors in any of their divisor pairs, and vice versa,

The form of the product of the two divisors in any of their divisor pairs reflects the form of odd number 819 as shown in the Table 1 below:

Odd perfect number if exists as any odd composite, so we have to work from the basing. First we have to work about the natural numbers, the positive natural numbers (n=1, 2, 3,...).

Following the Overview of Perfect Number are the some units:

- Unit 2. Natural number
- Unit 3. Even Perfect Number has not an end number of 0, 2, or 4
- Unit 4. No odd perfect number can be of the form 4k-1
- Unit 5. No odd perfect number can be of the form 6k-1
- Unit 6. There is no odd perfect number
- Unit 7. Argument of a Perfect Number Equation Model. There is no Odd Perfect Number

Unit 2: Natural Number

Natural number n=1, 2, 3, …, n

Consists 4 forms: 4k-3, 4k-2, 4k-1, 4k where k=1, 2, 3, …, n (Tables 2-8)

The form of odd number governs the form of the product of the two divisors in any of its division pairs, and vice versa, the form of the product of the two divisors in any of its division pairs reflects the form of odd number as shown in the table below with the odd number 819:

In other hand, we classify odd numbers into 3 forms:

form of 6k-1 (5, 11, 17, 23, 29, …), form of 6k-3 (3, 9, 15, 21, 27, …), and form of 6k-5 (1, 7, 13, 19, 25, …), with k=1…n.

The odd numbers of the form 6k-1 have the following special property:

Product of an odd number of the form 6k-5 and an odd number of the form 6k-1 is an odd number of the form 6k-1.

(6k’-5)(6k”-1)=36k’k”-6k’-30k”+5=6(6k’k”+k’-5k”+1)-1=6k-1

(k=6k’k”+k’-5k”+1 where k’=1…n, k”=1…n, so k=1…n)

Example: 13 are an odd number of the form 6k-5, 11 is an odd number of the form 6k-1, 13 × 11=143. So 143 is an odd number of the form 6k-1 [3,4].

An odd perfect number if exists, is an odd composite, and must be of the form 4k-3 or of the form 4k-1.

The product of two factors in any factor pair must follow the rules and properties of the odd number form.

* Odd number of the form 4k-1 has a property: (4k-1)=(4k-1)(4k-3)
* Odd number of the form 4k-3 has 2 properties: (4k-3)=(4k-3)(4k-3) and (4k-3)=(4k-1)(4k-1)
* Odd number of the form 6k-1 has a property: (6k-1)=(6k-5)(6k-1)
* Since then we have separate proofs:
* No odd perfect number can be of the form 4k-1 (see unit 4)
* No odd perfect number can be of the form 6k-1 (see unit 5)
* There is no odd perfect number. (see unit 6)

Unit 3: Even Perfect Number

Has not an end number of 0, 2, or 4.

The formula of Even Perfect Number $P=2^{p-1}(2^{p-1})$

When p=2, 3, 4, 5, …, 10, 11, 12, 13, … is when the last two numbers of p are in a number circle of the form of p 4k-2, 4k-1, 4k, 4k-3 where k=1…n.

The value of p at its form decides the end number of 2^{p-1}, the end number of Mersenne prime 2^{p-1} and the end number of Even Perfect Numbers $2^{p-1}(2^{p-1})$.

Since the combination of the end number of 2^{p-1} and the end number of 2^{p-1} in a multiplication $2^{p-1}(2^{p-1})$

So the end number of all Even Perfect Numbers is just 8 since …4 × …7 or 6 since …6 × …1, as shown here in the explanation (Tables 9-11).

From there Even Perfect Number has not an end number of 0, 2 or 4 [5].

Conclusion: All even perfect Number is ended by the end number 6 or 8. Even Perfect Number has not an end number of 0, 2 or 4. _______________

Unit 4: No Odd Perfect Number can be of the Form 4k−1

Perfect Number P Model Equation

The perfect number P as any composite, when appearing a factor pi then immediately appearing a factor P/p_i too, at a symmetrical position of p_i on the factor axis, in order to form a factor pair $(p_i, P/p_i)$ for a result $p_i \times P/p_i = P$. According to the definition of a perfect number P,

the sum of all factors of a perfect number P is 2P:

Table 1. Product of the two divisors in any of their division pairs reflects the form of odd number 819.

Divisior Pairs					
(1, 819)	(3, 273)	(7 , 117)	(9, 91)	(13, 63)	(21, 39)
1 × 819	3 × 273	7 × 117	9 × 91	13 × 63	21 × 39
(4k-3)(4k-1)	(4k-1)(4k-3)	(4k-1)(4k-3)	(4k-3)(4k-1)	(4k-3)(4k-1)	(4k-3)(4k-1)

Table 2. explanation of typical numbers 693, 834, 819, 8128.

