Global Journal of Technology & Optimization Volume 3, 2012

AN INNOVATIVE TECHNIQUE FOR DESIGN OPTIMIZATION OF CORE TYPE 3-PHASE DISTRIBUTION TRANSFORMER USING MATHEMATICA

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Abstract

In modern Industrial era the demand for electricity is increasing exponentially with each passing day. Distribution transformer is the most vital component for efficient and reliable distribution and utilization of electrical energy. With the increased demand in energy it has become essential for utilities to expand the capacity of their distribution networks significantly resulting in tremendous increase in demand of distribution transformers of various ratings. So the economic optimization by minimizing the mass of distribution transformer is of critical importance. This research paper focuses on the global minimization of the cost function of 3-phase core type oil immersed distribution transformer. The methodology used in this research work is based on nonlinear constrained optimization of the cost function subjected to various nonlinear equality and inequality constraints. The non-linear mathematical model comprising of the cost function and a set of constraints has been implemented successfully by using Mathematica software which provides a very robust and reliable computational tool for constrained nonlinear optimization that ensures the solution of the problem to be the global minimum. Finally, based on the above mentioned optimization technique, a 25 kVA 3-phase core type distribution transformer has been designed according to the latest specifications of PEPCO (Pakistan Electric and Power Company). It is found that the innovative optimization technique for transformer design that is developed during this research resulted in considerable cost reduction.

Keywords: Distribution Transformer, Global Optimization, Mathematica

1. Introduction

To meet the increased demand of oil immersed 3-phase distribution transformers in an economic way the cost optimization of the transformer design by reducing the mass of active part has become of vital importance.

In traditional transformer design techniques, designers had to rely on their experience and judgment to design the required transformer. Early research in transformer design attempted to reduce much of this judgment in favor of mathematical relationships [1].

Several design procedures for low-frequency transformers have been developed in past research. Mathematical models were also derived for computer- aided design techniques in an attempt to eliminate time consuming calculations associated with reiterative design procedures [2] - [4].

These previously developed design techniques were focused on maximizing the (VA) capacity of transformers or loss minimizations. Some techniques like unconstrained optimization, genetic algorithms and neural networks etc. also aimed to minimize the mass and consequently the cost of active part of the transformer but it does not ensures the global minimization of the cost function [5 - 11].

As far as global minimization of cost function of low frequency shell type dry transformer is concerned, adequate research work has been done which involves minimization of cost function by using geometric programming [12].

The optimization done by geometric programming always give the global minimum value of the cost function but the difficulty is that in practice, majority of mathematical formulae that are used for transformer design are non-linear and cannot be converted into geometric format.

Regarding the global design optimization of cost function for the active part (winding and core) of oil immersed 3-phase core type distribution transformer there is still a lot more room for further significant research.

In this research work the nonlinear mathematical model of 3-phase core type transformer comprising of the cost function and a set of nonlinear constraints all expressed in terms of certain primary variables has been used for non-linear global optimization by using Mathematica software. The main advantage of nonlinear optimization over geometric programming is that almost all the formulae which are used in design procedure for 3-phase core type transformer can easily be expressed in non-linear form. Moreover, by using Mathimatica the time consumed for cost minimization by non-linear optimization has been significantly reduced to a few seconds.

2. Non-linear mathematical model of transformer for cost optimization

Description of Basic Terms:

A. Design Variables:

J_s: Current Density in LV winding (A/mm^2)

Rc: Core Radius in mm (For circular core)

Rp: Mean Radius of HV winding in mm

Rs: Mean Radius of LV winding in mm

ts: Radial Build of LV winding in mm

tp: Radial Build of LV winding in mm

g: Gap between HV and LV winding in mm

hs: Height of LV winding without color in mm

Mc: Mass of core steel in mega grams (Mg)

In above mentioned primary variables the height of primary winding "hp" has not been considered as primary variable since it is usually a fraction of height of secondary winding "hs"[13], therefore mathematically we can write;

$$hp = \alpha \times hs \qquad (1)$$

Where normally $\alpha \approx 0.95$ (A fraction to be specified by User) "H" is the window height in mm and "T" the window width in mm and "X_stack" is the maximum stack width $\approx 2Rc$. These secondary variables can be expressed in terms of the other primary variables.

