Journal of Civil & Environmental Engineering

Zhu and Huang, J Civil Environ Eng 2016, 6:3
DOI: 10.4172/2165-784X.1000228

Research Article

Open Access

An Interval Parameter Two-Stage Stochastic Approach to Optimize Budget and Schedule in Construction Management

Jinxin Zhu and Gordon Huang*

Institute for Energy, Environment and Sustainable Communities, University of Regina, Canada

Abstract

An interval parameter two-stage stochastic model for project budgeting and scheduling is developed to dealing with the uncertainties residing in the project management. The model focuses on the probability distribution in activity durations and the uncertainties expressed as intervals in costs. It minimizes the inexact costs (direct costs, indirect costs and penalties) with reference to the specified project completion time and the durations of activities estimated from two-stage stochastic programming. The proposed approach for budgeting and scheduling is a hybrid of two stage stochastic programming and inexact optimization. Solutions obtained from the model will provide a reasonable crashing time plan to accomplish given projects on specific time and come up a lowest cost plan for tardiness. This approach can effectively reflect the interactive relationships among all the uncertain system components. The plan provides useful decision support for project managers through these post-optimality analyses. The developed model is applied to a case study to illustrate its feasibility of dealing the actual project management decision problems. The paper implements the model to a gas pipeline construction project with specified completion time and milestone allocating tasks as the case study. The proposed model provides a systematic framework that facilitates the decision making process and enable project managers to justify the range of the solutions when the decision variables are intervals.

Keywords: Cost; Crashing time allocation; Decision making; Inexact optimization; Project management; Two-stage stochastic programming; Uncertainty

Introduction

Estimating errors on management have been known as one major reason causing project failure [1-2] for a long time and Standish Group Reports summarized that projects tend to be failed other than successful given the inaccurate estimation [3]. The hardest part to achieve is the good allocation of task durations and measures of the total amount of costs when combined determine worker productivity. Bad estimation on activities may delay achieving milestone, disrupt the remaining schedule and increase the indirect costs of the project. The delay and disruption caused by estimating errors could lead to the project management failure, which is not delivering the project on time and within budget or at worst to project failure [4]. Plentiful articles provide evidences that link the project failure to an absence of good estimating for project planning and scheduling [5-7].

Traditionally, there are two divergent approaches applied to solve decision problems in project management: the program evaluation and review technique (PERT) and the critical path method (CPM). PERT is a probabilistic approach, and CPM is the deterministic one. Both of them use single-valued task durations to develop a baseline schedule. The real work against schedule must be accomplished. Various heuristics also have been developed to minimize the total completion time [8,9] Most of these methods cannot minimize the total project cost simultaneously [10]. Particularly, all these methods assume that the goals and method inputs are crisp or deterministic.

In practical project decision problems, the input data and parameters of the model (cost coefficients of objective functions and constraints, work forces and resources demand) are not deterministic because some related information is imprecise and given as intervals. Conventional heuristics, PERT, CPM and mathematical programming obviously cannot solve all the intervals programming problems because all of these methods are built on the foundation of crisp numbers. In particular, CPM determines the critical-path and PERT only considers durations of activities in the critical-path when it computes the probability of the project completion time. PERT ignores the near paths that possess a less significant possibility to be critical [11]. With durations represented by intervals, it is impossible to distinguish the critical path from near paths from the project network by using the CPM. Though, there are methods that have been developed for solving interval critical paths problems [2]. But one must test every possible combination for each deterministic numbers within the intervals [12]. Given a complex project network (e.g. a large-scale project), the number of combinations could be extremely large.

For PERT, it presumes that the beta distribution is applicable to all project activities. But many scholars have criticized this aspect of PERT [13,14]. No empirical study has been done to determine the activity durations' typical distribution. Without the particular probability distribution function, we can only obtain intervals from historical data. To deal with this uncertainty, best/worst case (BWC) model is commonly used. In BWC model, solutions are determined under best and worst scenarios, and are useful for testing responses of model solutions under two extreme situations. However, solutions are not adequate to generate decision alternatives in intervals [15].

Durations of activities are random variables. There will, obviously, be varying estimates with varying precision for each task. Because various factors such as unexpected events (Random acts of nature, vendor delays, incorrect shipments of materials, traffic jams, power failures, and sabotage), efficiency of work time, varying skill levels, mistakes and misunderstandings can cause the differences in the actual activity duration [16]. However, we are unable to know what factors will be operative when work is still under planning. Therefore, we

***Corresponding author:** Gordon Huang, Institute for Energy, Environment and Sustainable Communities, University of Regina, 3737 Wascana Parkway, Regina, SK, S4S 0A2, Canada; Tel: 306-585-4111; E-mail: huangg@uregina.ca

Received March 02, 2016; **Accepted** April 25, 2016; **Published** April 27, 2016

Citation: Zhu J, Huang G (2016) An Interval Parameter Two-Stage Stochastic Approach to Optimize Budget and Schedule in Construction Management. J Civil Environ Eng 6: 228. doi:10.4172/2165-784X.1000228

cannot know how long the activity will take. The delay and disruption caused by variance linked to estimates and their presumed certainty can significantly affect not only the duration of the whole project but also the total cost [17].

In response to the above complexities and uncertainties, two-stage stochastic programming (TSP) is effective for problems where decision scenarios analysis is needed and random variables are in present. The initial decision is made before the random events have taken place. The values of these events are unknown at this so called first-stage decision. After the random events have happened and their values have been obtained, a corrective decision would be made so as to minimize the total system cost. Apparently, this decision is the second-stage decision. To deal with the uncertainties existing as ambiguous intervals within the TSP framework, Huang and Loucks developed an inexact two-stage stochastic programming to tackle such uncertainties expressed as probability distribution and intervals [15]. Evidently, the interval parameter two-stage stochastic programming can deal with all the problems explained above, and account for the impacts of initial planning and corrective actions to total project costs.

Therefore, the objective of this proposed research is to develop an interval parameter two-stage model for project management and planning in a hypothetical case study. Through the case study, the applicability of the developed model will be tested. The results obtained will help the project managers better understand the key uncertainties that affect the project management, and generate decision alternatives.

Problem Formulation

The project management problem studied here can be described as follows. Assume that the project has m tasks that must be executed separately before the entire project can be finished in an uncertain environment. Uncertainties refer to the possible delays taking place in every activity. The tardiness of activities is random variable. Given a specified project completion time, the quantity of duration is allocated to the task and shown as a milestone. If the milestone is achieved on time, it will result in normal costs. However, if a milestone is not completed on time due to uncertainties, the days behind the schedule must be compensated by the crashing time. The crashing time dealing with these uncertainties results in incremental crashing costs. The tackled incremental crashing costs for all activities are imprecise. Also, the quality of information that can be obtained for normal costs of all activities is not good enough to be presented as deterministic numbers because the economic data has some intrinsic risks such as the inflation rate floating, labor costs increasing and shortage in fuel supply. Therefore, the crashing costs and normal costs are introduced into the approach as interval parameters. The goal of proposed approach is to minimize total costs with reference to normal costs, durations of activities, specified project completion time and crashing time. Besides, through the approach, the maximum allowed crash time can be obtained to minimizes the total cost while keep the whole project on time. Therefore, the tolerance of the crash time and the budget prepared for crashing activities are also determined.