Form	4k-3	4k-2	4k-1	4k
Each form has circle of 25 numbers with 25 number ends	01, 05, 09, 13, 17, 21, 25, 29, 33, 37, 41, 45, 49, 53, 57, 61, 65, 69, 73, 77, 81, 85, 89, 93, 97,	02, 06, 10, 14, 18, 22, 26, 30, 34, 38, 42, 46, 50, 54, 58, 62, 66, 70, 74, 78, 82, 86, 90, 94, 98,	03, 07, 11, 15, 19, 23, 27, 31, 35, 39, 43, 47, 51, 55, 59, 63, 67, 71, 75, 79, 83, 87, 91, 95, 99,	04, 08, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60, 64, 68, 72, 76, 80, 84, 88, 92, 96, 100
Properties	4k-3=(4k-1)(4k-1) 4k-3=(4k-3)(4k-3)	4k-3=(4k-1)(4k-1) 4k-3=(4k-3)(4k-3)	4k-1=(4k-1)(4k-3)	4k=(4k)(4k) 4k=(4k-2)(4k) 4k=(4k-1)(4k) 4k=(4k-3)(4k) 4k=(4k-2)(4k-2)
Example numb…	693	834	819	8128
Factors. XX, XX are 2 unique consequence divisors in the middle of the divisor sequence	1, 3, 7, 9, 11, 21, 33, 63, 77, 99, 231, 693	1, 2, 3, 6, 139, 278, 417, 834	1, 3, 7, 9, 13, 21, 39, 63, 91, 117, 273, 819	1, 2, 4, 8, 16, 32, 64, 127, 254, 508, 1016, 2032, 4064, 8128
Number of Fac…	12	8	12	14
Number of f pairs	6	4	6	7
Factor pairs. Attention to the last factor pair XX, XX	(1, 693), (3, 231), (7, 99), (9, 77), (11, 63), (21, 33)	(1, 834), (2, 417), (3, 278), (6, 139)	(1, 819), (3, 273), (7, 117), (9, 91), (13, 63), (21, 39)	(1, 8128), (2, 4064), (4, 2032), (8, 1016), (16, 508), (32, 254), (64, 127)
Product of 2 factors in each pair. All of products is equal to a value (693, 834, 819, 8128)	1 × 693 3 × 231 7 × 99 9 × 77, 11 × 63 21 × 33 33 × 21, 63 × 11 77 × 9, 99 × 7 231 × 3 693 × 1	1 × 834 2 × 417 3 × 278 6 × 139 139 × 16, 278 × 3 417 × 2 834 × 1	1 × 819 3 × 273 7 × 117 9 × 91, 13 × 63 21 × 39 39 × 273, 63 × 13 91 × 9, 117 × 7 273 × 3 819 × 1	1 × 8128 2 × 4064 4 × 2032, 8 × 1016, 16 × 508, 32 × 254, 64 × 127 127 × 64, 254 × 32 508 × 16, 1016 × 8 2032 × 4, 4064 × 2 8128 × 1
Property used	(4k-3)(4k-3)=(4k-3) (4k-1)(4k-1)=(4k-3)	(4k-2)(4k-3)=(4k-2) (4k-3)(4k-1)=(4k-2)	(4k-1)(4k-3)=(4k-1)	(4k)(4k)=4k (4k-3)(4k)=4k (4k-2)(4k)=4k (4k-1)(4k)=4k
Conclusion	Is an odd perfect number of the form 4k-3? No. (see unit 5, 6)	Is an even perfect number of the form 4k-2? Yes. Only 1, it is 6. (see unit 3)	Is an odd perfect number of the form 4k-1? No. (see unit 4)	Is an even perfect number of the form 4k? Yes. Today, 51 epns. (see unit 3)

Table 3. 693 is of the form 4k-3.

	Odd number of the form 4k-3			
Number of factors, 12	1, 3, 7, 9, 11 , 21, 33, 63, 77, 99, 231, 693			
Number of factor pairs, 6	(1, 693), (3, 231), (7, 99), (9, 77), (11, 63), (21, 33)			
The last factor pair	(21, 33)			
Product of 2 factors in each pair is form of … x form of…	1 ´ 693	3 ´ 231	7 ´ 99	9 ´ 77
	(4k-3)(4k-1)	(4k-1)(4k-1)	(4k-1)(4k-3)	(4k-3)(4k-1)
	11 ´ 63	21 ´ 33	33 ´ 21	63 ´ 11
	(4k-1)(4k-1)	(4k-3)(4k-3)	(4k-3)(4k-3)	(4k-1)(4k-1)
	77 ´ 9	99 ´ 7	231 ´ 3	693 ´ 1
	(4k-1)(4k-3)	(4k-3)(4k-1)	(4k-1)(4k-1)	(4k-1)(4k-3)
Property of the form 4k-3	(4k-3)=(4k-3)(4k-1)=(4k-1)(4k-3)= (4k-1)(4k-1)=(4k-3)(4k-3)			

Table 4. 819 is of the form 4k-1.

Odd number of the form 4k-1				
Number of factors, 12	1, 3, 7, 9, 13, 21, 39, 63, 91, 117, 273, 819			
Number of factor pairs, 6	(1, 819), (3, 273), (7, 117), (9, 91), (13, 63), (21, 39)			
The last factor pair	(21, 39)			
Product of 2 factors in each pair is form of … x form of…	1 × 819	3 × 273	7 × 117	9 × 91
	(4k-3)(4k-1)	(4k-1)(4k-3)	(4k-1)(4k-3)	(4k-3)(4k-1)
	13 × 63	21 × 39	39 × 21	63 × 13
	(4k-3)(4k-1)	(4k-3)(4k-1)	(4k-1)(4k-3)	(4k-1)(4k-3)
	91 × 9	117 × 7	273 × 3	819 × 1
	(4k-1)(4k-3)	(4k-3)(4k-1)	(4k-3)(4k-1)	(4k-1)(4k-3)
Property of the form 4k-1	(4k-1)=(4k-3)(4k-1)=(4k-1)(4k-3)			

Table 5. 834 is of the form 4k-2.

Odd number of the form 4k-2				
Number of factors, 8	1, 2, 3, 6, 139, 278, 417, 834			
Number of factor pairs, 4	(1, 834), (2, 417), (3, 278), (6, 139)			
The last factor pair	(6, 139)			
Product of 2 factors in each pair is form of … x form of…	1 × 834	2 × 417	3 × 278	6 × 139
	(4k-3)(4k-2)	(4k-2)(4k-3)	(4k-1)(4k-2)	(4k-2)(4k-1)
	139 × 6	278 × 3	417 × 2	834 × 1
	(4k-1)(4k-2)	(4k-2)(4k-1)	(4k-3)(4k-2)	(4k-2)(4k-3)
Property of the 4k-2	(4k-2)=(4k-3)(4k-2)=(4k-1)(4k-2)			

Table 6. 8128 is of the form 4k.

