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An Application of Stochastic Modelling: Describing Growth of COVID Infections

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Abstract

In this article it is tried to construct a stochastic model which looks a generalized stochastic version of Von Bertalanffy power law model and Richard's model and one can use to describe biological growth phenomena according to the appropriate situation and suitability of this model. It is mainly constructed to explain growth dynamics of patients infected by COVID-19 in South Korea. Here it is attempted to find the expression of variable of interest at time t and also the MLE of growth rate parameter is worked out. This model is applied to a real life data of infected patients by COVID-19 in South Korea after observing the growth pattern. This model could be used to the data sets of other countries, where no lockdown was imposed as a precautionary measure to deal with this situation. Then a comparative study is made between some well-known models and special cases of the model, described here. It is found that the special cases of the model that is described in this article fits better to the data than others.

Keywords: COVID-19 • Ito's lemma • Von bertalanffy model • Richard's model

Introduction

There are some variants of coronavirus that exists and scientists classified them into four subgroups-1). 229E(alpha), 2). NL63(alpha), 3). OC43(beta), 4). HKU1(beta) and there are three rare ones also and they are MERS-CoV, SARS-CoV, SARS-CoV-2. The third one under the rare category which is also known as COVID-19. It is mainly responsible for the pandemic situation that occured all around the world. It is actually an infectious disease, majorly causing respiratory problems [1]. It is observed that most of the infected persons by this virus has experienced a mild to moderate respiratory problems and many of them has recovered under normal treatments, but for aged persons who were suffering from CVD (Cardio Vascular Disease), severe respiratory problems, diabetes, cancer etc. i.e, persons with co-morbidity has faced serious trouble after getting infected (COVID positive). Since this is an infectious disease, it is natural to be interested about how it spreads. This virus (COVID-19) actually spreads itself *via* droplets of saliva or when a person infected by this virus sneezes or coughs, there is every possibility that surrounding persons may become infected by it. From previous paragraph one can classify this virus as a respiratory virus and it is important to have an idea about the possible ways of transmission of these respiratory virus [2].

Sources indicate that there are mainly three ways of transmission of such respiratory viruses, firstly it could be the transmitted *via* direct contact with infected person, secondly through droplet transmission and thirdly, through airborne transmission of smaller droplets and particles that stays in air for fair amount of time and can travel a significant amount of distance, also some Italian scientists collected outdoor air pollution samples and after studying it, they found gene highly specific to COVID-19 in multiple samples i.e, one may think it as airborne, though there is not much evidence of this fact. But one can consider it as partially air bone because of the fact that air current can help it to travel from one place to another and because of this reason, it is quite possible that some of the countries like South Korea, Sweden etc, haven't imposed any strict lockdown or curfew [3]. So far from the experience of patients who had an infection of this COVID-19 virus, it is described that there are some common symptoms observed when a person, possibly infected by this virus can be identified and the symptoms are fever (high temperature), cough, breathing problem, headache, sore throat, loss of smell or taste.

Literature Review

Observing the situation all around the world, one can understand the severity of this situation, not because of the fact that till now there is no such vaccine or specific treatments for the patients suffering from this disease. There are some clinical trials are going on to evaluate specific vaccine or treatment of this disease. This pandemic situation has a large adverse effect all around the world and the most one can observe it in world economy as well as in social life. According to IMF report the growth of world economy is under the worst recession since great depression in 1920s. It is not only world economy but it has huge impact on social life specially psychologial

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side of the society [4]. There are many research articles already available that focuses on this issue. According to the research this pandemic situation has significant impact on psychologial well-being of most of the exposed groups like children, college students, health workers etc, who are more likely to have post-traumatic stress, anxiety disorder, depression etc, it is also observed that the social distance and the precautionary measures has its impact on the relatioship among people and their response towards others.

Like most of the biological growth phenomena, growth dynamics of number of infected patients also shows sigmoidal pattern [5]. Now as it is known that at present there is no such specific vaccine or treatment for this disease so, to prevent the infection majority of the countries has taken few precautionary steps like social distancing, home quarantine, curfew etc. On the other hand few countries had their belief in herd immunity theory which states that it is a form of indirect protection from infection which occurs when a big part of the population has become immune to the infection. Keeping this in mind and using the knowledge about the models that already exists for describing Biological growth, here the growth of COVID infection may described by the following:

$$\frac{dx(t)}{dt} = r(x(t))^{a}[k - (x(t))^{\beta}] \qquad (1)$$

Observing the R.H.S the term outside the bracket indicates the growth part and the term within the bracket indicates the resistance or negative feedback part and clearly at any stage during the process they may oppose themselves with equal intensity or with different ones. Where r is the growth rate factor, k is the threshold level and x(t) is the amount of infection at time t. This model has a similar appearance as that of general power law model but it involves the threshold level. This article is contructed by the following sections. In section 2, a stochastic model is constructed by taking into account of the fact of time variability of the growth rate factor r. In section 3, the MLE of the model parameter (growth rate factor) is worked out. In section 4, analysis of the COVID-19 infections data of South Korea is presented. In section 5, Some Conclusion are made [6].