The proposed model in this work is based on the assumptions as follows:

1. All of the objective functions and constraints are linear.
2. The tasks studied are all located in the critical path. The uncertainties happening in every task are independent. So, the delay caused by uncertainties can be accumulated along the critical path.
3. All the fixed project costs which remain constant regardless of project duration under normal condition are not introduced into the model. In the model, only the costs which are affected by the activities' durations are considered. The total project costs obtained by this model are smaller than the actual total costs.
4. The probability of real time for completing the whole project under uncertainty subjects to a beta distribution.
5. The contractual penalty costs are incurred if the project is completed beyond the specified completion time. But if the project is finished ahead of the completion time, the contractual rewards revenues will be provided. The amount of costs and revenues are mostly determined by the economic values generated by the project. Each project has its own particular values. In the model, we are dealing with one project. Therefore, the value of the penalty is same with the value of the reward.

The assumption one implies that the linearity properties must be technically satisfied in order to solve an optimization problem through linear programming. For the sake of model facilitation, assumption two represents that any delay happening in each activity studied will end up with the delay of total project time in the same quantity. Assumption three concerns the simplicity of the model formulation. To achieve computational efficiency, we ignore all the costs that have not any relevance with activity's duration. Assumption four is made to reflect the uncertainties in the activity's duration. Regardless of the complexity to convert one project's time effectiveness into monetary value, assumption five represents that the amount of money losing for one day delay equals to the amount receiving from one day's ahead.

The following notation is used.

$Z^{\pm}$ = total project costs ($)

$NC_i^{\pm}$ = normal cost for task i ($/day)

T= contractual completion time (days)

$K_i^{\pm}$ = crashing costs for task i ($/day)

i = crashing time by which the finish time of task i is beyond the milestone when the actual completion time is a_j with probability p_j (days)

$Y_{iA}^{\pm}$ = crashing time for task i when the actual finished time is $A^{\pm}$ (days)

$A^{\pm}$ = actual completion time (days)

$a_j^{\pm}$ = actual completion time with probability taking value of j (days)

$D_i^{\pm}$ = planned duration of task i (days)

$D_{i\,\min}^{\pm}$ = minimum planned duration of task i (days)

$F^{\pm}$ = contractual penalty (when the project is completed beyond the specified completion time) or contractual reward (when the project is finished ahead of the completion time) ($/day)

ΔD_i = deviation between the upper bound value and lower bound value of $D_i^{\pm}$ (days)

x_i = scale plate variable $(0 \le x_i \le 1)$

$R_{opt}^{\pm}$ = optimal milestones allocation scheme (days)

$V_{opt}^{\pm}$ = optimal budget for tardiness ($)

Where $Z^{\pm}$, $NC_i^{\pm}$, $K_i^{\pm}$, $Y_{ij}^{\pm}$, $A^{\pm}$, $a_j^{\pm}$, $F^{\pm}$, $D_i^{\pm}$ and $D_{i\,\min}^{\pm}$ are interval parameters or variables. For instance, letting K_i^- and K_i^+ be the lower and upper bounds of $K_i^{\pm}$, correspondingly, we have $K_i^{\pm}=[K_i^-,K_i^+]$.

Modeling Development

The goal of proposed approach is to minimize total costs with reference to normal costs, crashing costs and contractual penalty/award. Moreover, all the parameters, except the contractual completion time, are expressed in intervals. Based on the above consideration, the objective function of the proposed model is as follows:

$$Min\,Z^{\pm}=\sum_{i=1}^{m}NC_i^{\pm}D_i^{\pm}+E\left(\sum_{i=1}^{m}K_i^{\pm}Y_{iA}^{\pm}\right)+F^{\pm}\left(\sum_{i=1}^{m}D_i^{\pm}-T\right) \tag{1a}$$

Constraint on actual completion time

$$\sum_{i=1}^{m}\left(D_i^{\pm}+Y_{iA}^{\pm}\right)\geq A^{\pm} \tag{1b}$$

Constraints on time allocation

$$D_i^{\pm}\geq D_{i\,min}^{\pm}\geq 0,\quad i=1,2,\ldots,m, \tag{1c}$$

Constraints on crashing time

$$D_i^{\pm}\geq Y_{iA}^{\pm}\geq 0,\quad i=1,2,\ldots,m. \tag{1d}$$

To solve this problem, the distribution of $A^{\pm}$ must be approximated by a discrete function. Letting $A^{\pm}$ takes the value $a_j^{\pm}$ with probability P_j for j=1,2,….,n, we have:

$$E\left(\sum_{i=1}^{m}K_i^{\pm}Y_{iA}^{\pm}\right)=\sum_{i=1}^{m}\sum_{j=1}^{n}K_i^{\pm}\left(p_jY_{ij}^{\pm}\right), \tag{2}$$

Thus, reformulation of the foregoing problem is:

$$Min\,Z^{\pm}=\sum_{i=1}^{m}NC_i^{\pm}D_i^{\pm}+\sum_{i=1}^{m}\sum_{j=1}^{n}K_i^{\pm}\left(p_jY_{ij}^{\pm}\right)+F^{\pm}\left(\sum_{i=1}^{m}D_i^{\pm}-T\right) \tag{3a}$$

$$s.t.\sum_{i=1}^{m}\left(D_i^{\pm}+Y_{ij}^{\pm}\right)\geq a_j^{\pm},\quad \forall j, \tag{3b}$$

$$D_i^{\pm}\geq D_{i\,min}^{\pm}\geq 0,\quad \forall i,j, \tag{3c}$$

$$D_i^{\pm}\geq Y_{ij}^{\pm}\geq 0,\quad \forall i,j. \tag{3d}$$

For each value of j, it is essential to repeat the basic constraint set which relates actual completion time a_j to allocated task durations D_i and crashing time Y_{iA}. This modeling formulation is so called a two-stage linear program (TSP) because task durations D_i are set at first stage before the actual completion time is known, while crashing time Y_{iA} are determined at second stage when the completion time is known and the targets are fixed. Apparently, model (3) can effectively reflect vibrations caused by uncertainties in the actual completion time. Moreover, an extended consideration for uncertainties in the parameters is applied in the model. Interval parameters introduced into TSP framework leads to the aforementioned hybrid interval-parameter TSP model, model (3).

As D_i are known, model (3) can be divided into two sets of deterministic submodels corresponding to the lower and upper bounds of the desired value of the objective function [18]. The process of transformation is based on an interactive algorithm, which is different from the process of normal best or worst case analysis [15]. The solution provides reliable intervals for the objective function and decision variables, which accounts for decision alternatives' generation. The detailed transformation process is as follows.