Odd number of the form 4k				
Number of factors, 14	1, 2, 4, 8, 16, 32, 64, 127, 254, 508, 1016, 2032, 4064, 8128			
Number of factor pairs, 7	(1, 8128), (2, 4064), (4, 2032), (8, 1016), (16, 508), (32, 254), (64, 127)			
The last factor pair	(64, 127)			
	1 8128	2 × 4064	4 × 2032	8 × 1016
Product of 2 factors in each pair is form of … x form of…	(4k-3)(4k)	(4k-2)(4k)	(4k)(4k)	(4k-3)(4k)
	16 × 508	32 × 254	64 × 127	127 × 64
	(4k)(4k)	(4k)(4k-2)	(4k)(4k-1)	(4k-1)(4k)
	254 × 32	508 × 16	1016 × 8	2032 × 4
	(4k-2)(4k)	(4k)(4k)	(4k)(4k)	(4k)(4k)
		4064 × 2	8128 × 1	
		(4k)(4k-2)	(4k)(4k-3)	
Property of the form 4k	(4k)=(4k-3)(4k)=(4k-2)(4k)=(4k)(4k)=(4k)(4k-1)			

$1+p_1+p_2+p_3+\ldots+p_i, \ldots+P/p_i, \ldots+P/p_3+P/p_2+P/p_1+P=2P$ With a factor pair (p_i, P/p_i) we symbolize cp_i for the factor P/p_i (c means couple of pi), so the factor pair (p_i, P/p_i) is (p_i, cp_i),

the factor pair (1, c1) is (1, P), the factor pair (p_1, P/p_1) is (p_1, cp_1), … from there the sum of all factors of a perfect number P is:

$1+p_1+p_2+p_3+\ldots+p_i, \ldots+cp_i, \ldots+cp_3+cp_2+cp_1+P=2P$

or perfect number P is: $1+p_1+p_2+p_3+\ldots+p_i, \ldots+cp_i, \ldots+cp_3+cp_2+cp_1=P$ (1)

The equation (1) is Perfect Number P Model Equation, the left side is the sum of 1 and an even number of factors [6,7].

No odd perfect number P can be of the form 4k-1

Supposing that the odd perfect number P is of the form 4k-1

Odd numbers have 2 forms: form of 4k-3 (1, 5, 9, 13, 17, …) and form of 4k-1 (3, 7, 11, 15, 19, …) with k=1…n.

The odd numbers have the following special property:

Product of an odd number of the form 4k-1 and an odd number of the form 4k-3 is an odd number of the form 4k-1.

$(4k'-3)(4k''-1)=16k'k''-12k''-4k'+3=4(4k'k''-3k''-k'+1)-1=4k-1$

($k=4k'k''-3k''-k'+1$ where $k'=1\ldots n$, $k''=1\ldots n$, so $k=1\ldots n$)

Example: 9 is an odd number of the form 4k-3, 11 is an odd number of the form 4k-1, 9 × 11=99. So 99 is an odd number of the form 4k-1.

Since P is of the form 4k-1, for each pair of factors (p_i, cp_i), when pi is of the form 4k-3, then cpi is of the form 4k-1, or vice versa, when pi is of the form

Table 7. Even composite, explanation of typical numbers 834, 8128.

834 is of the form 4k-2 Properties of the form 4k-2 is (4k-2)=(4k-3)(4k-2)=(4k-1)(4k-2)				**8128 is of the form 4k Properties of the form 4k is 4k=(4k-3)(4k)=(4k-2)(4k)=(4k-1)(4k)=(4k)(4k)**			
8 factors	4 factor pairs	Product of 2 fac.	Form of 2 factors	14 factors	7 factor pairs	Product of 2 facs	Form of 2 factors
1, 2, 3, 6, 139, 278, 417, 834	(1, 834)	1 × 834	(4k-3)(4k-2)	1, 2, 4, 8, 16, 32, 64, 127, 254, 508, 1016, 2032, 4064, 8128	(1, 8128)	1 × 8128	(4k-3)(4k)
	(2, 417)	2 × 417	(4k-2)(4k-3)		(2, 4064)	2 × 4064	(4k-2)(4k)
	(3, 278)	3 × 278	(4k-1)(4k-2)		(4, 2032)	4 × 2032	(4k)(4k)
	(6, 139)	6 × 139	(4k-2)(4k-1)		(8, 1016)	8 × 1016	(4k-3)(4k)
					(16, 508)	16 × 508	(4k)(4k)
					(32, 254)	32 × 254	(4k)(4k-2)
					(64, 127)	64 × 127	(4k)(4k-1)

Table 8. Odd composite, explanation of typical numbers 693, 819.

