An Implicit and Untested Premise of the Special Theory of Relativity

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Abstract
The derivation of Lorentz Transformation Equations in the Special Theory of Relativity, besides being explicitly based on the two postulates of that theory is also critically based on an implicit premise that the detection of a light signal/particle at a point in space at an instant of time is an event that is not exclusive to any inertial reference frame but it is capable of being measured by the observers in other inertial reference frames as well. This paper explains how this untested paradoxical premise vitiates the entire theory and suggests its replacement with a contrary postulate that the detection of light in an inertial reference frame is an event that is exclusive to that frame. This paper shows that if that contrary postulate is accepted, then the absoluteness of space and time declared by Newton can coexist with the absoluteness of the speed of light in vacuum declared by Einstein without any conflict between them.

Keywords: Lorentz transformation; Non-synchronization of moving clocks

Introduction
Of the happening of any observable event, it can be said that there is a near universal agreement about the fact that it has happened somewhere in the space at some point of time, even though there may be disagreements on where and when it happened. Indirectly, this indicates universal agreement on the existence of absolute space and absolute time notwithstanding the difficult, if not impossible, task of fixing an absolute inertial frame of reference from which the absolute measurements of space and time distances could be made. All observers in one inertial frame of reference, say S, agreeing on a common origin event, say O, would define that event with values of x-space coordinate and a time coordinate say (x,t). But the observers in another inertial frame of reference, say S’ moving with a uniform velocity v relative to S in the positive X-direction, would define that event (if it is observed by them also) with different values of x-space coordinate and a time coordinate say (x’,t’). But the observers in another inertial frame of reference, say S’ moving with a uniform velocity v relative to S in the positive X-direction, would define that event (if it is observed by them also) with different values of x-space coordinate and a time coordinate say (x’,t’) even though they had agreed on the same origin event O. This difference results from a universal “ignorance” that the observers in every frame of reference assume that their frame is at rest while only all other inertial frames are moving. Fortunately for us, the first postulate of the Special Theory of Relativity (STR) ensures that this ignorance does not hinder our scientific pursuits in any way. According to the STR, the aforesaid two sets of coordinates defining one and the same event (in the absolute space and time spectrum) by the observers in the frames S and S’ are connected by the following two equations, known as Lorentz Transformation Equations [1];

\[ x'=a(x-vt) \]
\[ t'=a(t-vx/c^2) \]

Where \( a=1/(\sqrt{1-v^2/c^2}) \)

It will be shown in the following sections that the derivation of the above equations is explicitly based on the following two postulates of the Special Theory of Relativity [2]:

(i) The laws of nature have the same mathematical form in all inertial reference frames;

(ii) The speed of light in vacuum is the same for all inertial reference frames.

An important fact is that Einstein, while deriving Lorentz Transformation Equation from the two postulates of the STR, had critically relied on a premise that the detection of a light signal/particle at a point in space at an instant of time is one and the same event that is observed by both observers—each moving with a uniform linear velocity relative to one another,—and they differ only in the values of the spatial location and the time of occurrence of that event measured by each of them. In other words, the detection of a light signal/particle at a point in space at an instant of time is an event that is not exclusive to any inertial reference frame but it is capable of being measured by the observers in other inertial reference frames as well. Presumably, Einstein would have intuitively taken it as an obvious truth that needed no express statement. As a matter of fact there would be necessity for a transformation equation only when one and the same event is observed by two observers and such an equation is obviously meaningless when the nature of the event is of such a manner that it is observable only by one observer and not by the other observer. The following sections of this article examine the validity of the said premise.

A Straight Forward Derivation of Lorentz Transformation Equations
Starting with Galilean Transformation Equation, which is in agreement with the first postulate of the STR, and making necessary alterations to make it agree with the Second Postulate also, Lorentz Transformation Equations can be easily derived by any novice without involving any advanced mathematics.

Suppose for an event of detection of a light signal at a particular point in space and at a particular instant of time by the Light Detector L(D), which is stationary in the frame S is assigned the coordinates (x,t) by a stationary observer S in that frame. It is obvious x=ct where c is the speed of light in vacuum explained in Figure 1.

