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Analysis on a Nonlinear Diffusion Equations using Numerical Scheme

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Perspective

A nonlinear fully implicit finite difference scheme with second-order time evolution for nonlinear diffusion problem is studied. The scheme is made with two-layer coupled discretization (TLCD) at each time step. It doesn't stir numerical oscillation, while permits large time step length, and produces more accurate numerical solutions than the other two well-known second-order time evolution nonlinear schemes, the Crank-Nicolson (CN) scheme and therefore the backward difference formula second-order (BDF2) scheme. By developing a replacement reasoning technique, we overcome the difficulties caused by the coupled nonlinear discrete diffusion operators at different time layers, and prove rigorously the TLCD scheme is uniquely solvable, unconditionally stable, and has second-order convergence in both space and time. Numerical tests verify the theoretical results, and illustrate its superiority over the CN and BDF2 schemes.

Nonlinear diffusion problems appear in radiation hydrodynamic, reservoir geo mechanics, astrophysics, and many other scientific fields. Efficient and accurate numerical simulations for them play an important role within the associated research areas. To solve nonlinear diffusion equations, people usually use time implicit discretization, since they do not suffer severe restrictions on the time step length while such restrictions are required by operator splitting methods and time explicit methods due to numerical stability. For instance, in Crank-Nicolson (CN) schemes are studied and predictor-corrector and extrapolation techniques are employed to linearize the numerical procedure. Within the backward Euler (BE) and linear CN schemes are studied for a few nonlinear diffusion systems in various applications. Compared with linear semi-implicit schemes, nonlinear fully implicit (FI) schemes are far more appealing since they will capture the transient status more precisely and avoid some nonphysical phenomena in case physical quantities vary violently with time advance.

Compared with the first-order time evolution approaches like the BE method, the second order ones are highly desirable to get efficient and accurate solutions. to unravel multi-dimensional heat conduction and radiation heat transfer problems, some numerical schemes with second-order accuracy in both space and time are developed. Specifically, in some efficient numerical methods with second-order time discretization are studied for non-equilibrium radiation diffusion problem. It shows that although being unconditionally stable in discrete L2 sense and second-order time accurate, the CN scheme causes nonphysical numerical oscillations when using large time step length; while the second order BE time discretization (in literatures it's sometimes mentioned as BDF2, backward difference formula second-order) doesn't present such oscillations, hence permits larger time step length, and has significantly higher accuracy and efficiency than the first-order BE time discretization. During a discrete scheme with two-layer coupled discretization (TLCD) at whenever step is presented, wherein additionally to primary unknowns at integer time layer, new primary unknowns at half time layer are introduced. Hence the TLCD scheme may be a two layer coupled discrete system at whenever step. Numerical tests show that it can avoid the apparent numerical oscillation, which differs from CN scheme.

To carry out theoretical studies on numerical methods for linear and nonlinear PDEs, some discrete functional analysis tools are developed. They're very useful in establishing fundamental properties of some discrete schemes and iterative methods for diffusion equations in divergence or non-divergence form. Some first-order BE schemes, CN and BDF2 schemes are analysed. Especially, by developing a temporal-spatial error splitting argument technique proposed. The properties of some semi-implicit (linear) schemes for nonlinear parabolic problems without mesh ratio requirement. However theoretical analysis on fully implicit (nonlinear) schemes for nonlinear diffusion problems remains rather rare. Within the analysis on nonlinear discrete schemes for diffusion operators in divergence form, the most difficulty lies within the estimate of the nonlinear diffusion term.

This paper contributes to presenting theoretical analysis on the numerical performance for the TLCD scheme. No such analysis result has been published to our knowledge. A new difficulty occurs, which is caused by the additional terms due to the coupling of two-layer discrete equations. They contain discrete diffusion operators whose coefficients are identical the difference between the values of the diffusion coefficient at different time layers. To specialise in illustrating the main ideas on the theoretical analysis, a finite difference method is taken for spatial discretization. Actually, other discrete ways like finite element and finite volume methods also are applicable. First, we prove the existence of the solution for the TLCD scheme with a hard and fast point theorem. A novelty during this procedure is that we tactfully bound the TLCD solution and therefore the difference between the solutions at two layers in proper discrete norms simultaneously. Next, with this boundedness result, a replacement reasoning technique is developed to beat the problem arising from the extra terms, and a rigorous proof for the convergence of the TLCD scheme is performed.

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