693 is of the form 4k-3 Properties of the form 4k-3 is				**819 is of the form 4k-1 Property of the form 4k-1 is**			
4k-3=(4k-1)(4k-1), 4k-3=(4k-3)(4k-3)				4k-1=(4k-1)(4k-3)			
12 Factors	6 factor pairs	Product of 2 fac.	Form of 2 factors	12 factors	6 factor pairs	Product of 2 facs	Form of 2 factors
1, 3, 7,	(1, 693)	1 × 693	(4k-3)(4k-3)	1, 3, 7,	(1, 819)	1 × 819	(4k-3)(4k-1)
9, 11,	(3, 231)	3 × 231	(4k-1)(4k-1)	9, 13,	(3, 273)	3 × 273	(4k-1)(4k-3)
21, 33,	(7, 99)	7 × 99	(4k-1)(4k-1)	21, 39,	(7, 117)	7 × 117	(4k-1)(4k-3)
63, 77,	(9, 77)	9 × 77	(4k-3)(4k-3)	63, 91,	(9, 91)	9 × 91	(4k-3)(4k-1)
99, 231,	(11, 63)	11 × 63	(4k-1)(4k-1)	117, 273,	(13, 63)	13 × 63	(4k-3)(4k-1)
693	(21, 33)	21 × 33	(4k-3)(4k-3)	819	(21, 39)	21 × 39	(4k-3)(4k-1)

Table 9. Form of odd number as shown in the table with the odd number 819.

Form of odd number					
(1, 819)	(3, 273)	(7, 117)	(9, 91)	(13, 63)	(21, 39)
1 × 819	3 × 273	7 × 117	9 × 91	13 × 63	21 × 39
(4k-3)(4k-1)	(4k-1)(4k-3)	(4k-1)(4k-3)	(4k-3)(4k-1)	(4k-3)(4k-1)	(4k-3)(4k-1)

4k-1, then cpi is of the form 4k-3, in order to always have:

$p_i \times cp_i$=(4k-3) × (4k-1)=(4k-1) × (4k-3)=4k-1.

So it is possible to arbitrarily give 2 factors of any factor pair (pi, cpi) such as p_1=(4k-3), cp_1=(4k-1); p_2=(4k-1), cp_2=(4k-3); p_3=(4k-1), cp_3=(4k-3); … and change in place into (1), we have:

1+(4k-3)+(4k-1)+(4k-1)+…+(4k-3)+(4k-1)+…+(4k-3)+(4k-3)+(4k-1)=P

In any factor pair (p_i, cp_i), whether the factor pair (4k-3, 4k-1) or the factor pair (4k-1, 4k-3) they both have the sum of the two factors in a pair

4k-3+4k-1 or 4k-1+4k-3, all is of the form 8k-4 or 4(2k-1) or 4k, and no matter how many pairs there are, their sum is always of the form 4k.

So the left side being a sum of numbers of the form 4k and the number 1 should have the form of 4k'+1 or 4k'+4-3 or 4(k'+1)-3 or form 4k-3.

The right side is the odd perfect number P of the form 4k-1.

The two sides are not the same in terms of the two end numbers, therefore unbalanced, so the right answer is

No odd perfect number P can be of the form 4k–1_________

Unit 5: No Odd Perfect Number can be of the Form 6k–1

Perfect Number P Model Equation

The perfect number P as any composite, when appearing a factor pi then immediately appearing a factor P/pi too, at a symmetrical position of pi on the factor axis, in order to form a factor pair (p_i, P/p_i) for a result $p_i \times P/p_i = P$. According to the definition of a perfect number P, the sum of all factors of a perfect number P is 2P:

$1+p_1+p_2+p_3+…+p_i, …,+P/p_i, …,+P/p_3+P/p_2+P/p_1+P=2P$ With a factor pair (p_i, P/p_i) we symbolize cpi for the factor P/p_i (c means couple of p_i), so the factor pair (p_i, P/p_i) is (p_i, cp_i), the factor pair (1, c1) is (1, P), the factor pair (p_1, P/p_1) is (p_1, cp_1), … from there the sum of all factors of a perfect number P is:

$1+p_1+p_2+p_3+…+p_i, …,+cp_i, …,+cp_3+cp_2+cp_1+P=2P$

or perfect number P is: $1+p_1+p_2+p_3+…+p_i, …,+cp_i, …,+cp_3+cp_2+cp_1=P$ (1)

The equation (1) is Perfect Number P Model Equation, the left side is the sum of 1 and an even number of factors.

No odd perfect number P can be of the form 6k-1

Table 10. End number of all Even Perfect Numbers is just 8 since …4 ×…7 or 6 since …6×…1.

Combination of the End Number				
The value of p				1
	2	3	4	5
	6	7	8	9
	10	11	12	13
	…	…	…	…
The form of p	4k-2	4k-1	4k	4k-3
The end number of 2p-1	2	4	8	6
Multiply	×	×	×	×
The end number of Mersenne prime 2p-1	3	7	5	1
Equal	=	=	=	=
The end number of P = 2p-1(2p-1)	6	…8	…0	6
Several Perfect Numbers	6	28, 8128		496, 33550336

Table 11. Data of several perfect numbers.

Rank	p	The form of P	The end number of 2p-1	×	The end number of Mersenne prime 2p-1	=	The end number of Perfect number 2p-1(2p-1)	Perfect number 2p-1(2p-1)
1	2	4k-2	2	×	3	=	6	6
2	3	4k-1	4	×	7	=	…8	28
3	5	4k+1	…6	×	31	=	…6	496
4	7	4k-1	…4	×	127	=	…8	8128
5	13	4k+1	…6	×	…1	=	…6	33550336
6	17	4k+1	…6	×	…1	=	…6	8589869056
7	19	4k-1	…4	×	…7	=	…8	1.37439E+11
…	…	…	…	…	…	…	…	…
51	82,589,933	4k+1	6	×	148894445742… 325217902591	=	6	110847779864… 7191207936

Supposing that the odd perfect number P has the form of 6k-1

We classify odd numbers into 3 forms: form of 6k-1 (5, 11, 17, 23, 29, …), form of 6k-3 (3, 9, 15, 21, 27, …), and form of 6k-5 (1, 7, 13, 19, 25, …), with k=1…n. The odd numbers of the form 6k-1 have the following special property:

Product of an odd number of the form 6k-5 and an odd number of the form 6k-1 is an odd number of the form 6k-1.