Suppose the observer S’ using a light detector L’(D’) that is stationary relative to him, detects the light signal at a distance x’ from him at the very same instant. We may conclude from Law of Constancy of the Speed of Light that x’=ct.

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The expectation of the author of this article is that the Light Detector L(D) stationary in the frame S and the Light Detector L'(D') stationary in the frame S' will receive the light at the same instant of time (t) but at different spatial locations separated by the distance vt. But, according to the STR, both Light Detectors L(D) and L'(D') will receive the light at the same instant of time (t) at the same place where Light Detector L(D) is located. While deriving Lorentz Transformation Equations from the two postulates of STR, Einstein proceeded on the basis that that particular light signal was capable of detection at the very same point in space and at the very same particular instant of time by the observers in both S and S'. In other words, the one and the same event of detection of light at a at a particular point in space and at a particular instant of time can be observed by the observers in S as well as by observers in S' even though the values of space and time coordinates (x',t') they would assign to that event would be different. This author prefers to give this assumption a name "Premise three" for future references in this article. Though this premise has not been expressly stated anywhere in the Special Theory of Relativity, it forms the core of the STR.

According to thePremise three, which claims that the same event can be detected by the stationary observers stationed in the frame S' also and the Galilean Transformation

\[ x' = x - vt \]

Since \( x = ct \) for the event under consideration, the above equation becomes

\[ x = ct - vt \]

The Second Postulate requires that when the same event is expressed by the coordinates \((x',t')\) in the frame S', then they should satisfy the equation \(x' = ct'\) in obedience to the law of constancy of the velocity of light in all inertial reference frames, which is the Second Postulate. This means

\[ t' = x'/c \]

\[ = (ct - vt)/c \]

\[ = t - (vt/c) \]

\[ = t - vx/c^2 \] [because \( t = x/c \)]

Now the following two equations

\[ x' = x - vt \]

\[ t' = t - vx/c^2 \]

can serve as Transformation Equations satisfying the second postulate as well as the Premise three. If one does not miss Physics for Mathematics, one will note that the above expression for \( t' \) gives only the part of the total time \( t \) that was taken by the light signal to travel the distance beyond the origin point of the frame S' (that is the distance \( x' \) in S') to reach the point at a distance \( x \) in S. But in the STR Physics was sacrificed for the sake deriving a mathematical equation that correctly transforms events of detection of light signals from one inertial frame to the other and it was taken that \( t' \) gives the time between the origin event and the measured event as measured by the observers in S' whereas \( t \) gives the time between the very same events as measured by the observers in S.

The consequence of taking \( t' \) as the counterpart of \( t \) in the frame S, instead of taking it as a part of \( t \), is that, according to an observer in frame S, the clocks attached to S' will be showing different time at an instant of time depending on their relative distances from the origin point whereas all the clocks attached to S will be showing an identical time; the equation \( t' = t - vx/c^2 \) gives the time shown by the clock attached to S' that is at a distance \( x \) from the origin point of S as measured in S at an instant of time which is shown as \( t \) by all clocks attached to S. We may call this consequence "Non-synchronization of moving Clocks"

Introducing "Premise three" with its inevitable consequence of Non-synchronization of moving Clocks, Einstein satisfied the second postulate of the STR. But that has cost him the first postulate. Einstein crossed this hurdle in an ingenious way. Adding any constant factor, say \( a \), on the RHS of both equations would not disturb the only requirement \( x'/t' = x/t = c \) that is needed for upholding the second postulate of the STR. So we can modify the equations as

\[ x' = a(x - vt) \] and

\[ t' = a(t - vx/c^2) \]

Let us now choose a suitable value for \( a \) so that the equations satisfy the first postulate of the STR also.