Step 1 This step is to determine values for decision variables and cost coefficients, which correspond to the desired bound of the objective function value. For model (3), Z_- is desired since the objective is to be minimized.

$D_i^{\pm}$ can be expressed in a deterministic value form of $D_i^-+\Delta D_ix_i$, where $\Delta D_i=D_i^+-D_i^-$ and $0\leq x_i\leq 1$. Then, we can transform model (3) into:

$$Min\,Z^{\pm}=\sum_{i=1}^{m}NC_i^{\pm}\left(D_i^-+\Delta D_ix_i\right)+\sum_{i=1}^{m}\sum_{j=1}^{n}K_i^{\pm}\left(p_jY_{ij}^{\pm}\right)+F^{\pm}\left(\sum_{i=1}^{m}D_i^{\pm}-T\right), \tag{4a}$$

$$s.t.\sum_{i=1}^{m}\left(D_i^-+\Delta D_ix_i+Y_{ij}^{\pm}\right)\geq a_j^{\pm},\quad \forall j, \tag{4b}$$

$$D_i^-+\Delta D_ix_i\geq D_{i\,min}^{\pm}\geq 0,\quad \forall i,j, \tag{4c}$$

$$D_i^-+\Delta D_ix_i\geq Y_{ij}^{\pm}\geq 0,\quad \forall i,j, \tag{4d}$$

$$0\leq x_i\leq 1,\quad \forall i. \tag{4e}$$

For objective function, we have its lower bound as follows:

$$Z^-=\sum_{i=1}^{m}NC_i^-\left(D_i^-+\Delta D_ix_i\right)+\sum_{i=1}^{m}\sum_{j=1}^{n}K_i^-\left(p_jY_{ij}^-\right)+F^-\left(\sum_{i=1}^{m}D_i^-+\Delta D_ix_i-T\right). \tag{5a}$$

To interpret the process of converting more clearly, we put all decision variables at the constraints' left-hand sides and rewrite (4b), (4c) and (4d) as follows:

$$s.t.\sum_{i=1}^{m}\left(\Delta D_ix_i+Y_{ij}^{\pm}\right)\geq a_j^{\pm}-\sum_{i=1}^{m}D_i^-,\quad \forall j,z \tag{5b}$$

$$\Delta D_ix_i\geq D_{i\,min}^{\pm}-D_i^-,\quad \forall i, \tag{5c}$$

$$Y_{ij}^{\pm}-\Delta D_ix_i\leq D_i^-\quad \forall i,j, \tag{5d}$$

$$Y_{ij}^{\pm}\geq 0\quad \forall i,j. \tag{5e}$$

Based on the model (5), when $D_i^{\pm}$ approach their lower bounds (i.e. $x_i=0$,), low cost could be obtained if the durations' allocations are satisfied, but increasing number of crashing activities may have to be executed (and thus higher crashing costs) when the planned time allocations (milestones) are not achieved. Conversely, when $D_i^{\pm}$ approach their upper bounds (i.e. $x_i=1$,), we may have a higher cost but, meanwhile, lower risk of violating the planned milestones (and thus lower crashing costs). Therefore, it is difficult to determine whether D_i^- or D_i^+ will correspond to the lower bound of the total project costs (i.e. Z^- ,).

If $D_i^{\pm}$ are considered as uncertain input parameters, an inexact two-stage stochastic programming can be used directly to solve these problems [18]. In the model, an optimized set of target values can be obtained by having decision variables as x_i in model (5). In this study, this optimized set corresponding to the highest possible project benefit is applied to give the uncertain milestones allocations.

Step 2 According to the interval programming method proposed by Huang in 1996 [15] when the constraints' right-hand sides are also uncertain, the sub model that corresponds to Z^- should be associated with lower bounds of the right-hand sides (assuming that $\leq$ relationships exist). Thus, we have the sub model for Z^- as follows:

$$Min\ Z^-=\sum_{i=1}^{m}NC_i^-\left(D_i^-+\Delta D_ix_i\right)+\sum_{i=1}^{m}\sum_{j=1}^{n}K_i^-\left(p_jY_{ij}^-\right)+F^-\left(\sum_{i=1}^{m}D_i^-+\Delta D_ix_i-T\right). \tag{6a}$$

$$s.t.\sum_{i=1}^{m}\left(\Delta D_ix_i+Y_{ij}^-\right)\geq a_j^- -\sum_{i=1}^{m}D_i^-,\quad \forall j, \tag{6b}$$

$$\Delta D_i x_i \geq D_{i\,min}^- - D_i^-, \qquad \forall i, \tag{6c}$$

$$Y_{ij}^- - \Delta D_i x_i \leq D_i^- \qquad \forall i, j, \tag{6d}$$

$$Y_{ij}^- \geq 0 \qquad \forall i, j, \tag{6e}$$

$$0 \leq x_i \leq 1, \qquad \forall i, \tag{6f}$$

where Y_{ij}^- and x_i are decision variables. Solution for Z^- gives the extreme lower bound of total project costs with reference to uncertain inputs of milestones allocation.

Step 3 Let $Y_{ij\,opt}^-$ and $x_{i\,opt}$ be solutions of model (6). Then we can have the optimized tasks' durations as follows:

$$D_i^{\pm} = D_i^- + \Delta D_i x_{i\,opt}, \qquad \forall i. \tag{7}$$

Step 4 According to the interval programming method proposed by Huang in 1996 [15] we have the sub model for Z^+ as follows:

$$Min\ Z^+ = \sum_{i=1}^{m} NC_i^+ \left(D_i^- + \Delta D_i x_{i\,opt}\right) + \sum_{i=1}^{m}\sum_{j=1}^{n} K_i^+ \left(p_j Y_{ij}^+\right) + F^- \left(D_i^- + \Delta D_i x_{i\,opt} - T\right). \tag{8a}$$

$$s.t. \sum_{i=1}^{m} \left(\Delta D_i x_{i\,opt} + Y_{ij}^+\right) \geq a_j^{\ +} - \sum_{i=1}^{m} D_i^-, \qquad \forall j, \tag{8b}$$

$$\Delta D_i x_{i\,opt} \geq D_{i\,min}^+ - D_i^-, \qquad \forall i, \tag{8c}$$

$$Y_{ij}^+ - \Delta D_i x_{i\,opt} \leq D_i^- \qquad \forall i, j, \tag{8d}$$

$$Y_{ij}^+ \geq Y_{ij\,opt}^- \qquad \forall i, j, \tag{8e}$$

where $_{ij}$ are decision variables. Sub model (6) and (8) are deterministic linear programming problems. Based on a grey linear programming approach by Huang et al. [19] we have solutions for model (3) under the optimized tasks' duration allocation as follows:

$$Z_{opt\,opt}^{\pm} = \left[Z_{opt}^-, Z_{opt}^+\right], \tag{9a}$$

$$Y_{ij\,opt}^{\pm} = \left[Y_{ij\,opt}^-, Y_{ij\,opt}^+\right], \qquad \forall i, j, \tag{9b}$$

where Z_{opt}^- and $Y_{ij\,opt}^-$ are solutions of sub model (6), and Z_{opt}^+ and $Y_{ij\,opt}^+$ are those of sub model (8). Therefore, the optimal milestones allocation scheme is:

$$R_{ij\,opt}^{\pm} = D_{i\,opt}^{\pm} + Y_{ij\,opt}^{\pm}, \qquad \forall i, j. \tag{10}$$

