

Analysis of Efficient Beamforming and Power Optimization in Wireless Communication

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Abstract

In cellular communication the use of multiple antennas at the base station is the building block of the communication system. The concept of resource allocation especially beam-forming is basically the efficient use of the transmitted power in between or among the users by maintaining an effective cost in term of all the involved constraints and with respect to the economic implications. The computation of optimal users beam-forming is not an easy task but the calculation becomes simple and easy when we compute it with respect to the single or only one design parameter with respect to only one user. The calculation for single user is providing an optimal solution for the power and optimization. We have explained a structure with optimal linear transmit beam-forming with respect to an efficient number of parameters to not to lose the optimal value. Beam forming vector for each user predict its channel. Further power is maximized with this and leakages are reduced. Low complexity schemes available in this regard are providing a comprehensive solution when tested in various scenarios. Some of the optimal solution is provided in this paper with the help of some simple example using variable antennas. This may be extended to designate various constraints in any specific cellular network in consideration.

Keywords: Beamforming; Cellular communication; Economic implications; Resource allocation; Optimal users

Introduction

Small packet services to a large number of user is an effective and efficient technological advancement. It is not mandatory that the number of user are equivalent to the number of antennas. In current system of communication the existence of large number of user is common practice. For handling large users even large number of antennas may exist or may not exist but the utilization of the existing structure is an efficient tool in this regards. It is the optimal utilization of the resources to yield maximum output. One such important technique is the utilization of beams in an optimal way. It means to utilize beam forming techniques in such a way that the parameter provide us an effective result. The results efficacy is further elaborated with the help of different dimensions in this paper. Where we are utilizing the the single or per user strategy which may further be divided depending upon the number if receiver and transmitters to a avail the average achievable sum information rate [1]. Now in wireless communication system there different resources some of which are pre allocated to the users in a non-competitive fashion, or certain for small and random packets, the reservation-based approach is inefficient in resource utilization due to irregularity of the packets [1]. This is of great practical significance for networks offering small packet services to a large number of users [2]. Considering the importance of pilot contamination mitigation to guarantee the good performance of massive MIMO, many schemes, ranging from optimal pilot design and allocation, advanced channel estimation, to downlink pre coding have been proposed to deal with this problem [3]. Mobile user terminals (UT's) receive wireless services from a base station situated inside each cell, within the coverage region. There exists a networking connection in between all Base Stations (BSs) with a wired pivotal network. There exists some problems such as path loss and inter cell interference (ICI) etc. These two factors further give rise to different elements. Such as both reduces the signal to interference-plus-noise ratio (SINR), and hence achievable data rates of the user terminals [4,5]. The computation of optimal users beam-forming is not an easy task but the calculation becomes simple and easy when we compute it with respect to the single or only one design parameter with respect to only one user. With this simple techniques

there exist a number of heuristic beam-forming techniques and schemes. At low SNR optimal beam-forming helps to maximize the received signal power and the value of interference leakage is minimized at high SNRs and an interesting feature is that at intermediate ranges of SNRs it creates a balance between the conflicting goals. We extend this optimal beam-forming structure to a more practical multicell scenario as described in the thesis below [6]. We will provide a combination of massive MIMO and Small cell access points to improve the overall system performance. We will model multi-cell system of different types with some different parameter of the system and will provide an analysis of the system performance though simulation results with respect to individual and system performance criterion. Ideology is based on the structural constraints and resource allocation. The resource allocation concept is basically the efficient use of the transmitted power in between or among the users by maintaining an effective cost in term of all the involved constraints and economic implications. While deploying SCA and massive MIMO there arises a problem of inter user interference and in a complex network where a large number of users exist and large number of antennas are deployed at the BS [7].

Small Packets in Massive MIMO System and Optimization Techniques

While keeping in view the large number of antennas at base station there exist different orthogonal links from user to base station. A large amount of when the number of antennas at BS is large, the different propagation links from the users to the BS tend to be orthogonal, and

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the large amount of spatial degrees of freedom is useful for mitigating the effect of fast fading. Overall, massive MIMO technique provides higher data rate, better spectral and energy efficiencies. All these advantages make massive MIMO a promising technique. This is of great practical significance for networks offering small packet services to a large number of users [8].