From Fig.1 It is evident,

$$H = hs + slacks \qquad (2)$$

Where "$slacks$", is a slack distance in the window which depends on the voltage or BIL of the winding and is a constant for the unit under consideration, As Shown in Fig.1 mathematically we can write:

$$slacks = (UpperGap2Yoke + LowerGap2Yoke) + (2 \times LV\_collar)$$

Where,

$UpperGap2Yoke$: Distance of LV winding (with collar) from top yoke in "mm"

$LowerGap2Yoke$: Distance of LV winding (with collar) from bottom yoke in "mm"

Similarly from Fig.1 it is clear that:

$$T = 2(Rp + tp/2 + g0 - Rc) \qquad (3)$$

B. Design input parameters:

There are a number of input design parameters which are to be specified by the user. These parameters are also called performance parameters and are described below:

kVA: Power rating of the 3-phase transformer to be designed

Z: Per unit impedance of the 3-phase transformer

$FeLoss$: Iron (or core) loss in kW specified by the user

$CuLoss$: Copper loss in kW specified by the user

C. Constants for design procedure:

dn: density of copper in g/cm^3, i.e. 8.9 g/cm^3

dfe: density of core steel(iron) in g/cm^3,i.e. 7.65 g/cm^3

rho: resistivity of copper in ohm-m/mm^2, i.e. 21×10^{-9}

$RCuEnr$: Rate of Copper in Rs/Kg for HV winding

$RCuIns$: Rate of Copper in Rs/Kg for LV winding

The mathematical formulation (for global optimization of Core type 3-Phase Transformer) is done in terms of certain primary design variables, which are given as:

B: Flux Density in Tesla

Stk: Core stacking Factor

(Usually taken as 0.95 for safe design)

pfc: Geometric filling factor of core (i.e. ratio of core area to the area of circum scribing circle)

$ecfs$: Eddy current factor for LV winding (It is specified by user)

$ecfp$: Eddy current factor for HV winding (It is specified by user)

pfs: Fill factor of secondary winding, it is defined as the ratio of copper volume in LV winding to the whole volume of the LV winding

pfp: Fill factor of secondary winding, it is defined as the ratio of copper volume in HV winding to the whole volume of the HV winding

g_0: Half of the clearance between the two phases in mm (User specified)

D. Geometric Illustration of Design variables:

In order to further elaborate some of the above listed design variables a clear geometry of core type transformer is provided in Fig. 1. It is evident that X_stack, H and T/2 are secondary design variables and can easily be expressed in terms of above listed primary variables.

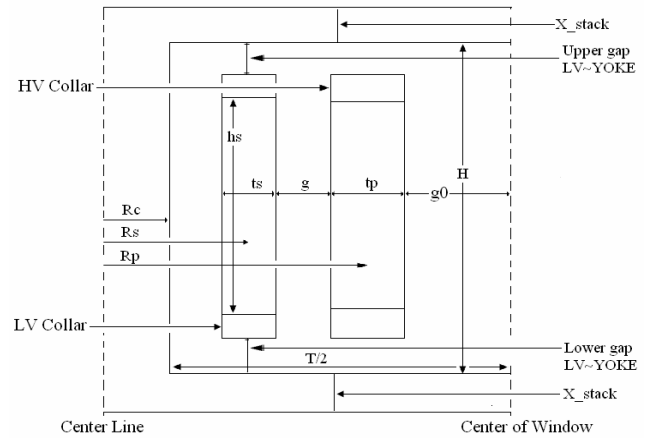Fig.1. Geometry of core type transformer

3. Formulation of nonlinear cost function in terms of primary variables

The objective of the optimization of 3-phase transformer design is to minimize the total cost of active part which comprises of the cost of copper used in windings and the cost of the iron used in core. The cost of the copper in both windings and the cost of core will be calculated in million Rs (Rupees) and therefore the total cost will also be in million Rs (Rupees). The derivation of non-linear cost function in terms of primary design variables for the active part of transformer is as under:

Cost of copper in LV=Mass of copper in LV (Kg) $\times RCuInR \times 10^{-6}$

"10^{-6}" is multiplied to convert the cost in million rupees.