(6k'-5)(6k"-1)=36k'k"-6k'-30k"+5=6(6k'k"+k'-5k"+1)-1=6k-1

(k=6k'k"+k'-5k"+1 where k'=1…n, k"=1…n, so k=1…n)

Example: 13 is an odd number of the form 6k-5, 11 is an odd number of the form 6k-1, 13 × 11=143. So 143 is an odd number of the form 6k-1.

Since P is of the form 6k-1, so each pair of factors (p_i, cp_i),

When p_i is of the form 6k-5 then cp_i is of the form 6k-1, or vice versa, when pi is of the form 6k-1 then cp_i is of the form 6k-5, in order to always have:

$p_i \times cp_i$=(6k-5) × (6k-1)=(6k-1) × (6k-5)=6k-1.

So it is possible to arbitrarily give any 2 factors of the factor pair (p_i, cp_i) as p1=(6k-5), $cp_{1=}$(6k-1); p_2=(6k-1), cp_2=(6k-5); p_3=(6k-1), cp_3=(6k-5); … and change in place into (1), we have:

1+(6k-5)+(6k-1)+(6k-1)+…+(6k-5)+(6k-1)+…+(6k-5)+(6k-5)+(6k-1)=P

In any factor pair (p_i, cp_i), whether the factor pair (6k-5, 6k-1) or the factor pair (6k-1, 6k-5) they both have the sum of the two factors in the pair

6k-5+6k-1 or 6k-1+6k-5, all is of the form 12k-6 or 6(2k-1) or 6k, and no matter how many pairs there are, their sum is always of the form 6k [8].

So the left side being a sum of numbers of the form 6k and the number 1 should have the form of 6k+1 or 6k'+6-5 or 6(k'+1)-5 or form 6k-5.

The right side is the odd perfect number P of the form 6k-1.

The two sides are not the same in terms of the two end numbers, therefore unbalanced, so the right answer is

No odd perfect number P can be of the form 6k–1_________

Unit 6: There is No Odd Perfect Number

We classify odd numbers into 2 forms: form of 4k-3 and form of 4k-1 with k=1…n.

Form of 4k-3: 01, 05, 09, 13, 17, 21, 25, 29, 33, 37, 41, 45, 49, 53, 57, 61, 65, 69,73, 77, 81, 85, 89, 93, 97 (a circle of 25 numbers, 2 end numbers)

Form of 4k-1: 03, 07, 11, 15, 19, 23, 27, 31, 35, 39, 43, 47, 51, 55, 59, 63, 67, 71, 75, 79, 83, 87, 91, 95, 99, (a circle of 25 numbers, 2 end numbers).

Odd numbers of the form 6k-1 have a circle of 50 numbers, noted only 2 end numbers: 05, 11, 17, 23, 29, 35, 41, 47, 53, 59, 65, 71, 77, 83, 89, 95, 101, 107, 113, 119, 125, 131, 137, 143, 149, 155, 161, 167, 173, 179, 185, 191, 197, 203, 209, 215, 221, 227, 233, 239, 245, 251, 257, 263, 269, 275, 281, 287, 293, 299.

The following circles are 305, 311, …, 599; 605, 611, …, 899; …

The odd numbers of the form 6k-1 noted only 2 end numbers are also the last 2 end numbers alternating of the odd numbers of the form 4k-3 and the odd numbers of the form 4k-1 [9-12].

There are already 2 separate proofs:

* No odd perfect number can be of the form 4k–1 (unit 4) and
* No odd perfect number can be of the form 6k–1 (unit 5).

The separate proof No odd perfect number can be of the form 4k-1 reinforces the proof No odd perfect number can be of the form 6k–1.

Since the circle of 50 odd numbers of the form 6k-1 noted only 2 end numbers are also the last 2 end numbers alternating of the odd numbers of the form 4k-3 and the odd numbers of the form 4k-1, so the proof

No odd perfect number can be of the form 4k-1 and the form 4k-3.

_________As a result there is no odd perfect number __________

Unit 7: Argument of a Perfect Number Equation Model

There is no odd perfect number

A perfect number is a positive integer that is equal to the sum of its positive divisors, excluding the number itself. Here, a perfect number is usually denoted P.

Currently only even perfect numbers are to be known. It is not known if any odd perfect numbers exist or not.

In this article, the author analyzes, argues, builds up any even or odd perfect number model and proves there is no odd perfect number [13,14].

Analyzing some perfect numbers

Hereafter are some even perfect numbers and theirs explanations.

6=1+2+3=2 × 3

28=1+2+4+7+2 × 7=4 × 7

496=1+2+4+8+16+31+2 × 31+4 × 31+8 × 31=16 × 31

8128=1+2+4+8+16+32+64+127+2 × 127+4 × 127+8 × 127+16 × 127+32 × 127=64 × 127

Attention to the bold numbers, we see the product of 2 bold numbers has the value of the corresponding perfect number 2 × 3=6; …; …; 64 × 127=8128.

Just from one explanation form of perfect number 8128: 1+2+4+8+16+32+64+127+2 × 127+4 × 127+8 × 127+16 × 127+32 × 127=8128

1+2+4+8+16+32+64+127+2 × 127+4 × 127+8 × 127+16 × 127+32 × 127=64 × 127

$1+2+2^2+2^3+2^4+2^5+2^6+127+2 \times 127+2^2 \times 127+2^3 \times 127+2^4 \times 127+2^5 \times 127=2^6 \times 127$

$2^{n-1}+(2^{n-1})\ 2^{n-1}(2^{n-1})$

We pay special attention to the unique pair of consecutive factors 64, 127 in the middle of the sequence.