From the above equations it can be derived

\[ x = [1/a(1-v^2/c^2)](x + vt') \]

\[ t = [1/a(1-v^2/c^2)](t' + vx/c^2) \]

So to satisfy the first postulate

\[ a = 1/(a(1-v^2/c^2)) \]

\[ a^2 = [1/(1-v^2/c^2)] \]

Therefore

\[ a = \pm 1/(1-v^2/c^2)^{1/2} \]

The negative value is ignored and it is taken

\[ a = 1/(1-v^2/c^2)^{1/2} \]

The final forms of the equations are

\[ x' = a(x - vt) \] and

\[ t' = a(t - vx/c^2) \]
Which are equivalent to
\[ x = a(x' + vt') \] and
\[ t = a(t' + vx/c^2) \]
Where \( a = 1/\sqrt{1 - v^2/c^2} \)

Here again, if one does not miss Physics for Mathematics, one will note that this addition of factor \( a \) to RHS of the equations is responsible for the physical consequences of

(i) \( \text{Length Contraction} \) \( (x' = x/a \text{ when } t' = 0) \); and

(ii) \( \text{Time Dilation} \) \( (t' = t/a \text{ when } x' = 0) \)

[As already stated, "Premise three" is responsible for the physical consequence of Non-synchronization of moving Clocks.]

The above derivation of Lorentz Transformation Equations may appear to be crude, unsophisticated and even artificial. But careful dissection of derivations of those equations given by various authors including the one given by Einstein himself in his original 1905 German-language paper published as zur Electrodynamik bewegter Korper, in Annalen der Physik 17:891, 1905) [3] would show that those derivations followed, in essence, the same logic that we have used in the above derivation even though the derivations of those authors may have been clothed in sophisticated formats reflecting the scholarliness and mathematical geniuses of those authors. This straightforward derivation of Lorentz Transformation Equations has been preferred in this article because it shows not only the respective parts played by the two postulates of the STR in that derivation but also how those equations critically depend on Premise three.

It may also be seen that out of infinite events that have been taking place at infinite points of space at infinite instants of time, only the events of the light signal being at a particular point in space at a particular instant of time during its transmission alone were considered. It has been generalised that the same equations would govern even the other infinite events that have nothing to do with the transmission of light. Obviously, the validity of this generalization depends on experimental verification of the predicted consequences of the STR. It may also be noted that the cause-effect relations between events, which constitute the bedrock of all natural laws, has been left to the mercy of the impossibility of communication at a speed greater than that of light.

**Physical Consequences of Lorentz Transformation**

For a better understanding of the physical consequences of Lorentz Transformation Equations, they can be rewritten in the following format;

\[ x = (x/a) - vt \] (Or) \[ x = (x'/a) + vt \]
\[ t = (t/a) - (vx/c^2) \] (Or) \[ t = (t'/a) + (vx/c^2) \]

The first term in RHS of the First Line Equations indicates **Length Contraction**.

The first term in RHS of the Second Line Equations indicates **Time Dilation**.

The second term in RHS of the Second Line Equations indicates **Non-synchronization of moving Clocks**

The second term in RHS of the First Line Equations indicates **Relative Uniform Motion between the frames**.

**Premise Three is Untested**

Though there have been claims of experimental verification of Time Dilation, so far no one has claimed to have verified Non-synchronization of moving Clocks, which was a direct consequence of Premise three. Even in the famous Mu-Meson Experiment [4], only the Time Dilation of moving mu-mesons/clocks was claimed to have been verified and it was not verified whether at a given time instant in earth frame, the moving mu-mesons/clocks showed different times i.e., different stages of decay. Premise three can be said to have been experimentally verified only if a single event of detection of a light signal at a particular point in space and at a particular instant of time is observed by both of the two observers belonging to different inertial frames of reference.

It does not seem to be possible for any observer measure the location of a photon at different instants during its movements over a distance and draw a trajectory of its transition. It is only possible for any observer to know the location of a photon as and when it impinges on a light detector. The spatial location of that photon at the instant when it impinges on the light detector is the spatial location of the light detector at that instant; and the time of such impinging is the time as per the clock fitted with that light detector. Obviously a light detector can be stationary only in one inertial frame of reference and its clock is synchronised with the stationary clocks of that frame. Let us assume that the light detector L stationary in the frame S and another light detector L' stationary in frame S' happen to be at one spatial point at the instant of time when a ray of light reaches that point. Now according to Premise three, the detection of light by the light detector L and the t detection of light by the light detector L' must be simultaneous events as viewed an any frame of reference since both the spatial distance and time difference between the two events were zero. Only if that simultaneity is experimentally proved, Premise three can be said to have been proved.