Now it is clear that:

Mass of copper in LV (Kg)= $(3 \times dn \times pfs \times 2 \times \pi \times Rs \times hs \times ts \times 10^{-6})$

Therefore we can write:

Cost of copper in LV= $(3 \times dn \times pfs \times 2 \times \pi \times Rs \times hs \times ts \times 10^{-6}) \times RCuInR \times 10^{-6}$

Similarly:

Cost of copper in HV= $(3 \times dn \times pfp \times \alpha \times 2 \times \pi \times Rp \times hs \times tp \times 10^{-6}) \times RCuEnR \times 10^{-6}$

Where *pfp* and *pfs* are fill factors of HV and LV winding respectively and usually assumed as 0.5 in order to account for the adequate insulation and thermal ducts for cooling of both windings. Now,

Cost of core=Mass of core in Kg $\times FeR \times 10^{-6}$

Since Mc is in mega grams therefore:

Mass of core in Kg = $(Mc \times 10^6) \times 10^{-3}$

Hence we can write:

Cost of core= $Mc \times FeR \times 10^{-3}$

Now we denote the objective function by "*Cost*" which is the total cost of core and windings and is given as:

$$Cost = ((3 \times dn \times pfs \times 2 \times \pi \times Rs \times hs \times ts \times 10^{-6}) \times RCuInR + (3 \times dn \times pfp \times \alpha \times 2 \times \pi \times Rp \times hs \times tp \times 10^{-6}) \times RCuEnR) \times 10^{-6} + Mc \times FeR \times 10^{-3} \qquad (4)$$

Eq. (4) gives the standard form of the cost function (in terms of primary design variables) that will be implemented in Mathematica.

4. Non-linear constraints

There are a number of different nonlinear equality and inequality constrains which are imposed on the cost function for its accurate global minimization such that the optimized transformer design not only satisfy all the customer specifications but also full fill the required performance measures. These constraints play an unavoidable role in nonlinear optimization to determine the global minimum value of the cost function of active part of transformer and the values of primary design variables at which the minimum value of cost function will occur. A detailed derivation in standard normalized form of such constraints in terms of primary variables has been carried out in [13]. A detailed explanation of all these constraints is given as:

4.1. Copper loss Constraint

The total copper loss in the LV (or secondary) winding is denoted by "*wsCu* (in kW)" and mathematically we can write the simplified expression as given below:

$$wsCu = rho \times (1+ecfs) \times (Js)^2 \times V_s$$

Where "*rho*" is the copper resistivity (in ohm-m/mm^2) which is evaluated at the appropriate reference temperature, "*ecfs*" is the eddy current factor which is due to stray flux and depends on the type of wire or cable making up the winding. V_s (in mm^3) is the copper volume in LV winding which can be expressed as

$$V_s = (3 \times pfs \times 2 \times \pi \times Rs \times hs \times ts)$$

By putting the value of "V_s" in the expression for "*wsCu*" we get:

$$wsCu = rho \times (1+ecfs) \times (Js)^2 \times (3 \times pfs \times 2 \times \pi \times Rs \times hs \times ts) \quad (5)$$

Similarly the total copper loss "*wpCu* (in kW)" can also be expressed by using the following simplified expression:

$$wpCu = rho \times (1+ecfp) \times (Jp)^2 \times V_p$$

Where V_p (in mm^3) is the copper volume and is given as:

$$V_p = (3 \times pfs \times 2 \times \pi \times Rs \times hs \times ts)$$

Therefore we can write:

$$wpCu = rho \times (1+ecfp) \times (Jp)^2 \times (3 \times pfs \times 2 \times \pi \times Rs \times hs \times ts) \qquad (6)$$

