

# An Enhanced Method for Tail Index Estimation

Jim Kate\*

Department of Applied statistics, University of Zadar, Croatia

## Commentary

Extreme events in earthquakes, wind speed, among others are rare but may cause catastrophic effects on humans and therefore the environment. The first parameter within the estimation of such rare events is that the tail index which measures the tail heaviness of an underlying distribution. Since extreme events are rare, the presence of missing observations may further cause flawed. In sight of this, there's a growing effort by researchers to deal with this problem. However, the prevailing methods of estimating the tail index use only the available non-missing data. Thus, if the missing observations are influential values, ignoring them could introduce more bias and better mean square error (MSE) within the tail index estimation and subsequently other extreme event--estimators like high quantiles and little exceedance probabilities. During this study, we propose imputation of the missing observations before applying some standard estimators (Hill and geometric-type) to estimate the tail index. Through a simulation study, we assess the performance of the quality estimators under the proposed data enhancement method and therefore the existing modified estimators of the tail index. The results show that the improved estimators have relatively lower bias and MSE. The estimation method was illustrated with a practical dataset on wind speed with missing values. Therefore, we recommend imputation mechanism as viable for enhancing the performance of tail index estimators within the case where there's missingness.

Statistics of extremes is a branch of Statistics that deals with the estimation of parameters of rare events. It enables the assessment of the probability of events that are more extreme than any previous observation from a sample of random variables Coles. Accordingly, the occurrence of rare events in phenomena like earthquakes, hurricanes, wind speed, sea waves, and floods can have a catastrophic impact on human beings and therefore the environment. Modelling the occurrence of such events aids in getting to reduce or prevent the impact of such events.

Recent developments during this area specialise in modelling and predicting rare events to mitigate their negative impact on humans and properties. The first parameter of interest is that the tail index or extreme value index (EVI), which measures the tail heaviness of an underlying distribution. One key challenge researchers encounter in their quest to model rare events or estimate the tail index is that the amount of observations available is typically very small to none thanks to their unusual occurrence. Therefore, having missing observations can affect the tail index estimates computed from the sample, thereby resulting in unreliable estimates for exceedance probabilities, high quantile, and return period which are the goals of utmost value analysis. During this study, we propose an enhanced method of estimating the tail index of underlying distributions, where the missing values are estimated and replaced within the data via an imputation method.

\*Address for Correspondence: Jim Kate, Department of Applied statistics, University of Zadar, Croatia, E-mail: jim.k@gmail.com

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Many researchers conducted in statistics of extremes are limited to estimating the tail index and other parameters of extremes with complete dataset. The reduced-bias estimators for the tail index parameter on complete data. within the case of presence of censoring, provide techniques for incorporating censoring within the estimation of the tail index, employed an existing adjusted empirical likelihood method, to construct confidence intervals of the tail index so on achieve a far better accuracy. Through a simulation study, they found their method to be superior in terms of the coverage probability and length of confidence intervals. Within the case of the presence of covariate information, the empirical likelihood-based statistics to construct confidence regions for the parametric statistic of the parametric tail index regression model. Also, studies several estimators of the tail index within the presence of both censoring and covariate information. However, of these estimators don't take under consideration missing data and hence are somewhat challenged within the presence of missing observations.

Since missing data is a common problem in statistics, some authors worked on the estimation of the exponent of the regular variation with the utilization of incomplete data samples. They proved the asymptotic consistency of Capitol Hill estimator. Under the idea of weak dependency, they proved the consistency of their proposed Hill-type estimator of the tail index supported an incomplete sample.

In addition, focused on extreme value analysis without the most important values. The study revealed that the presence of missing extremes makes the selection of threshold for the highest order statistics problematic. They simultaneously considered the amount of missing extremes with the tail index and other parameters and proposed a functional version of Capitol Hill estimator and named it Hill Estimator without Extremes (HEWE). The estimator was found to be robust to missing extremes on light-tailed dataset.

Furthermore, the simple tail index estimation in the case of heterogeneous and dependent data samples with missing values. Their study was on the asymptotic behaviour of the median estimator and its robustness against deviations of the slowly varying function. Although under small deviations from the assumed parametric model, the proposed method provided a reliable tail index, the highest values of the sample weren't considered.

However, the existing methods for estimating the tail index use reduced sample size since portions of the dataset within the order statistics that are missing are ignored. Using only portions of the data may result in estimators with large bias and/or variance especially if the missing observations are influential within the top order statistics which are of interest in statistics of extremes.

Therefore, in the quest to reduce bias and variance in tail index estimation in the presence of missing observations; we propose imputation of the missing observations before applying standard tail index estimators (such as Hill and geometric-type), instead of using the modified estimators in the literature.

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