The number of active users can be even more than the number of antennas at BS. This is of great practical significance for networks offering small packet services to a large number of users. The major challenge is the application of the different available schemes for the utilization of resources and proper communication. For example in power systems a technique based on CS for meter reading in smart grid is proposed in, and its consideration is limited to single antenna systems. Besides, a novel neighbour discovery method in wireless networks with Reed-Muller Codes has been proposed in where CS technique is also adopted [9,10]. Again in case of Base station practically, the BS does not have perfect CSI. Instead, it estimates the channels as usual. Conventionally it is done by using uplink pilots. Since both the time frequency resources allocated for pilot transmission and the channel coherence time are limited, the number of possible orthogonal pilot sequences is limited too, and hence, the pilot sequences have to be reused in neighboring cells of cellular systems. Therefore, channel estimates obtained in a given cell get contaminated by the pilots transmitted by the users in other cells. This cause pilot contamination that is, the channel estimate at the base station in one cell becomes polluted by the pilots of the users from other cells. Considering the importance of pilot contamination mitigation to guarantee the good performance of massive MIMO, many schemes, ranging from optimal pilot design and allocation, advanced channel estimation, to downlink pre coding have been proposed to deal with this problem [11]. Beamforming transmission is a multifunction technique signal for the transmission of a signal to one to multiple numbers of users from an array of N-antennas. The basic purpose in wireless communication system is to reduce interference to non-intended users and increase power of signal at intended user.

In case of a limited number of antennas only limited results are achieved such as we achieve a limited spatial directivity. It predicts that there is interference which is due to the energy leakage between the users. But it is quite a complicate task to manage a balance between signal power maximization and minimization of the interface leakages. Keeping in view the scenario of the optimization of multiuser transmit beam-forming we conclude that it is generally a nondeterministic polynomial-time (NP) hard problem. Nevertheless, this paper shows that the optimal transmit beam-forming has a simple structure with very intuitive properties and interpretations. This would further provide a theoretical foundation for practically low-complexity beam-forming schemes for the implementation in the multiple scenarios.

In massive MIMO and small scale networks there are a large number of improvements in dynamic parts but due to complexity and dense arrays it required more hardware and hence size of system increases. So to fulfill the demand it is needed to be installed in a reasonable and efficient way with respect to an optimized energy. In an overall view if we look at massive MIMO techniques these are having much higher data rate, and spectral and energy efficiencies are also remarkable. It may increase the number of active users in an effective way even more than the number of antennas and offer large number of services to large user. The significant way to improve data rate is spatial multiplexing which is done without the usage of much frequency resources and keeping the same total transmit power.

Optimal Beamforming and Multiuser Transmit Beamforming Parameters

There are multiple techniques and schemes available for the attainment of high signal power is to transmit same data signals from all antennas with different amplitudes and phases [11]. Unfortunately, the finite number of transmit antennas only provides a limited amount of spatial directivity, which means that there are energy leakages between the users which act as interference. In term of total power constraints values as we can first see simple power minimization problem and further will derive the structure of optimal beam forming. Let us consider a scenario in downlink channel in which BS is equipped with N antennas and can communicate with users K, using SDMA and single antenna [12,13] in this condition is that the data signal to K users is shown as; $1. S_k \in \mathbb{C}$, further is normalized to unity power and the vector $h^k \in \mathbb{C}^{N \times 1}$ elaborate the vector channel. Using linear beamforming K signals of different data can be separated such as beamforming vectors $w_1, \dots, w_k \in \mathbb{C}^{N \times 1}$, where, w_k is associated with K user.

It is normalized version is as $\frac{w_k}{\|w_k\|}$ is beamforming direction and in N dimension vectors space direction is pointed out by this. It is to be noted that in LoS scenarios this points only physical direction. Power transmitted to user K can be pointed out by a squared norm.