It has the product 64 × 127 of exactly equal to the perfect number 64 × 127=8128.

We call it the last factor pair:

$(1, 8128), (2, 4064), (2^2, 2032), (2^3, 1016), (2^4, 508), (2^5, 254), (2^6, 127)$

Looking at the bold numbers, in general we have immediately drawn out the formula for even perfect numbers: $P=2^{n-1}(2^{n-1})$

This is the general formula of an even perfect number P to be formed by 2 primes 2 and 2^n-1, when 2^n-1 is a Mersenne prime.

Composite number 840 and perfect number 8128

1. Composite Number 840 has 32 positive divisors including itself (840). They are listed from 1 to 32 in the following sequence:

1, 2, 3, 4, 5, 6, 7, 8, 10, 12, 14, 15, 20, 21, 24, 28, 30, 35, 40, 42, 56, 60, 70, 84,

105, 120, 140, 168, 210, 280, 420, 840

The number of divisors of 840 including 840: n=32

Sum of divisors of 840 without 840: s=2040

The number of pairs of factors of 840: m=0.5n, so m=16

32 divisors of 840 including 840 form 16 pairs of factors:

(1, 840), (2, 420), (3, 280), (4, 210), (5, 140), (6, 168), (7, 120), (8, 105)

(10, 84), (12, 70), (14, 60), (15, 56), (20, 42), (21, 40), (24, 35), (28, 30)

We call pair 28, 30 the last factor pair, the 16th. The product of 2 factors in each pair is the same:

1 × 840=2 × 420=3 × 280=4 × 210=5 × 140=6 × 168=7 × 120=8 × 105=

10 × 84=12 × 70=14 × 60=15 × 56=20 × 42=21 × 40=24 × 35=28 × 30=840.

Especially the last factor pair, the 16th, is a pair of 2 unique consecutive factors 28, 30 in the middle of the sequence.

2. Perfect number 8128 has 14 positive divisors including itself (8128)

They are listed from 1 to 14 in the following sequence:

$1, 2, 2^2, 2^3, 2^4, 2^5, 2^6, 127, 254, 508, 1016, 2032, 4064, 8128$

The number of divisors of 8128 including 8128: n=14

Sum of divisors of 8128 without 8128: s=8128

The number of pairs of factors of 8128: m=7 since m=0.5n.

14 divisors of 8128 including 8128 form 7 pairs of factors:

$(1, 8128), (2, 4064), (2^2, 2032), (2^3, 1016), (2^4, 508), (2^5, 254), (2^6, 127)$

We call pair 2^6, 127 the last factor pair, the 7th pair. The product of 2 factors in each pair is the same:

$1 \times 8128=2 \times 4064=2^2 \times 2032=2^3 \times 1016=2^4 \times 508=2^5 \times 254=2^6 \times 127=8128$.

Specially the last factor pair, the 7th, consists of 2 unique consecutive factors 2^6, 127 in the middle of the sequence, that has the product $2^6 \times 127$ of exactly equal to the perfect number, $2^6 \times 127=8128$, with a note, 127 is a Mersenne prime.

3. The perfect number as well as the composite number

They always have a last factor pair m, m+1, consists of 2 unique consecutive factors in the middle of the sequence.

The product of 2 factors of the last factor pair m, m+1, is always exactly equal to the value of the perfect number or the composite number.

However, in order to appear a perfect number, the mth factor is a composite number (except 2 is a prime in the even perfect number 6), but the (m+1)th factor must be a Mersenne prime.

Argument of a perfect number equation model

1. Argument of factors of a perfect number

Supposing that, there is any even or odd perfect number P.

The perfect number has n factors 1 … n as in the increasing order sequence:

1, 2, 3, 4, …, m, m+1, …, n-3, n-2, n-1, n (1)

where m=0.5n.

We add before factor m two factors m-1, m-2 and after factor m+1 two factors m+2, m+3 into the order sequence (1):

1, 2, 3, 4, …, m-2, m-1, m, m+1, m+2, m+3, …, n-3, n-2, n-1, n (2)

Convert the order sequence of n factors from 1 … n into the symbol sequence of n factors from $p_1 \dots p_n$:

$p_1, p_2, p_3, p_4, \dots, p_{m-2}, p_{m-1}, p_m, p_{m+1}, p_{m+2}, p_{m+3}, \dots, p_{n-3}, p_{n-2}, p_{n-1}, p_n$ (3)

2. Argument of the value of factors

The first factor p_1 must be 1.

The factor p_2 must be a prime number. The factor p_2 cannot be a composite, because if p_2 is a composite, it must have 2 smaller factors before it, but before it is only 1. So p_2 must be a prime number.

If we let $p_2=2$, that is when we consider the cases where even perfect numbers appear. If we consider p_2 to be a prime number other than 2, then we consider cases where odd perfect numbers appear.

The factor p_3 must be a composite. The factor p3 cannot be a prime number, because if p_3 is a prime number then the same p_4, p_5, p_6, …, p_{m-2}, p_{m-1} is a prime number too. And so there is no pair of central consecutive factors p_m, p_{m+1}, where pm is a composite, p_{m+1} is a prime number.

So the factor p_3 must be a composite.

Supposing that, p_3 is a composite and $p_3=p_u p_v$, so before p_3 there must be two smaller factors p_u, p_v but actually before p_3 is p_2, so p_3 can only be

$p_3=p_2p_2=p_2^2$. So the factor p_3 must be a composite p_{22}. Analogical argument we have p_4, …, p_m, each is a composite: $p_4=p_2^3$, $p_5=p_2^4$…, $p_{m-2}=p_2^{m-3}$, $p_{m-1}=p_2^{m-2}$, $p_m=p_2^{m-1}$

The factor p_{m+1}. In order to appear a perfect number, the factor m+1 in the sequence (2) and (3) or the factor p_{m+1} in the sequence (4) must be a prime number.