It may also be seen that since the time \( t' \) measured by the moving observers corresponds to the time \( t \) measured by the moving observers is the counterpart of \( t \), the Clocks of the moving observers, which were found to have been synchronized before the commencement of the relative motion between the two systems, became non-synchronized even at the very commencement of the relative motion between the two systems. [When \( t = 0, t' = -vx/c^2 \) for all values of \( x' \)]. It is our experience that any physical change takes place either gradually or in small quantum jumps; and hence the alleged instant change in the times shown by clocks in the range of \( 0 \) to \( \infty \) (\( x' \) ranges from \( 0 \) to \( \infty \)) appears to be unrealistic and improbable. Suppose the small time taken to change the velocity of the moving observer from zero to a non-zero value is also taken into consideration. The alleged change in consequence of time changing from zero to values between \( +\infty \) to \( -\infty \) in a very small time required to accelerate the moving observer from the velocity zero to \( v \), appears to be in conflict with the prediction of the STR that no cause can have its effect at a place whose distance from the cause is more than the distance that light may travel during the time interval between the said cause and the said effect.

**Premise Three is Paradoxical**

We have seen that Non-synchronization of moving Clocks is a direct consequence of Premise three. Let us imagine two infinitely long rulers, say S and S' lying parallel to one another. Let us imagine that an observer with a clock is sitting on each mark on both rulers. Let us synchronize those clocks so that all clocks held by the observers in both rulers show the same time at any instant of time. Let us now impart a constant velocity \( v \) to one of the two rulers. Now those two rulers with
their respective clocks would constitute two inertial frame of reference say S and S’.  

Lorentz Transformation Equations derived on the basis of Premise three predicts that the observers in the frame S at a given instant of time i.e., when all their clocks show the same time the clocks in the other frame S’ would show different times ranging from \(-\infty\) to \(+\infty\), the time difference between two clocks separated by a distance ‘x’ would be equal to \(-vx/c^2\).

Similarly, the observers in the frame S’ at a given instant of time i.e., when all their clocks show the same time the clocks in the other frame S would show different times ranging from \(-\infty\) to \(+\infty\), the time difference between two clocks separated by a distance ‘x’ would be equal to \(+vx/c^2\).

Since clocks include all kinds of clocks including biological clocks, the human observers sitting on the marks of the rulers are also clocks. This conceptualisation gives rise to a Multiple Twin Paradox. Let us place one-month old twins–one on a ruler-mark in S and the other on the coinciding ruler-mark in S’ when both rulers are at rest in relative to one another. Similarly, let us place a twin on each ruler-mark of S and his twin brother on the coinciding ruler-mark in S’. Let us assume that all twins so placed on the rulers are of equal age, say one month. Now, as soon as a uniform relative velocity imparted between the two rulers the following paradoxical situation would arise, as a direct consequence of Premise three.

While all babies in S will observe that they are still 1 month old, they will observe that ‘babies’ in the frame S’ have attained different ages ranging from \(-\infty\) to \(+\infty\) depending on the spatial distance between one another; the age difference between two ‘babies’ separated by a distance ‘x’ would be equal to \(-vx/c^2\). (Thus the observers in S will be equal to “demy-gods’ seeing the entire past, present and future of one stream of events as an infinite time spectrum).

Similarly, the babies in the frame S’ at a given instant of time i.e., when all of them are of the same age the ‘babies’ in the other frame S would be at different ages ranging from \(-\infty\) to \(+\infty\) depending on the spatial distance between one another; the age difference between two ‘babies’ separated by a distance ‘x’ would be equal to \(+vx/c^2\). (Thus the observers in S’ will be equal to “demy-gods’ seeing the entire past, present and future of one stream of events as an infinite time spectrum).