Because the ampere-turns of the primary and secondary are equal under balanced conditions the current density in HV "Jp (in A/mm^2)" can be expressed in terms of "Js (in A/mm^2)" as given below:

$$Jp = (Js \times pfs \times ts)/ (\alpha \times pfp \times tp)$$

By putting this value in (6) we get:

$$wpCu = rho \times (1+ecfp) \times ((Js \times pfs \times ts)/ (\alpha \times pfp \times tp))^2 \times (3 \times pfs \times 2 \times \pi \times Rs \times hs \times ts) \qquad (7)$$

Now "wCu (in kW)" is the total copper loss and is given as:

$$wCu = rho(1+ecfs) \times (Js)^2 \times (3 \times pfs \times 2 \times \pi \times Rs \times hs \times ts) + (1+ecfp) \times ((Js \times pfs \times ts)/(\alpha \times pfp \times tp))^2 \times (3 \times pfs \times 2 \times \pi \times Rs \times hs \times ts)) \qquad (8)$$

Since the total copper loss of the transformer should less than the copper loss specified by the user, i.e. *CuLoss* therefore:

$$wCu \leq CuLoss$$

Or,

$$wCu/Culoss-1 \leq 0 \qquad (9)$$

Here (9) is the standard form of copper loss constraint that will be implemented in Mathematica. If we denote the copper loss constraint as "*ConsCu*" then

$$ConsCu = wCu/Culoss-1 \qquad (10)$$

And $ConsCu \leq 0$ \qquad (11)

4.2. Core loss Constraint

For accurate calculation of core loss, the core data include mainly the magnetization curve and the core loss at different values of flux density and frequency. An expression for core loss (in Watts/Kg at 50 Hz) that works well from practical point of view for M4 grade cores [14] is given as:

$$wcperKG = (B^4 - (3.91 \times B^3) + (5.88 \times B^2) - (3.35)) \qquad (12)$$

And   *Core Loss (In kW)* $= BF * wcperKG * Mc$

Where "BF" the core building factor that accounts for higher losses due to non-uniform flux in the corners of the core, due to building stresses, and other factors. The core loss of transformer should be less than or equal to FeLoss for desired performance, therefore:

$$Core\ Loss \leq FeLoss$$

Or $(BF * wcperKG * Mc / FeLoss)-1 \leq 0$

Or $ConsWc \leq 0$ \qquad (13)

Where,

$ConsWc = (BF * wcperKG * Mc / FeLoss) - 1$ (14)

4.3. Power transfer Constraint

The total power (in MVA) transferred per phase should be equal to "P", where

$$P\ (In\ MVA\ per\ Phase) = kVA / (3 \times 1000)$$

The per phase power constraint "$ConsPower$" in its standard explicit form is given by the expression:

$$ConsPower = Stk \times 10^{-12} \times pfc \times pfs \times Rc^2 \times B \times Js \times hs \times ts \times \frac{\sqrt{2} \times \pi^2 \times f}{P} - 1$$

(15)

Where $f = frequency\ (For\ Pakistan\ it\ is\ 50\ Hz)$

And the per phase power transfer constraint is equality therefore:

$$ConsPower = 0 \qquad (16)$$

4.4. Impedance Constraint

The per unit impedance of the between the primary and secondary windings of transformer should be less than the per unit impedance "Z" specified by the customer. As it is clear that per unit impedance of the transformer is comprised of per unit resistance and reactance of that transformer, therefore the impedance constraint is sub divided into two constraints, i.e. the resistance constraint and reactance constraint.

We know that maximum per unit resistance "Rdc" of a transformer is given as:

$$Rdc = wCu / kVA \qquad (17)$$

Therefore it is evident that maximum reactance X_max will be:

$$X\_max = (Z^2 - Rdc^2)^{1/2} \qquad (18)$$

Now a mathematical expression for reactance constraint "ConsReactance" in terms of primary variables is derived in [8], but this expression utilizes the British system of units (i.e. inches etc.), by converting the expression in our standard units that are used throughout the mathematical modeling we get:

$ConsReactance =$

$$\left(\frac{7.5606 \times 10^{-8} \times \frac{f}{25.4}}{\left(10^{-9} \times \sqrt{2} \times \pi^2 \times f\right)^2 \times X_{max} \times (pfc^2 \times Rc^4 \times B^2 \times hs)} \right) \times \left(Rs \times \frac{ts}{3} + Rp \times \right.$$
$$\left. \frac{tp}{3} + Rs \times \frac{g}{2} + Rp \times \frac{g}{2} + ts \times \frac{g}{4} - tp \times \frac{g}{4} \right) - \frac{1+a}{2} - 0.32 \left(\frac{Rp}{hs} \right) - 0.16 \times \frac{tp}{hs} + 0.32 \times \frac{Rc}{hs}$$

(19)

And $ConsReactance = 0$ (20)

4.5. Constraint for mass of core

Mc (Mass of core in Mega grams) is our primary variable but can be expressed in other primary variables, an equality constraint "$ConsMc$" in standard (normalized form) is given by the expression:

$$ConsMc = 10^{-9} \times dfe \times \pi \times \left(\frac{pfc \times Rc^2}{Mc} \right) \times \left(3 \times hs + 8 \times Rp + 4 \times tp + 4 \times Rc + 0.30235 \times \frac{Rc}{pfc} + 3 \times slacks + 8 \times g0 \right) - 1$$

(21)

And $ConsMc = 0$ (22)

4.6. Miscellaneous Constraints

We treated Rp as an independent variable since it appears in many formulas. However, it can be expressed in terms of other primary variables as evident from Fig.1:

$$Rp = Rs + \frac{ts}{2} + g + \frac{tp}{2} \qquad (23)$$

By converting (21) into standard form we get:

$$ConsRadius = \frac{Rs}{Rp} + \frac{ts}{2 \cdot Rp} + \frac{g}{Rp} + \frac{tp}{2 \cdot Rp} - 1 \qquad (24)$$

As $ConsRadius$ is an equality constraint therefore:

$$ConsRadius = 0 \qquad (25)$$

An inequality constraint is imposed on the mean radius of the LV winding since it must not drop below a minimum value. From Fig.1 it is clear that:

$$Rc \geq Rc + gc + \frac{ts}{2} \qquad (26)$$

If we denote the inequality constraint given in (24) by "$First$" then it can be written in standard form as:

$$First = \frac{Rs}{Rc} - \frac{ts}{2 \times Rc} - \frac{gc}{Rc} - 1 \qquad (27)$$

And $First > 0$

The HV-LV gap g must not fall below a minimum value given by voltage or BIL (Basic Insulation Level) considerations. Calling this minimum gap "$gmin$", leads to the inequality

$$g > gmin \qquad (28)$$

In standard form (26) can be written as:

$$Second = \frac{g}{gmin} - 1 \qquad (29)$$

And $Second > 0$ (30)

The flux density B is limited above by the saturation of iron or by a lower value determined by overvoltage or sound level considerations. Calling the maximum value $Bmax$ leads to the inequality in standard form:

$$\frac{Bmax}{B} - 1 > 0 \qquad (31)$$

If the constraint in (29) is denoted by "$Third$" then it can be written as:

$$Third = \frac{Bmax}{B} - 1 \qquad (32)$$

And $Third > 0$ (33)

The current density Js should be less than a certain maximum value $Jmax$ there imposing an inequality constraint on current density in standard form we can write:

$$\frac{Jmax}{Js} - 1 > 0 \qquad (34)$$

We denote the inequality constraint given in (33) by "$Fourth$" and in standard form it is given as:

$$Fourth = \frac{Jmax}{Js} - 1 \qquad (35)$$

And $Fourth > 0$ (36)

It is worth mentioning here that temperature rise constraint is not used because in case of oil immersed core type 3-phase distribution transformers the mathematical expression for winding thermal gradients are very complex and are rather difficult to express in terms of primary variables. Therefore to avoid this

difficulty without disturbing the accuracy of minimization of cost function, the transformer is optimized by assuming 0.5 fill factor in both windings. Since 0.5 fill factors adequately accounts for the space required for thermal cooling ducts in both windings, therefore once the optimized design of transformer with 0.5 fill factor is done the ducts in both windings are increased one by one until the temperature gradient of both windings fall below the maximum permissible limit.