3. Argument of the sequence n factors

From above arguments, the symbol sequence of n factors (3) will convert into the preliminary value sequence of n factors:

$1, p_2, p_2^2, p_2^3,\ldots, p_2^{m-3}, p_2^{m-2}, p_2^{m-1}, p_m^{+1}, p_m^{+2}, p_m^{+3}, \ldots, p_{n-3}, p_{n-2}, p_{n-1}, p_n$ (4)

Factor p2 and factor p_{m+1} are prime numbers. For an ease observation,

We can re-symbolize p for p_2 and q for p_{m+1}, then we have: $1, p, p^2, p^3,\ldots, p^{m-3}, p^{m-2}, p^{m-1}, q, p_{m+2}, p_{m+3}, \ldots, p_{n-3}, p_{n-2}, p_{n-1}, p_n$
(5)

The sequence of n factors of a perfect number P including P forms m=0.5n pairs of factors:

$(1, p_n), (p, p_{n-1}), (p^2, p_{n-2}), (p^3, p_{n-3}),\ldots, (p^{m-3}, p_{m+3}), (p^{m-2}, p_{m+2}), (p^{m-1}, q)$ and the product of two factors in each pair has the same value $p^{m-1} \times q$:

$1 \times p_n=p \times p_{n-1}=p^2 \times p_{n-2}=p^3 \times p_{n-3}=\ldots=p^{m-3} \times p_{m+3}=p^{m-2} \times p_{m+2}=p^{m-1} \times q$ Then the values of $p_{m+2}, p_{m+3}, \ldots, p_{n-3}, p_{n-2}, p_{n-1}, p_n$ in the sequence (4) are:

$p_n=p^{m-1} \times q$; $p_{n-1}=p^{m-1} \times q/p=p^{m-2} \times q$; $p_{n-2}=p^{m-1} \times q/p^2=p^{m-3} \times q$;

$p_{n-3}=p^{m-1} \times q/p^3=p^{m-4} \times q$; …; $p_{m+3}=p^{m-1} \times q/p^{m-3}=p^2 \times q$; $p_{m+2}=p^{m-1} \times q/p^{m-2}=p \times q$

From there the value sequence (5) will be the value sequence (6):

$1, p, p^2, p^3, \ldots, p^{m-3}, p^{m-2}, p^{m-1}, q, p \times q, p^2 \times q, \ldots, p^{m-4} \times q, p^{m-3} \times q, p^{m-2} \times q, p^{m-1} \times q$ (6) Specially the last pair of factors (p^{m-1}, q) consists the unique 2 consecutive factors pm-1, q in the middle of the sequence that has the correct product pm-1 × q=P.

Thus, when arguing about factors of a perfect number P, we have calculated all factors of P only by the 2 prime numbers p and q. Order, symbol, value of n factors and unique value of product of 2 factors of each pair (Table 12).

4. General Equation Model of a Perfect number from the value sequence:

$1, p, p^2, p^3,\ldots, p^{m-3}, p^{m-2}, p^{m-1}, q, p \times q, p^2 \times q, \ldots, p^{m-4} \times q, p^{m-3} \times q, p^{m-2} \times q, p^{m-1} \times q$ (6)

And from the Definition of a Perfect number we have

A General Equation Model of a Perfect number P (i) or (ii)

$$P=1+p+p^2+p^3+\ldots+p^{m-3}+p^{m-2}+p^{m-1}+q+pq+p^2q+\ldots+p^{m-4}q+p^{m-3}q+p^{m-2}q=p^{m-1}q \quad \text{(i)}$$

$$2P=1+p+p^2+p^3+\ldots+p^{m-3}+p^{m-2}+p^{m-1}+q+pq+p^2q+\ldots+p^{m-4}q+p^{m-3}q+p^{m-2}q+p^{m-1}q \quad \text{(ii)}$$

As a result,

We have a General Equation Model of Perfect number P (even or odd) to be formed by only the 2 prime numbers p and q.

The Model has a pair of 2 unique consecutive factors p^{m-1}, q in the middle of the sequence, that has the product $p^{m-1} \times q$ of exactly equal to that perfect number P: $p^{m-1} \times q=P$.

The product of 2 factors of any pair is the same value, $p^{m-1}q$, so we have

The general formula for a perfect number: $P=p^{m-1}q$ (P)

5. Finding the relationship of p and q

Passing $q+pq+p^2q+\ldots+p^{m-4}q+p^{m-3}q+p^{m-2}q$ in equation (ii) to the right side

$$1+p+p^2+p^3+\ldots+p^{m-3}+p^{m-2}+p^{m-1}=p^{m-1}q-(q+pq+p^2q+\ldots+p^{m-4}q+p^{m-3}q+p^{m-2}q)$$

Drag q to be the common factor on the right side

$$1+p+p^2+p^3+\ldots+p^{m-3}+p^{m-2}+p^{m-1}=p^{m-1}q-(q+pq+p^2q+\ldots+p^{m-4}q+p^{m-3}q+p^{m-2}q)$$

Table 12. Table of order, symbol, value of n factors and unique value of product of 2 factors of each pair.