Paradoxically the babies in each frame would claim that all babies in their frame continue to be of the same age while those in the other frames have suddenly acquired different ages ranging from \(-\infty\) to \(+\infty\) years depending on the relative spatial differences between them.

It may be seen that the above observed phenomenon of one-month old babies in a moving frame acquiring ages ranging from \(-\infty\) to \(+\infty\) is not a gradual process it happens almost instantaneously at the very moment when a relative velocity becomes operative between the frames as a result of acceleration given to one of the frames for a very short time. After the relative motion is settled with a uniform velocity, there will be no more sudden jump in ages and each baby will age at the same uniform rate though the uniform rate of their clocks will be slower than that of the other frame by a factor equal to \(1/a\).

Suppose one baby, say B stationary in the frame S acquires acceleration and starts moving with a constant velocity \(v\). Now B no longer belongs to the frame S and has become a new member of the frame S’. What will be its age after this change of frame? If we apply Lorentz Transformation Equations, the age of B will make an instantaneous jump from \(t\) to \(a(t−vx/c^2)\).

The usual Twin Paradox presented in books on STR is only a case of one particular pair–one of the pair say A on a ruler-mark in S and its counterpart say A’ on the coinciding ruler-mark in S’ when both rulers are at in relation to one another. After the imparting of uniform relative velocity \(v\) between the rulers, at any instant of time, they will be separated by a distance of \(vt\) for the observers in S and \(v't\) for the observers in S’. The observers in S including A would observe at any point of time, say \(t\) that A is younger than them by \((t−t'/a)\) units. But the observers in S’ including A’ would observe at any point of time, say \(t’\) that A’ is younger than them by \((t’−t'/a)\) units. Who is really younger A or A’? This is a paradox. (Incidentally, many authors of books on STR extend the story still further and imagine that A’ makes a u-turn and travels back with the same speed to meet A. To deal with this extended story let us imagine a third ruler say S’ which is moving with a uniform velocity \(-v\) relative to S. Now A’ has to jump to the ruler S’ to return back to meet A. If A makes that jump when the time in S is \(t\), than the ruler-mark in S’ to which he jumps would read \(2at\) and the clock on that ruler-mark would be showing time as \((2at−t')/a\), according to Lorentz Transformation Equations. This means that instantaneously the age of A’ will increase from \((t'/a)\) to \((2at−t')/a\). This is at least unrealistic, if not a paradox. A’ will take further time \((t'/a)\) to reach A. So when A’ returns back to A, his age will be \(2at\) and the age of A will be \(2t\). At the time of their reunion A’ will be older than A. But in many books on STR it has been claimed that A’ will be younger than A at the time of their reunion).

A Third Postulate Suggested in Lieu of Premise Three

It is suggested that in the place of the untested and paradoxical Premise Three, the following statement may be adopted as the third postulate of the STR.

“\(The\ detection\ of light\ by\ an\ inertial\ reference\ frame\ is\ an\ event\ that\ is\ exclusive\ to\ that\ frame.\)"

The above postulate will make it clear that there is no question measuring the same event by another frame and hence there is no necessity to derive transformation equation for that event. Only the absence of this postulate led Einstein to start on a premise that one reference frame can measure the instant of the detection of light signal in another reference frame.

A corollary of the above postulate may be derived to be the following.

“\(The\ speed\ of\ light\ relative\ to\ any\ inertial\ reference\ frame\ cannot\ be\ measured\ by\ any\ observer\ that\ is\ not\ stationary\ in\ that\ frame.\)"

The above corollary is important since it directly disproves Lorentz Transformation.

With the inclusion of the above third postulate in the STR, we can retain the Galilean Transformation

\[
x' = x - vt; \quad y' = y; \quad z' = z; \quad t' = t
\]

and Galilean Velocity transformation formula

\[
u_s = u_s - v
\]

with an addition of an exception clause that the above formula will be \(u'_s = u_v\) in a special case where \(u_v = c\), the speed of light in vacuum.