Considering the budget prepared for risks causing completion time beyond the contractual limitation, we can have the optimal crashing costs and contractual penalty (budget for tardiness) as follows:

$$V_{opt}^{\pm} = \sum_{i=1}^{m}\sum_{j=1}^{n} K_i^{\pm} \left(p_j Y_{ij\,opt}^{\pm}\right) + F^{\pm} \left(\sum_{i=1}^{m} D_i^{\pm} - T\right). \tag{11}$$

Implementation

The following hypothetical problem can be used to illustrate the proposed approach. A construction manager is in charge of allocating milestones into a gas pipeline construction project with specified completion time T. The pipeline project has four tasks carried out in sequence, namely, excavating, laying, welding and backfilling. Each sector's executing team needs to know how much time they can get. If they take longer time than planned to finish the task, they will crash the activity to catch up with the schedule. The contractual penalty costs are incurred if the project is completed beyond the specified completion time. But if the project is finished ahead of the completion time, the contractual rewards revenues are provided. Tables 1 and 2 show the related time limitations and economic data. The framework of the model is demonstrated in the Figure 1.

The proposed approach is suitable to deal with the system with uncertainties in its components. In this study, uncertainties in milestone's allocation, economic data as well as distribution data for actual total completion time are provided as intervals. Let $D_i^{\pm}$ for i=1,2,3,4.. be the quantity of time that allocated to each task i. If the task is finished in the allocated time, normal cost to the task per day is estimated to be $NC_i^{\pm}$. However, if the allocated time is exceeded, the exceeded time must be made up by crashing the task so as to stick to the schedule. Most used options for team leaders to crash an activity are increasing labors and equipment, and/or working overtime. But these options can result in a increasing of costs to task i with $K_i^{\pm}$ per day crashed ($K_i^{\pm} > NC_i^{\pm}$).

Based on model (4), the project manager's decision problem can be formulated as an interval parameter two-stage stochastic problem as follows:

$$\begin{aligned} Min\,Z^{\pm} =& [40,75]\times[200,250]+[30,40]\times[180,220]+[10,20]\times[300,400]+[20,40]\times[190,230]+ \\ & 0.1\times[800,1000]\times Y_{11}^{\pm}+0.65\times[800,1000]\times Y_{12}^{\pm}+0.25\times[800,1000]\times Y_{13}^{\pm}+0.1\times[900,1200]\times Y_{21}^{\pm}+ \\ & 0.65\times[900,1200]\times Y_{22}^{\pm}+0.25\times[900,1200]\times Y_{23}^{\pm}+0.1\times[1200,1600]\times Y_{31}^{\pm}+0.65\times[1200,1600]\times Y_{32}^{\pm}+ \\ & 0.25\times[1200,1600]\times Y_{33}^{\pm}+0.1\times[850,1100]\times Y_{41}^{\pm}+0.65\times[850,1100]\times Y_{42}^{\pm}+0.25\times[850,1100]\times Y_{43}^{\pm}+ \\ & [100,200]\times\left([40,75]+[30,40]+[10,20]+[20,40]-150\right), \end{aligned} \tag{12a}$$

$$s.t.\ [40,75]+[30,40]+[10,20]+[20,40]+Y_{11}^{\pm}+Y_{21}^{\pm}+Y_{31}^{\pm}+Y_{41}^{\pm} \geq [125,135], \tag{12b}$$

$$[40,75]+[30,40]+[10,20]+[20,40]+Y_{12}^{\pm}+Y_{22}^{\pm}+Y_{32}^{\pm}+Y_{42}^{\pm} \geq [135,165], \tag{12c}$$

$$[40,75]+[30,40]+[10,20]+[20,40]+Y_{13}^{\pm}+Y_{23}^{\pm}+Y_{33}^{\pm}+Y_{43}^{\pm} \geq [165,185], \tag{12d}$$

$$0 \leq Y_{11}^{\pm} \leq [40,75], \tag{12e}$$

$$0 \leq Y_{12}^{\pm} \leq [40,75], \tag{12f}$$

$$0 \leq Y_{21}^{\pm} \leq [30,40], \tag{12g}$$

$$0 \leq Y_{21}^{\pm} \leq [30,40], \tag{12h}$$

$$0 \leq Y_{22}^{\pm} \leq [30,40], \tag{12i}$$

$$0 \leq Y_{23}^{\pm} \leq [30,40], \tag{12j}$$

$$0 \leq Y_{31}^{\pm} \leq [10,20], \quad (12k\ 0 \leq Y_{32}^{\pm} \leq [10,20], \tag{12l}$$

$$0 \leq Y_{33}^{\pm} \leq [10,20], \tag{12m}$$

$$0 \leq Y_{33}^{\pm} \leq [10,20], \tag{12n}$$

$$0 \leq Y_{42}^{\pm} \leq [20,40] \tag{12o}$$

$$0 \leq Y_{43}^{\pm} \leq [20,40]. \tag{12p}$$

Model (12) can then be converted into two sets of deterministic sub models as follows:

Step 1 Let $D_i^{\pm}$ have a deterministic value form of $D_i^- + \Delta D_i x_i$ where $0 \leq x_i \leq 1$ and $0 \leq x_i \leq 1$. Therefore, $D_1^{\pm}$, $D_2^{\pm}$, $D_3^{\pm}$ and $D_4^{\pm}$' respectivelyhavevaluesof $40+(75-40)x_1, 30+(40-30)x_2, 10+(20-10)x_3$ and 25+(40-20)x_4. According to model (4), we can covert model (12) to:

$$\begin{aligned} Min\,Z^{\pm} =& (40+35x_1)\times[200,250]+(30+10x_2)\times[180,220]+(10+10x_3)\times[300,400]+(20+10x_4)\times \\ & [190,230]+0.1\times[800,1000]\times Y_{11}^{\pm}+0.65\times[800,1000]\times Y_{12}^{\pm}+0.25\times[800,1000]\times Y_{13}^{\pm}+0.1\times \\ & [900,1200]\times Y_{21}^{\pm}+0.65\times[900,1200]\times Y_{22}^{\pm}+0.25\times[900,1200]\times Y_{23}^{\pm}+0.1\times[1200,1600]\times Y_{31}^{\pm}+0.65\times \\ & [1200,1600]\times Y_{32}^{\pm}+0.25\times[1200,1600]\times Y_{33}^{\pm}+0.1\times[850,1100]\times Y_{41}^{\pm}+0.65\times[850,1100]\times Y_{42}^{\pm}+0.25\times \\ & [850,1100]\times Y_{43}^{\pm}+[100,200]\times\left([40,75]+[30,40]+[10,20]+[20,40]-150\right), \end{aligned} \tag{13a}$$