5. Global Optimization using mathematica

Mathematica is computational software that is accompanied with a very powerful and reliable nonlinear global optimization tool "*Minimize*". The "*Minimize*" function attempts to globally minimize any non-linear objective function subject to set of non-linear constraints. Therefore Global optimization problems can be solved exactly by using "*Minimize*". The default algorithm that is used by "Minimize" is "Nelder-Mead" which is based on direct search, but if "Nelder-Mead" does poorly, it switches to "differential evolution" [15].

The implementation of the cost function and constraints using Mathematica to find the global minimum value of the cost function is comprised of the sequence of the following steps:

➢ First of all initialize the user specifications and design constants in Mathematica note book.
➢ Write the expression for the cost function derived in (4) in the same Mathematica note book.
➢ Implement the expressions for all the equality and inequality constraints given in (8),(14),(15),(19),(21),(24),(29),(32) and (35)
➢ Use "*Minimize*" to globally optimize the cost function with the following syntax:

Minimize [{Cost, Rc>5&& B>1.5 && hs> 100 && g>8 && Rs> 10 && Rp>10 && tp>5 && ts> 5 && Mc>0.01 && Js>1 && ConswCu<=0 && ConsWc<=0 && ConsPower==0 && ConsReactance==0 && ConsMc==0 && ConsRadius==0 && First>0 && Second>0&& Third>0 && Fourth>0},{Rs,ts,Rp,tp,hs,Mc,Js,B,Rc,g}]

The output of above command will be the minimum value of the cost function subjected to the given constraints and the values of primary variables at which the minimum value of cost function occurs.

6. Example design of core type distribution transformer

The optimization methodology that has been developed in this research can be used for rating from 15kVA to 10MVA 3-phase core type transformer. However in order to check the validity of the approach presented in this paper a 3-phase oil immersed core type distribution transformer has been designed according to the latest specifications of PEPCO which are as under:

Power Rating = 25 kVA
CuLoss=0.512 kW
FeLoss=0.099 kW
Z=0.04 per unit
Voltage Rating:11000/435 volts
Temperature rise:40/50 ^{0}C

BIL=95 kV

The rates of LV, HV copper and core materials are as follow:

RCuEnR=748 Rs./kg (HV copper)
RCuInR=671 Rs./kg (LV Copper)
FeR=252 Rs./kg

The output from Mathematica is given in Table 1 as:

Table 1. Output Results from Mathematica

Description	Value
Cost (Million Rs.)	0.0566003
Mean radius of LV (mm)	58.7925
Radial build of LV (mm)	13.9787
Mean radius of HV (mm)	82.278
Radial build of HV (mm)	16.9922
Height of LV (mm)	198.601
Mass of core (Mega grams)	0.0838097
Current density of LV (A/mm^2)	2.21496
Flux density (Tesla)	1.5
Radius of core (mm)	49.8031
Gap b/w LV & HV (mm)	8.0

When the above specified 25 kVA core type transformer was designed using unconstrained optimization design techniques, the cost of active part was found to be 70,000 Rs. However, when the same transformer was designed according to the values of primary design variables given in table1 obtained from Mathematica, the cost was reduced to 56,000 Rs, which is about 21% less than the cost of design from the conventional method, this reduction in cost indicates a very significant achievement in economic optimization of active part of 3-phase core type distribution transformer.

7. Conclusions

Today the most important challenge for the transformer industry is the economic optimization of the distribution transformers to meet the increased demand. This paper presents an innovative and robust version of non-linear constrained optimization implemented by means of Mathematica that ensures significant cost reduction of active part of oil immersed 3-phase core type distribution transformer. It also has the advantage that it can incorporate any form of non-linear formulae that is required for accurate design of the 3-phase core type transformer. The accuracy, fastness and reliability of Mathematica are also another advantage. This is a very practical and futuristic technique that will really help the designers to design more economical transformers.

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