A Hypothesis Suggested to Conceptualise the Third Postulate

A hypothesis that may help one to conceptualise the aforesaid third postulate may be that each inertial frame has its own space with all such spaces of inertial frames overlapping over one another. When a
light source emits light, the light spreads in all these spaces and the speed of light in each space is \( c \) relative to the inertial frame attached to that space; and any observer/object stationary in an inertial frame can detect light that spreads in his/its inertial frame and he/it will be insensitive to the light spreading in the spaces attached to other inertial frames.

**An Experiment Suggested to Prove/Disprove the Premise Three**

The following is an extract of a thought experiment stated by Einstein in Chapter 9 of his book "Relativity–The Special and the General Theory" to "prove" that the events which are simultaneous with reference to one inertial frame are not simultaneous with respect to another inertial frame explained in Figure 2 [5].

"Are two events (e.g., the two strokes of lightning A and B) which are simultaneous with reference to the railway embankment also simultaneous relative to the train? We shall show directly that the answer must be in the negative.

When we say that the lightening strokes A and B are simultaneous with respect to the embankment, we mean that the rays of light emitted at the places A and B, where the lightening occurs, meet each other at the mid-point \( M \) of the length A B of the embankment. But the events A and B also correspond to positions A and B of the train. Let \( M' \) be the mid-point of the distance A and B on the travelling train. Just when the flashes (as judged from the embankment) of lightning occur, the point \( M' \) naturally coincides with the point \( M \) but it moves towards the right in the diagram with the velocity \( v \) of the train. If an observer sitting in the position \( M' \) in the train did not possess this velocity, then he would remain permanently at \( M \) and the light rays emitted by the flashes of lightning at A and B would reach him simultaneously, i.e., they would meet just where he is situated. Now in reality (considered with reference to the railway embankment) he is hastening towards the beam of light coming from B, whilst he is riding on ahead of the beam of light coming from A. Hence the observer will see the beam of light emitted from B earlier than he will see that emitted from A. Observers who take the railway train as their reference-body must therefore come to the conclusion that the lightening that the lightening flash \( B \) took place earlier than the lightening flash \( A \). We thus arrive at the important result:

Events which are simultaneous with reference to embankment are not simultaneous with respect to the train, and vice versa (relativity of simultaneity). *Every reference-body (co-ordinate system) has its own particular time;* unless we are told the reference-body to which the statement of time relates, there is no meaning in a statement of the time of an event.

In the opinion of this author that if the above experiment is actually conducted it will disprove the conclusion of Einstein that Simultaneity of events is relative. In other words it will disprove Premise three. The experiment will reveal that contrary to Einstein’s prediction the rays of light emitted at the places A and B will meet each other at the mid-point \( M' \) (Let us call it Event \( M' \)) in the train also besides meeting at the mid-point \( M \) of the embankment. (Let us call it Event \( M \)). Both events \( M \) and \( M' \) will happen simultaneously since light travels with the same speed \( c \) in both frames. But they will happen at different places.

To verify the above conclusion one has to place two light detectors fitted with accurate clocks at the mid-point \( M' \)-one to receive light ray from A and another to receive light ray from B and let those clocks record the exact time of receipt of light at the light detector attached to it. Similarly one has to place two light detectors fitted with accurate clocks at the mid-point \( M \)-one to receive light ray from A and another to receive light ray from B and let those clocks record the exact time of receipt of light at the light detector attached to it. It will be seen that the rays of light emitted at the places A and B will meet each other at the mid-point \( M' \) in the train also besides meeting at the mid-point \( M \) of the embankment. Thus one can experimentally disprove Premise three. The underlying logic is that the ray of light observed in frame are different from that observed in the other frame though both rays move with the same speed \( c \) in their respective frames. Contrary to Einstein’s statement "*Every reference-body has its own particular time*," the observable truth is that "*Every reference-body has its own transmission of light.*"

**Conclusion**

The absoluteness of space and time declared by Newton can coexist with the absoluteness of the speed of light in vacuum declared by Einstein without any conflict between them provided that we reconcile ourselves to the observed fact that a light detector can detect only light signals that impinge on that detector with a velocity \( c \) relative to that detector, where \( c \) is the universal constant denoting the speed of light in vacuum measurable by any observer.

**References**