$$s.t.\ 100+35x_1+10x_2+10x_3+20x_4+Y_{11}^{\pm}+Y_{21}^{\pm}+Y_{31}^{\pm}+Y_{41}^{\pm} \geq [125,135], \tag{13b}$$

$$100+35x_1+10x_2+10x_3+20x_4+Y_{12}^{\pm}+Y_{22}^{\pm}+Y_{32}^{\pm}+Y_{42}^{\pm} \geq [135,165], \tag{13c}$$

$$100+35x_1+10x_2+10x_3+20x_4+Y_{13}^{\pm}+Y_{23}^{\pm}+Y_{33}^{\pm}+Y_{43}^{\pm} \geq [165,185], \tag{13d}$$

$$0 \leq Y_{11}^{\pm} \leq [40,75], \tag{13e}$$

Task	Minimum allowable allocation ($D^{\pm}_{i\ \min}$)	Task Duration allocation ($D_i^{\pm}$)	Normal costs when task duration is not exceeded ($NC_i^{\pm}$)	Crashing costs when task duration is exceeded ($K_i^{\pm}$)
Excavate ($i=1$)	34	[40, 75]	[200, 250]	[800, 1000]
Lay ($i=2$)	25	[30, 40]	[180, 220]	[900, 1200]
Weld ($i=3$)	7	[10, 20]	[300, 400]	[1200, 1600]
Backfill ($i=4$)	19	[20, 40]	[190, 230]	[850, 1100]
Pipeline project	Specified completion time (T)		Contractual penalty/reward ($F^{\pm}$)	
	150		[100, 200]	

Table 1: Time limitations and economic data.

	Actual completion time ($a_j^{\pm}$)	Probability (p_j)
Low risk ($j=1$)	[125, 135]	0.1
Medium risk ($j=2$)	[135, 165]	0.65
High risk ($j=3$)	[165, 185]	0.25

Table 2: Description of actual completion time distribution.

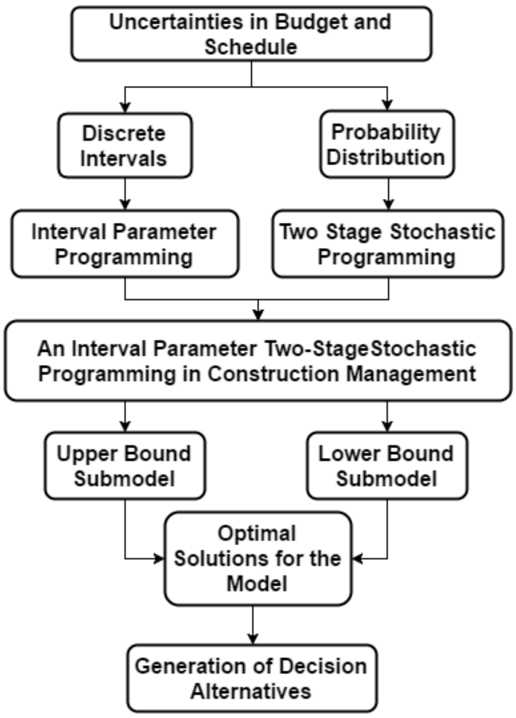

Figure 1: Framework of the model.

$$0 \le Y_{12}^{\pm} \le [40,75], \tag{13f}$$
$$0 \le Y_{13}^{\pm} \le [40,75], \tag{13g}$$
$$0 \le Y_{21}^{\pm} \le [30,40], \tag{13h}$$
$$0 \le Y_{22}^{\pm} \le [30,40], \tag{13i}$$
$$0 \le Y_{23}^{\pm} \le [30,40], \tag{13j}$$
$$0 \le Y_{31}^{\pm} \le [10,20], \tag{13k}$$
$$0 \le Y_{32}^{\pm} \le [10,20], \tag{13l}$$
$$0 \le Y_{33}^{\pm} \le [10,20], \tag{13m}$$
$$0 \le Y_{41}^{\pm} \le [20,40], \tag{13n}$$
$$0 \le Y_{42}^{\pm} \le [20,40], \tag{13o}$$
$$0 \le Y_{43}^{\pm} \le [20,40], \tag{13p}$$
$$0 \le x_i \le 1, \qquad i=1,2,3. \tag{13q}$$

Step 2 From model (6), we can have sub model Z^- as follows:

$$\begin{aligned} Min\, Z^- = {} & (40+35x_1)\times 200+(30+10x_2)\times 180+(10+10x_3)\times 300+(20+20x_4)\times 190+0.1\times 800\times \\ & Y_{11}^- + 0.65\times 800\times Y_{12}^- + 0.25\times 800\times Y_{13}^- + 0.1\times 900\times Y_{21}^- + 0.65\times 900\times Y_{22}^- + 0.25\times 900\times Y_{23}^- + 0.1\times \\ & 1200\times Y_{31}^- + 0.65\times 1200\times Y_{32}^- + 0.25\times 1200\times Y_{33}^- + 0.1\times 850\times Y_{41}^- + 0.65\times 850\times Y_{42}^- + 0.25\times 850\times \\ & Y_{43}^- + 100\times(100+35x_1+10x_2+10x_3+20x_4-150), \end{aligned} \tag{4a}$$

$$100+35x_1+10x_2+10x_3+20x_4+Y_{12}^-+Y_{22}^-+Y_{32}^-+Y_{42}^- \ge 135, \tag{14b}$$
$$100+35x_1+10x_2+10x_3+20x_4+Y_{12}^-+Y_{22}^-+Y_{32}^-+Y_{42}^- \ge 135, \tag{14c}$$
$$100+35x_1+10x_2+10x_3+20x_4+Y_{13}^-+Y_{23}^-+Y_{33}^-+Y_{43}^- \ge 165, \tag{14d}$$
$$0 \le Y_{11}^- \le 40, \tag{14e}$$
$$0 \le Y_{12}^- \le 40, \tag{14f}$$
$$0 \le Y_{13}^- \le 40, \tag{14g}$$
$$0 \le Y_{21}^- \le 30, \tag{14h}$$
$$0 \le Y_{22}^- \le 30, \tag{14i}$$
$$0 \le Y_{23}^- \le 30, \tag{14j}$$
$$0 \le Y_{31}^- \le 10, \tag{14k}$$
$$0 \le Y_{32}^- \le 10, \tag{14l}$$
$$0 \le Y_{33}^- \le 10, \tag{14m}$$
$$0 \le Y_{41}^- \le 20, \tag{14n}$$
$$0 \le Y_{42}^- \le 20, \tag{14o}$$
$$0 \le Y_{43}^- \le 20, \tag{14p}$$
$$0 \le x_i \le 1, \qquad i=1,2,3. \tag{14q}$$