We can model a receive signal that is as $r_k \in \mathbb{C}$ at user k

$$\gamma_k = h_k^H \left(\sum_{i=1}^k w_i s_i \right) + n_k \quad (1)$$

Where as it is to be noted that with respect to zero mean and variance, n_k pretends value of additive receiver noise so the signal to noise interference ratio at user K is

$$SINR_k = \frac{|h_k^H w_k|^2}{\sum_{i \neq k} |h_k^H w_i|^2 + \sigma^2} = \frac{1}{\sigma^2} \frac{|h_k^H w_k|^2}{\sum_{i \neq k} \frac{1}{\sigma^2} |h_k^H w_i|^2 + 1} \quad (2)$$

The basic purpose and main goal is to determine the noise impact. By transmit beam-forming the maximization of metric of performance utility can be achieved where as it is the general function of SINR [14,15] We are more interested in the designation of structure of optimal beam-forming.

Average achievable sum information rate

We have considered a scenario in which we have checked the performance of prescribed factors by assuming a systematic design with respect to following constraints. Suppose that we have transmitter $K_t=4$ with four number of antennas. There exist $K_r=4$ users. The channel link existing between transmitter j and user k is generated as an uncorrelated Rayleigh fading. So the average channel gain $E = \{ \|h_{j,k}\|_2^2 \}$ equals four for serving transmitters and two for the transmitters which are interfering. To elaborate the pattern and behavior of various heuristic beam-forming techniques we have considered a scenario with 4-user MISO interference channels with $N_j=4$ antennas with per base station specifications and with global interference coordination. The overall values of channel vectors h_{jk} are generated as uncorrelated - Rayleigh - fading and the value of average channel gain is as $E = \frac{\{ \|h_{j,k}\|_2^2 \}}{\delta_k^2}$ equals the $\frac{N_j}{2}$ for the all of the existing base stations.

The results are shown in the figure below for the average value of achievable sum information rate. MRT is good at the value of a very less SNR, and the XFBF is much good at high SNR. However the main factor that is needed to be emphasized more is that SLNR-MAX is the

strategy which is much more versatile technique and has a competitive strategy for design at different schemes. It actually combines the overall fruits of MRT and ZF asymptotically. Further it amazingly performs them at an intermediate value of SNR by having a close value oriented remarkably closely to the optimal solution.

Simple Power Minimization Problem

By looking into the simple power minimization problem as given below it is not a complicated task to use this to

derive optimal beam forming structure.

$$\max_{w_1, \dots, w_k} \text{imize} = f(SINR_1, \dots, SINR_K) \sum_{k=1}^K \|w_k\|^2 \leq P$$

Subject to $SINR_k \geq \gamma_k \dots (A1)$

Here in this expression the parameters such as $\gamma_1 \dots \gamma_k$ are the SINR parameters that will be achieved by all users as per depending on number users for optimum of the equation above. These γ -parameters elaborate the SINRs which is needed to obtain some certain value of data rates. The values of the γ -parameters are not variable in the equation above and this gave clear impact to optimal beam-forming solution, but there is no variation in the solution structure.

Here the cost function $\sum_{k=1}^K \|w_k\|^2$ is clear depiction of convex function of beam-forming vector.

The basic purpose and main goal is to determine the noise impact. By transmit beam-forming the maximization of metric of performance utility can be achieved where as it is the general function of SINR [14,15]. We are more interested in the designation of structure of optimal beam-forming.

Problem of optimization of general transmit beamforming

Our purpose in this thesis is to analyze carefully problem of optimization of general transmit beam-forming. The task is to maximize the arbitrary utility function $f(SINR_1 \dots SINR_K)$ which is increasing in the value of SINR of every user and there exist a limitation in the total transmit power i.e. limited by P. Mathematically it can be stated as:

$$\max_{w_1, \dots, w_k} \text{imize} = f(SINR_1, \dots, SINR_K)$$

Subject to

$$\sum_{k=1}^K \|w_k\|^2 \leq P \quad (A2)$$

It is a quite complicated task to solve the problem A2 however suppose that if we consider that we know the values of SINR, i.e. $SINR^*_1 \dots SINR^*_k$ that have obtained by an optimal solution to the problem A2. Than what would be the outcome result in case of $\gamma_k = SINR^*_k$, for the values $k=1 \dots K$. with it if solve (A1) for the parameter specifically with particularity of γ -parameter. With respect to this simple scenario it is very clear that the particular constraints that will solve A1 will also be same for providing the solution to A2 and will solve it definitely. It can be specifically described as follow that A1 finds the beam-forming vectors that are solving it achieves the SNR values; $SINR^*_1 \dots SINR^*_k$. The solution for the problem A1 must satisfy the total values of power constraints in the problem A2. The basic reason is that the problem A1 gives the beam-forming which further achieve the given SINRs by using the minimal value of the power. So it can be depicted clearly that the beam-forming vectors from A1 are feasible for A2 and got optimal SINRs values and these are the optimal solution for the problem A2 also.