Order of factor	Symbol of factor	Value of factor 1 … n	×	Value of factor n … 1	=	Same value of product
1	p1	1	×	pm-1q	=	The product of 2 factors of each pair is the unique value: pm-1q
2	p2	P	×	pm-2q	=	
3	p3	p2	×	pm-3q	=	
4	p4	p3	×	pm-4q	=	
…	…	…		…	=	
m-2	pm-2	Pm-3	×	p2q	=	
m-1	pm-1	Pm-2	×	Pq	=	
M	Pm	pm-1	×	Q	=	pm-1q
m+1	pm+1	Q	×	pm-1	=	
m+2	pm+2	Pq	×	pm-2	=	
m+3	pm+3	p2q	×	pm-3	=	
…	…	…		…	=	pm-1q
n-3	pn-3	pm-4q	×	p3	=	
n-2	pn-2	pm-3q	×	p2	=	
n-1	pn-1	pm-2q	×	P	=	
N	pn	pm-1q	×	1	=	

Add and minus with the same value pm-1 into the square brackets on the right side and pass –pm-1 into the parentheses, then reduce we have:

$$1+p+p^2+p^3+...+p^{m-3}+p^{m-2}+p^{m-1}=\left[p^{m-1}+p^{m-1}-p^{m-1}-\left(1+p+p^2+...+p^{m-4}+p^{m-3}+p^{m-2}\right)\right]q$$

$$1+p+p^2+p^3+...+p^{m-3}+p^{m-2}+p^{m-1}=2p^{m-1}-(1+p+p^2+...+p^{m-4}+p^{m-3}+p^{m-2}+p^{m-1})q$$

$$From\ there:\ q=\frac{1+p+p^2+p^3+...+p^{m-3}+p^{m-2}+p^{m-1}}{2p^{m-1}-(1+p+p^2+...+p^{m-4}+p^{m-3}+p^{m-2}+p^{m-1})}$$

Multiply the numerator and the denominator with the same value p-1 and simplify:

$$q=\frac{(p-1)\left(1+p+p^2+p^3+...+p^{m-1}\right)}{(p-1)2p^{m-1}-(p-1)\left(1+p+p^2+p^3+...+p^{m-1}\right)}$$

$$q=\frac{p^m-1}{2p^m-2p^{m-1}-\left(p^m-1\right)}=\frac{p^m-1}{2p^m-2p^{m-1}-p^m+1}=\frac{p^m-1}{p^m-2p^{m-1}+1}$$

$$Fromtherewecountqbyp: q=\frac{p^m-1}{p^m-2p^{m-1}+1}$$

This is the formula that indicates the relationship between the two primes p and q of any perfect number P.

6. The perfect number depends only on the value of prime number p.

$$Change\ q=\frac{p^m-1}{p^m-2p^{m-1}+1}to\ (P), wehave:\ p=p^{m-1}\times\frac{p^m-1}{p^m-2p^{m-1}+1}(p^*)$$

This is the general formula of any perfect number P (even or odd) counted by 2 primes p and q, that $q=\frac{p^m-1}{p^m-2p^{m-1}+1}$

The formula $p=p^{m-1}\times q(\mathrm{p})$ for any perfect number P (even or odd) to be formed by the 2 prime numbers p and q, now is totally counted by the value of the only prime p in the formula $p=p^{m-1}\times\frac{p^m-1}{p^m-2p^{m-1}+1}(p^*)$

7. The even perfect number

When we change p=2 into (P*), we have: P=2m-1 × 2^m-1/2^m-2 × 2^m-1+1

Because 2m=2 × 2m-1, then the denominator

$$2^m-2\times2^{m-1}+1=2^m-2^m+1=1, we\ have: p=2^{m-1}\left(2^m-1\right)$$

Here, we have again the formula for even perfect number

$p=2^{m-1}\left(2^m-1\right)$, when 2m-1 is a Mersenne prime.

8. There is no odd perfect number

When p ≥ 3, is there or not a prime q with q>p

And $q=\frac{p^m-1}{p^m-2p^{m-1}+1}$, along with the prime p in order to form an odd

To answer this question, herein below are some little changes: c

Because

$$p^m=p\times p^{m-1}-1$$

then Numerator:

$$p^m-1=p\times p^{m-1}-1,$$

Denominator:

$$p^m-2p^{m-1}+1=p\times p^{m-1}-2p^{m-1}+1=(p-2)p^{m-1}+1,$$

So

$$q=\frac{p^m-1}{p^m-2p^{m-1}+1}$$

to be changed to:

$$q=\frac{p\times p^{m-1}-1}{(p-2)p^{m-1}+1},\ Let\ p^{m-1}=x,$$

we have:

$$q=\frac{px-1}{(p-2)x+1}$$

We see that When

So

$$q=lim_{x\to\infty}\frac{px-1}{(p-2)x+1}=\frac{(px-1)'}{((p-2)x+1)'}=\frac{p}{p-2}$$

From there:

When m→∞, then

$$q=\frac{p}{p-2}$$

When p=3, m→∞, q→3;

When p=5, m→∞, $q=\frac{5}{3}$

When p=7 m→∞, $q\to\frac{7}{5}$,

When p=11, m→∞, $q=\frac{11}{9}$;............;

When p→∞, m→∞, q→1

For every value of p ≥ 3, q is always less than p, even q ⟶ 1. As a result, we have an answer to the question above:

For every value of p ≥ 3, when m ⟶ ∞, there is not a prime q, with q>p, along with the prime p in order to form an odd perfect number P.

Then there is no Odd Perfect Number.

Conclusion

The perfect number as well as the composite number always have the last factor pair (m, m+1), that consists the unique 2 consecutive factors m, m+1 in the middle of the sequence.

The product of those two factors is equal to the value of the perfect number or the composite number. Particularly at the perfect number m must be a composite number and m+1 must be a prime number Mersenne.

The Argument of the Perfect Number Equation Model led to

A General Formula of Perfect Number and showed that only Even perfect numbers exist and Odd perfect numbers do not exist. Even though the odd perfect number does not exist, it still has a name for millennia.

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