Sub model (14) is a deterministic linear programming problem. Its solution is given as follows:

$$Z_{opt}^- = 31300 \quad x_{1opt} = 0.1428571 \quad x_{2opt} = 1 \quad x_{3opt} = 0 \quad x_{4opt} = 1 \quad Y_{11opt}^- = Y_{12opt}^- = Y_{21opt}^- = Y_{22opt}^-$$
$$= Y_{23opt}^- = Y_{31opt}^- = Y_{32opt}^- = Y_{33opt}^- = Y_{41opt}^- = Y_{42opt}^- = Y_{43opt}^- = 0 \quad Y_{13opt}^- = 30$$

Step 3 We then have the optimized milestones allocation as follows:

$$D_{1opt}^{\pm} = 40 + 35\times 0.1428571 = 45,$$
$$D_{2opt}^{\pm} = 30 + 10\times 1 = 40,$$

$$D^{\pm}_{3opt} = 10 + 10 \times 0 = 10,$$

$$D^{\pm}_{4opt} = 20 + 20 \times 1 = 40.$$

Step 4 According to model (8) and solutions above, we can have the submodel Z^+ as follows:

$$Max\,Z^{-} = 45\times 250 + 40\times 220 + 10\times 400 + 40\times 230 + 0.1\times 1000\times Y^{+}_{11} + 0.65\times 1000\times Y^{+}_{12} + 0.25\times 1000\times Y^{+}_{13} + 0.1\times 1200\times Y^{+}_{21} + 0.65\times 1200\times Y^{+}_{22} + 0.25\times 1200\times Y^{+}_{23} + 0.1\times 1600\times Y^{+}_{31} + 0.65\times 1600\times Y^{+}_{32} + 0.25\times 1600\times Y^{+}_{33} + 0.1\times 1100\times Y^{+}_{41} + 0.65\times 1100\times Y^{+}_{42} + 0.25\times 1100\times Y^{+}_{43} + 200\times(135-150), \quad (15a)$$

$$s.t.\ 135 + Y^{+}_{11} + Y^{+}_{21} + Y^{+}_{31} + Y^{+}_{41} \geq 135, \quad (15b)$$

$$135 + Y^{-}_{12} + Y^{-}_{22} + Y^{-}_{32} + Y^{-}_{42} \geq 165, \quad (15c)$$

$$135 + Y^{-}_{13} + Y^{-}_{23} + Y^{-}_{33} + Y^{-}_{43} \geq 185, \quad (15d)$$

$$Y^{+}_{11} \geq 0, \quad (15e)$$

$$Y^{+}_{12} \geq 0, \quad (15f)$$

$$Y^{+}_{13} \geq 30, \quad (15g)$$

$$Y^{+}_{21} \geq 0, \quad (15h)$$

$$Y^{+}_{22} \geq 0, \quad (15i)$$

$$Y^{+}_{23} \geq 0, \quad (15j)$$

$$Y^{+}_{31} \geq 0, \quad (15k)$$

$$Y^{+}_{32} \geq 0, \quad (15l)$$

$$Y^{+}_{33} \geq 0, \quad (15m)$$

$$Y^{+}_{41} \geq 0, \quad (15n)$$

$$Y^{+}_{42} \geq 0, \quad (15o)$$

$$Y^{+}_{43} \geq 0, \quad (15p)$$

Solution of submodel (15) is as follows:

$Z^{+}_{opt} = 62375 \quad Y^{+}_{11opt} = Y^{+}_{21opt} = Y^{+}_{22opt} = Y^{+}_{23opt} = Y^{+}_{31opt} = Y^{+}_{32opt} = Y^{+}_{33opt} = Y^{+}_{41opt} = Y^{+}_{42opt} = 0 \quad Y^{+}_{12opt} = 30 \quad Y^{+}_{13opt} = 45 \quad Y^{+}_{43opt} = 5$

According to model (9), we have the solutions for model (12) under the optimized milestone allocation (i.e., when $x_{1opt} = 0$, $x_{2opt} = 1, x_{3opt} = 0$ and $x_{4opt} = 1$) as follows:

$Z^{\pm}_{opt} = [31300, 62375] \quad Y^{\pm}_{11opt} = Y^{\pm}_{21opt} = Y^{\pm}_{22opt} = Y^{\pm}_{23opt} = Y^{\pm}_{31opt} = Y^{\pm}_{32opt} = Y^{\pm}_{33opt} = Y^{\pm}_{41opt} = Y^{\pm}_{42opt} = 0 \quad Y^{\pm}_{12opt} = [0,30] \quad Y^{\pm}_{13opt} = [30,45] \quad Y^{\pm}_{43opt} = [0,5]$

Thus, the optimal milestone allocation scheme referring to model (10) is:

$R^{\pm}_{11opt} = 40 + 0 = 40 \quad R^{\pm}_{12opt} = 40 + [0,30] = [40,70] \quad R^{\pm}_{13opt} = 40 + [30,45] = [70,85] \quad R^{\pm}_{21opt} = 30 + 0 = 30 \quad R^{\pm}_{22opt} = 30 + 0 = 30 \quad R^{\pm}_{23opt} = 30 + 0 = 30 \quad R^{\pm}_{31opt} = 10 + 0 = 10 \quad R^{\pm}_{32opt} = 10 + 0 = 10 \quad R^{\pm}_{33opt} = 10 + 0 = 10 \quad R^{\pm}_{41opt} = 20 + 0 = 20 \quad R^{\pm}_{42opt} = 20 + 0 = 20 \quad R^{\pm}_{43opt} = 20 + [0,5] = [20,25]$

Results and Findings

A more detailed presentation of the above solutions is provided in Table 3. From the table, we can see that solutions for the objective function and all of the non-zero decision variables are intervals in general. Under the situation that the completion time is beyond the time limited by the contract, first the allotment of crashing time to task one and then, if necessary the allotment to task four, should be increased. Task two and three's planned durations are better not crashed since they bring the higher normal costs when the activities are finished in time and are subjected to higher crashing costs if the activities are delayed. In comparison, the task one and four are subject to lower normal costs and crashing costs (Table 1). In Figures 1-3 Gantt charts are drawn regarding optimal milestones allocation which is associated with every rectangular bar.

	$i=1$	$i=2$	$i=3$	$i=4$
Milestones ($D^{\pm}_{i\ opt}$)	45	40	10	40
Crashing time ($Y^{\pm}_{ij\ opt}$)				
$j=1$	0	0	0	0
$j=2$	[0,30]	0	0	0
$j=3$	[30,35]	0	0	[0,5]
Optimized milestones ($R^{\pm}_{ij\ opt}$)				
$j=1$	40	30	10	20
$j=2$	[40,70]	30	10	20
$j=3$	[70,85]	30	10	[20,25]
Objective ($Z^{\pm}_{opt}$)	[31300,62375]			

Table 3: Solutions under optimized milestones.