We, to find out the values of $SINR^*_1 \dots SINR^*_k$ we need to solve

A2, and it is a quite complicated task because the values of SINR are predefined in A1 and are to be calculated in A2. However we have an optimal beam-forming value of A2 as follow

$$w_k^* = \sqrt{p_k} \frac{[I_N + \sum_{i=1}^k \frac{\lambda_i}{\delta^2} h_i h_i^H]^{-1} h_k}{\| [I_N + \sum_{i=1}^k \frac{\lambda_i}{\delta^2} h_i h_i^H]^{-1} h_k \|} \quad \text{for } k=1, \dots, K \quad (3)$$

However, the importance of (16) is that it provides a simple structure for the optimal beam-forming. Although the matrix above in eqn. (3) is same for all of the users, the matrix with respect to the optimal beam-forming vectors and these can be written in the very compact form. The optimal beam-forming direction in the above eqn. (3) consist of two main parts. The channel vector h_k which between the base station BS and the intended users k and 2 is the matrix;

$$[I_N + \sum_{i=1}^k \frac{\lambda_i}{\delta^2} h_i h_i^H]^{-1} \quad (4)$$

As with respect to the beam-forming in the same direction from the expression of the channel as

$$\tilde{W}_k^{MRT} = \frac{h_k}{\|h_k\|}$$

Is the matched filtering or Maximum ratio transmit (MRT). The maximization of the received signal power, $p_k |h_k^H \tilde{W}_k|^2$ and due to this selection at the respective intended user. As

$$\arg \max \tilde{w}_k : \| \tilde{w}_k \|^2 = 1$$

$$|h_k^H \tilde{w}_k|^2 = \frac{h_k}{\|h_k\|} \quad (5)$$

The expression shows with respect to above mentioned fact for MRT is due to the Cauchy-schwarz-inequality. The optimality in beam-forming directions for the values as $K=1$. Further it is not as when there exist a large number of users because of the inter-user-interference is unaccounted in it for MRT (Figure 1).

As the value before normalization $[I_N + \sum_{i=1}^k \frac{\lambda_i}{\delta^2} h_i h_i^H]^{-1}$ when multiplied with h_k , take care of the respective values and it rotates the MRT for reducing the value of interference which is due to the co-user-directions, as $h_1 \dots h_{k-1}, h_{k+1} \dots h_k$ and it is shown in below figure. In the figure the optimal-beam-forming as it lies in between the MRT and the vector is orthogonal to all of the co-user channels. The direction of optimal beam-forming is depended at final on the utility function [16-24]. Finally we discuss some heuristic beam-forming techniques out for some special cases where it is truly optimal but in some special cases only. For example, as we have considered a scenario that is symmetric where the specifications of channels are as that they are uniformly and equally strong with a specific uniform and well directed and specific directivity characteristics. As from the previous scenario in A2 the utility function is too symmetric w.r.t. $SINR_1 \dots SINR_k$ here from the symmetry of λ -parameters can also be made in a suitable and sensible manner. It can further be concluded that $\lambda_k = \frac{P}{K}$ and for all the values of "k" $\sum_{i=1}^k \lambda_i = P$.

Power constraints in practical system

In term of total power constraints values as we can first see simple power minimization problem and further will derive the structure of optimal beam forming. Let us consider a scenario in downlink channel in which BS is equipped with N antennas and can communicate with users K, using SDMA and single antenna [22]. In this condition is that

the data signal to K users is shown as; $s^k \in \mathbb{C}$, further is normalized to unity power and the vector $h^k \in \mathbb{C}^{N \times 1}$ elaborate the vector channel. Using linear beam-forming K signals of different data can be separated. Such as beam-forming vectors $w_1, \dots, w_K \in \mathbb{C}^{N \times 1}$, where, w_k is associated with K user. In the scenario we have described, we have considered the user K with different values and the sum rate utility function as

$$f(\text{SINR}_1, \dots, \text{SINR}_K) = \sum_{k=1}^K \log_2(1 + \text{SINR}_k). \quad (6)$$

It can be elaborated that the simulation results lies at the random circular symmetric complex and averaged with respect to Gaussian Channel realization. At low SNR optimal beam-forming helps to maximize the received signal power and the value of interference leakage is minimized at high SNRs and an interesting feature is that at intermediate ranges of SNRs it creates a balance between the conflicting goals. We extend this optimal beam-forming structure to a more practical Multicell scenario as described.