Given milestone allocation, the interval solutions for $Y^{\pm}_{ij}$ reflect potential system condition variations caused by uncertain inputs of $NC^{\pm}_i$, $K^{\pm}_i$ and $a^{\pm}_j$. The solution of $Y^{\pm}_{(11opt)} = 0$, $Y^{\pm}_{12opt} = [0,30]$ and $Y^{\pm}_{13opt} = [30,45]$ means that, for task 1 (i.e., excavate), there will be no crashing time at low risk but a crashing time of 0 to 30 days at medium risk (with a probability of 65%) and a crashing of 30 to 45 days at high risk (with a probability of 25%). The solution of $Y^{\pm}_{41opt} = 0$, $Y^{\pm}_{43opt} = [0,5]$ and $Y^{\pm}_{43opt} = [0,5]$ demonstrates that, for task 4 (i.e., backfill), there will be no crashing activity needed at the low and medium risk. However, a crashing time of 0 to 5 days will be carried out at high risk (with a probability of 25%) if necessary. Under advantageous conditions (e.g., when the other activities do not violet the time limitations and/or the actual completion time $a^{\pm}_4$ approaches its lower bound), the crashing time may move to the value of 0; however, under risky condition, the crashing time may become as high as 5.

The values of Z^{-}_{opt} and Z^{+}_{opt} provide two extremes of the total project cost. As actual values of the parameters and variables changing from their lower bounds to upper bounds, the project cost may correspondingly vary between Z^{-}_{opt} and Z^{+}_{opt} with different reliability levels [15]. The solution presented in Table 3 was obtained under the optimized milestones allocation (i.e. $D^{\pm}_{1opt} = 45, D^{\pm}_{3opt} = 10$ and $D^{\pm}_{4opt} = 40$). While this solution may lead to an extremely low project cost under advantageous situation, high crashing costs may have to be paid when the completion time is beyond the time limitation. This uncertainty results in a wide interval between the lower and upper bounds of $Z^{\pm}$. Solutions under other scenarios of milestones allocation can also be obtained by letting $D^{\pm}_i$ have different deterministic values. For example, solution when $D^{\pm}_1$ reaches D^{-}_1 (i.e., let $x_1 = 0$ in submodel (13)) represents a situation when the project manager is optimistic of activity duration of task 1, and thus promises a lower bound value.

Table 4 describes solutions under different scenarios of milestone allocation. The solution when all $D^{\pm}_i$ reach their lower bounds represents a situation when the manager is optimistic of tasks' durations, and thus plans the lower bound allocation values to all tasks. This action leads to a plan with both lower total normal costs and lower contractual costs, but a higher risk of crashing tasks. The net benefit under this condition is [48775, 85900]. Conversely, the solution when

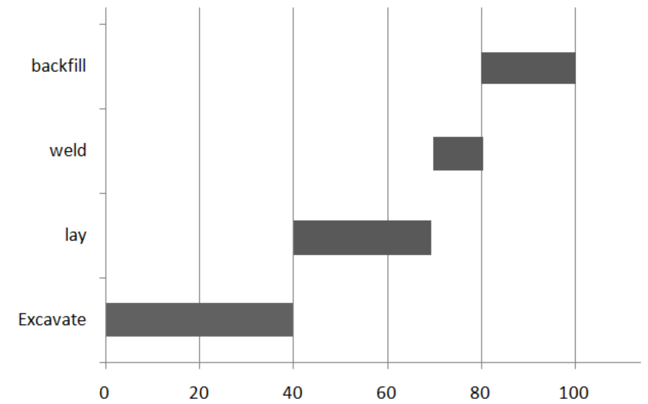

Figure 2: Gantt chart for optimal milestones in low risk.

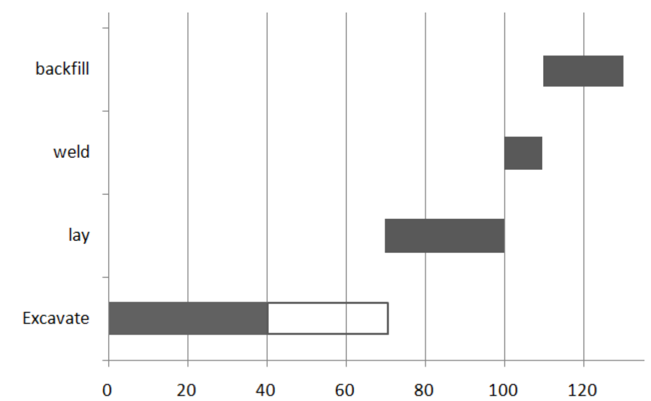

Figure 3: Gantt chart for optimal milestones in medium risk.

all $D_i^{\pm}$ reach their upper bounds implies that the project manager is conservative of tasks' durations.

Thus, a plan with both higher total normal costs and contractual costs is generated, where risks of crashing tasks are reduced. At the allocation's lower bounds, the resulting plan will be effective in low risk situation when all allocated milestones are not violated, but it will become risky in medium and high risk situations due to over-optimistic on tasks' durations and the crashing costs. When all $D_i^{\pm}$ equal their mid-values $x_i = 0.5$ corresponds to a situation when project manager stands between optimistic and conservative conditions.

Decision alternatives can be created through change of the crashing values and thus optimized allocation values in the intervals for decision variables. The project managers could also adjust the allocation scheme through the adjustment between lower and upper bounds of non-zero $Y_{ij}^{\pm}$ when they are not satisfy with the recommended alternatives. Therefore, the intervals for $Y_{ij}^{\pm}$ are useful for them to justify the generated alternatives directly.

Figure 4 and Table 5 present reduced costs of decision variables, and indicate the economic impacts of variations in crashing time allocation. No matter when the risk of disturbing events happening in the whole project is in a low, medium or high level, task three has more impacts on the project costs than any other three tasks, which means the project manager should be cautious on the crashing of task three. The crashing of task three could cause higher increment on the project costs. $Y_{11}^{\pm}$ and $Y_{12}^{\pm}$ have lowest impacts on project costs at their lower bounds. $Y_{13}^{\pm}$ and $Y_{43}^{\pm}$ have lowest impacts on project costs at either bounds. Project manager should mainly consider crashing these tasks with reference to different risk level. This study is consistent with the above results of the relationship between crashing time and project costs. The post-optimality analysis conducted for solutions reflects the relationship between economic consideration and the task duration scheduling.

In Table 5, it shows the sensitivity of crashing time value for project costs at their lower and upper bounds. In practical decision making problems, the project manager could adjust the values of decision variables continuously between the two bounds of their solutions. Through this process, the decision maker could implement more implicit knowledge into the problem. Eventually, more applicable and satisfactory decision schemes could be obtained.