Illustration with simulations: (For 2-antennas): Consider by keeping in view the scenario described above we are going to simulate under following parameters. Suppose that we have transmit antenna $K_t=2$, with number of antenna per Base Station $N_t=2$ having total number of user $K_r=2$ than the results are as follow and the Comparison of MRT, ZF, SLNR-Max and Optimal beam-forming is as under (Figure 2).

Illustration with simulations: (For 5-antennas): Consider by keeping in view the scenario described above we are going to simulate under following parameters. Suppose that we have transmit antenna $K_t=5$, with number of antenna per Base Station $N_t=5$ having total number of user $K_r=5$ than the results are as follow;

Comparison of MRT, ZF, SLNR-Max and Optimal beam-forming is as under Figure 3.

Illustration with simulations: (For 10-antennas): Consider by keeping in view the scenario described above we are going to simulate under following parameters. Suppose that we have transmit antenna $K_t=10$, with number of antenna per Base Station $N_t=10$, having total number of user $K_r=10$ than the results are as follow (Figure 4):

There exists a comparison maximization of sum rate with respect to linear beam-forming with the different heuristic strategies of beam-forming. These are SLNR-MAX, MRT and ZFBF the three strategies. Over channel realization the value of average sum information rate in a channel of 4-MISO [16] interference channel is presented as a function of transmit power. It is depicted clearly from the diagram that ZFBF and MRT are showing a quite good behavior and are good beam-forming direction at low and high SNR level or values and the remarkable factor is that SLNR-MAX is showing a good performance in the whole range of SNR [17].

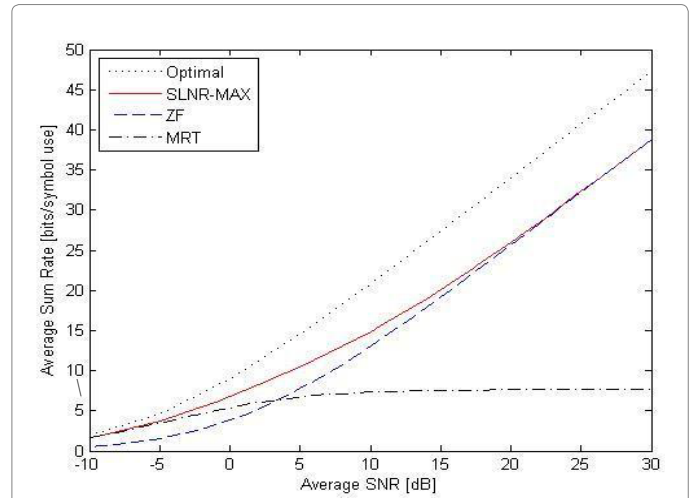
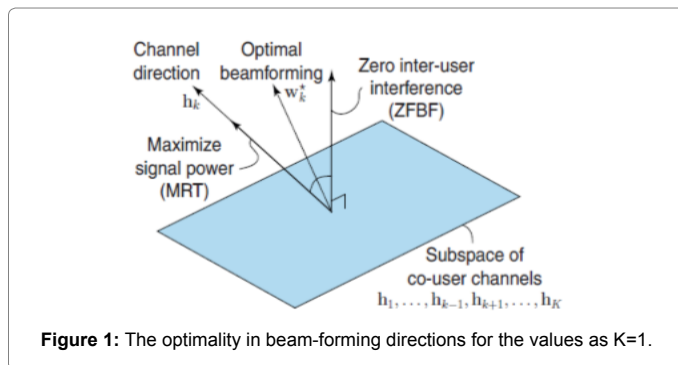


Figure 2: Illustration with simulations: (for 2-antennas).