Traditional method (i.e., CPM) tackled task duration scheduling problem can only deal with the deterministic values. When the reliable data is not available, either the method cannot be applied, or it will be solved by letting all interval parameters be equal to their mid-values [2]. The mid-values for all parameters is an input scenario and will generate only one of many alternatives from the CPM. Although further sensitivity analysis can provide an individual response to variations of the uncertain inputs for each possibility, it can hardly reflect numerous possibilities in these uncertain inputs and interactions among them.

Similarly, if PERT method is applied, only solutions with considering the three estimated durations (the optimistic duration, the pessimistic duration and the most likely duration) are obtained. They are useful for judging the construction teams' capabilities to complete the project on time, but cannot efficiently deals with variations in estimating durations. In another word, the solutions obtained from PERT fail to reflect the uncertainties in the task duration's estimation. Therefore, the PERT method is not able to construct a set of stable intervals for decision variables [13].

Compared with existing approaches for solving project management problems, the interval parameter two-stage stochastic approach has its strengths in data availability, the computational requirement and solution algorithm [20]. In practical problems, the availability of information is often not good enough to be acquired as deterministic numbers or probability distributions to support decision making. In most situations, the data is obtained in intervals. These intervals can be processed by interval parameter two-stage stochastic model, which is particularly relevant to the project management area because uncertain parameters are often an obstacle to good estimates of lower and upper bounds.

In fact, even if the quality of information is good enough to be presented as a probability distribution, a large multi-stage programming model is still hard to solve when its parameters are uncertain and expressed as probability density functions. In contrast with it, the proposed approach is able to effectively communicate these uncertain parameters, presented as intervals and probability, into its optimization framework. Moreover, the approach reduces computational requirements by its simplified sub models.

Based on the illustrative purposes, a very simple project is chosen to be a worked example. For large-scale project management problems, especially those including project networks, the optimization formulation could be too complicated to solve. For instance, additional stages often need to be considered to describe how a problem unfolds over time because of the project's dynamic feature. However, more than three stages could make the optimization model become too

	$D_i^{\pm} = D_i^{-}$				$D_i^{\pm} = D_i^{+}$				$D_i^{\pm} = D_i^{mid}$			
	$i=1$	$i=2$	$i=3$	$i=4$	$i=1$	$i=3$	$i=3$	$i=4$	$i=1$	$i=2$	$i=3$	$i=4$
$D_i^{\pm}$	40	30	10	20	75	40	20	40	57.5	35	15	30
$Y_{ij}^{\pm}$												
$j=1$	[25, 35]	0	0	0	0	0	0	0	0	0	0	0
$j=2$	[35, 40]	[0, 5]	0	[0, 20]	0	0	0	0	[0, 27.5]	0	0	0
$j=3$	40	[5, 25]	0	20	[0, 10]	0	0	0	27.5	0	0	[0, 20]
$j=$												
$j=1$	[65, 75]	30	10	20	75	40	20	40	57.5	35	15	30
$j=2$	[75, 80]	[30, 35]	10	[20, 40]	75	40	20	40	[57.5, 85]	35	15	30
$j=3$	80	[35, 55]	10	40	[75, 85]	40	20	40	85	35	15	[30, 50]
$Z^{\pm}$	[48775, 85900]				[38300, 52250]				[32250, 62725]			

Table 4: Solutions under different scenarios of milestones allocation.

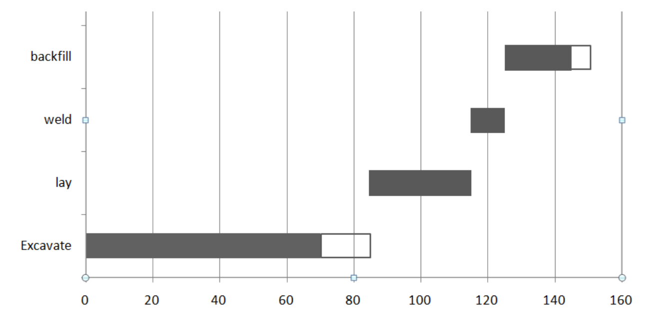

Figure 4: Gantt chart for optimal milestones in medium risk.

	Y_{11}	Y_{12}	Y_{13}	Y_{21}	Y_{22}	Y_{23}	Y_{31}	Y_{32}	Y_{33}	Y_{41}	Y_{42}	Y_{43}
Lower	80	420	0	90	485	25	120	680	100	85	452.5	12.5
Upper	0	0	0	20	130	25	60	390	125	10	65	0

Table 5: Reduced costs of decision variables.

large to justify. In addition, many project management problems are complicated because the manager needs to take adequate account of persistence in historical data. To quantify the future project completion time, the handled conditional probabilities may lead to non-linearity in the modeling and violation of the linear assumption in the proposed approach [21].

In general, the proposed approach can deal with the uncertain parameters, intervals for approximating probabilistic information, through simplifying a real-world project problem and generating several alternatives. However, it has many limitations for handling large and complicated projects. Given such a condition, more other models hybrid with the proposed approach could be developed to obtain improved applicability in project management [22,23].

In real-life project management decisions, a perfect project's total costs are the sum of normal costs, crashing costs and contractual penalties. The input data or related parameters, such as cost coefficients

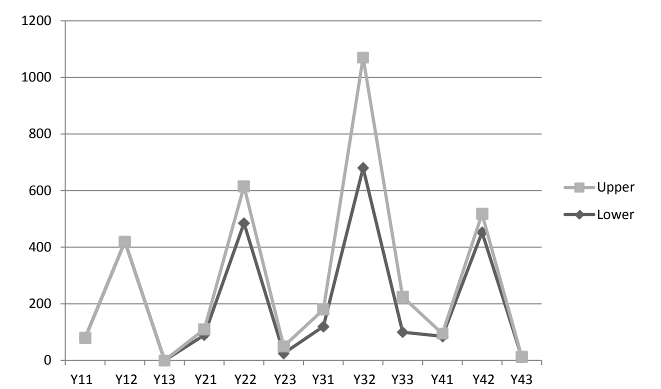

Figure 5: Reduced costs of decision variables.

and time limitation of the objective function and constraints are frequently imprecise, because some relevant information is incomplete or unavailable. This paper developed an interval parameter two-stage stochastic approach for solving the project management decision problems in an uncertain environment.

The proposed model minimizes the total costs with reference to normal costs, crashing costs and penalty costs, allocated task duration, specified project completion time and crashing time. The model yields an interval optimal solution and several decision alternatives in different scenarios. Moreover, the proposed model provides a systematic framework that facilitates the decision making process and enable project managers to justify the range of the solutions when the decision variables are intervals. Consequently, the interval parameter two-stage stochastic approach is practically applicable for dealing with project management problems.

The main limitation of the proposed model is that it cannot handle the large and complicated system. The studied system in this paper is a pipeline construction project. For large-scale construction management problems, adequate account of persistence in historical data and dynamic system features can raise the major problem for the linear assumption in the approach. It is recommended that more complex and hybrid model based on the proposed approach should be developed for improving its applicability.

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