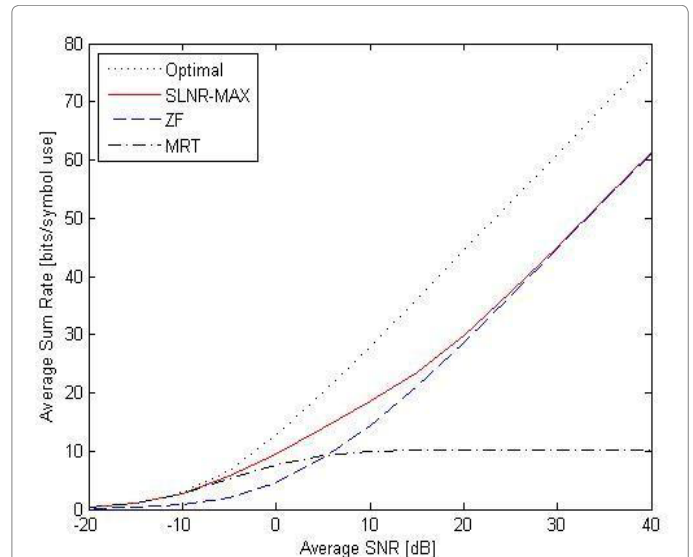


Figure 3: Comparison of Optimal, SLNR-Max, MRT and ZF Beam-forming.

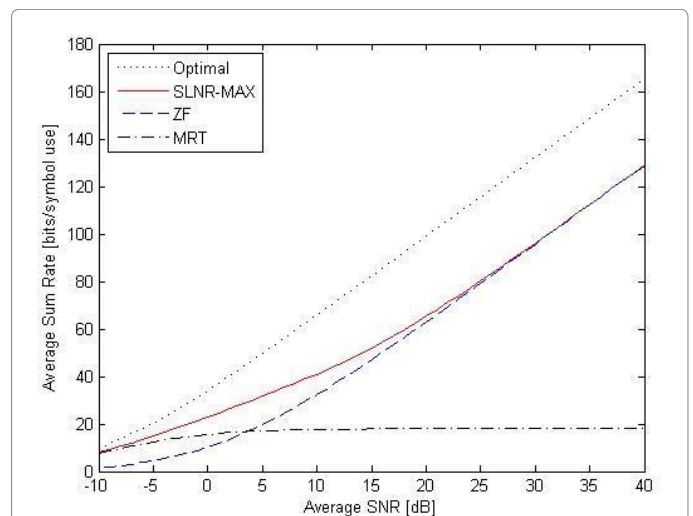


Figure 4: Comparison of Optimal, SLNR-Max, MRT and ZF Beam-forming.

Conclusion and Future Research

In the scenario we have described, we have considered the user K with different values and the sum rate utility function [18]. It can be elaborated that the simulation results lies at the random circular symmetric complex and averaged with respect to Gaussian Channel realization as also prescribed above [19]. The figures above shows Simulation results for the scenario as for transmit antennas with the numbers prescribed. The diagrams illustrate scenarios separately and collective comparative results are also elaborated with multiple numbers of antennas. As the number N increases MRT become optimal at the rate at which SNR range is low. For ZFBF it asymptotically seems optimal at high SNR range. For Transmit MMSE the result is near optimal in the whole range of SNR and transmit MMSE combine the asymptotic properties of MRT and ZFBF. Fine tuning is needed to fulfill optimality gap. Figures elaborate with large number of antenna array we don't need fine tuning i.e. in case of massive MIMO. In case of large number of antennas the optimality increases automatically because of the large availability of the resources but the above described scenario can be extended for further for base stations small cells and users in a specific region [20]. Advantage of optimal beam-forming is the maximization of received signal power at a low rate value of SNR, minimization of interference leakage at high value of SNR range and creation of a balance in between the confliction at the moderate or intermediate SNRs ranges. It could be extended in future to challenges like multi streams and imperfect robust CSI s for the multi users. There were also some existing problems such as path loss and inter cell interference (ICI) etc. Where the user terminals are located farther away from BS are often probable to receive strongly attenuated signals which causes problem of signals attenuation and give rise to path loss [21]. ICI is caused by the transmission from BS in other cells. The multi cell scenario for this is also an optimal resource and research direction.

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