Stochastic extension of the model

As it is stated that in this section the variation of growth rate factor with time will be con- sidered. Now, it is already stated that except for few countries like South Korea, Taiwan, Sweden etc, who believed in herd immunity theory but most of the countries imposed pre-cautionary measures to slow the rate of increase of number of infected patients. In real life situations it is witnessed that the growth dynamics includes oscillatory phenomena and it appears in several application areas like biological growth data, population growth dynamics etc, but out of these biomedicine stand out. In these cases where such oscillatory phenomena is witnessed, stochastic models based on deterministic ones by introducing multiplicative noise term to it play an important role [7]. Many authors considered this set up just mentioned above and this brings us to the stochastic version of the model, described by equation 1 as:

$$dx(t) = (r(x(t))^{a}[k - (x(t))^{\beta}])dt + \sigma.x(t)dw(t) \qquad (2)$$

Where w(t) is the standard wiener process and the differential is meant in Ito's sense. As it appears just above that it is a stochastic differential equation of Ito type. So it suits better to solve it using Ito's lemma. Now, to apply Ito's lemma, let us apply the following transformation;

$$F(x, t) = x^{1-a}$$

$$F^{\prime}(x, t) = (1 - \alpha)x^{-a}$$

$$F^{\prime\prime}(x, t) = -\alpha(1 - \alpha)x^{-a-1}$$

Now applying Ito's lemma we get,

$$f = r(1 - \alpha)\mu.y^{\lambda} - \frac{\sigma^2\alpha(1 - \alpha)}{2}y$$

$$g = \sigma(1 - \alpha)y$$

This gives us the reduced stochastic differential equation as,

$$dy(t) = [r(1 - \alpha)\mu.(y(t))^{\lambda} - \frac{\sigma^2\alpha(1 - \alpha)}{2}y(t)]dt + \sigma(1 - \alpha)y(t)dw(t)$$

It is again a stochastic differential equation in Ito's form and it is to be further reduced using Ito's lemma so that one can have an explicit solution of x(t). Now to apply Ito's lemma once again we apply the following transformation.

$$F_1(y, t) = y^{1-\lambda}$$

$$F_1^{\prime}(y, t) = (1 - \lambda)y^{-\lambda}$$

$$F_1^{\prime\prime}(y, t) = -\lambda(1 - \lambda)y^{-\lambda-1}$$

$$f_1 = (1 - \lambda)[r(1 - \alpha)\mu - \frac{\sigma^2\alpha(1 - \alpha)}{2}z]$$

$$g_1 = \sigma(1 - \alpha)(1 - \lambda)z$$

Therefore the SDE reduces to

$$dz(t) = \frac{\sigma^2}{2}(\alpha-1)(1-\lambda)(\alpha-(\alpha-1)\lambda)z(t)+r(1-\alpha)(1-\lambda)\mu]dt-\sigma(\alpha-1)(1-\lambda)z(t)dw(t)$$

$$dz(t) = [(A - B)z(t) + M]dt - Cz(t)dw(t)$$

It is a SDE in standard form and the solution of z(t) is of the form,

$$z(t) = exp([\frac{A}{2} - B]t - Cw(t))[z(0) + M\int_0^t exp([B - \frac{A}{2}]s + Cw(s))ds]$$

$$x(t) = [exp([\frac{A}{2} - B]t - Cw(t))[(x(0))^{(1-a)(1-\lambda)} + M\int_0^t exp([B - \frac{A}{2}]s + Cw(s))ds]]^{\frac{1}{(1-\alpha)(1-\lambda)}} \qquad (3)$$

$$A = \frac{\sigma^2\alpha(\alpha - 1)(1 - \lambda)}{2}$$

$$B = \frac{\sigma^2(\alpha - 1)^2(1 - \lambda)\lambda}{2}$$

$$M = r(1 - \alpha)(1 - \lambda)\mu$$

$$C = \sigma(\alpha - 1)(1 - \lambda)$$

MLEs of the growth rate parameter

Now to obtain the MLE of a and b on the basis of continuous records available one may assume that the diffusion coefficient terms are known. So this part does not involve any unknown parameters. Using Girsanov theorem the infill log-likelyhood becomes;

$$l(a, b) = A - \frac{1}{2}B$$

$$A = \int_0^T \frac{r(x(t))^a(k - (x(t))^\beta)}{\sigma^2(x(t))^2}dx(t)$$

$$B = \int_0^T \frac{(r(x(t))^a(k - (x(t))^\beta))^2}{\sigma^2(x(t))^2}dt$$

This gives the MLE of growth rate parameter as,

$$\hat{r} = \frac{I_1 - I_2}{I_3}$$

$$I_1 = \frac{k(x(T))^{a-1}}{(\alpha - 1},$$

$$I_2 = \frac{(x(T))^{a+\beta-1}}{(\alpha + \beta - 1)}$$

$$I_3 = \int_0^T (x(t))^{2a}(k - (x(t))^\beta)^2 dt$$

Analysis of real life data

Here, a real life data set of no. of infected persons in South Korea by COVID-19 at different time points are taken (data source: Worldometer). In this data set no. of infected persons at every 7th day are given. Data points are available from February 15, 2020 to April 25, 2020. It is known from different sources that South Korea didn't impose any curfew or lockdown situation [8]. Here, we tried to fit this model to the data set and compare the results with some of the famous models that are already established models to describe biological growth phenomena, one thing that should be mentioned here is that if one replaces α and β as,

$$\alpha = \frac{2}{3}$$

$$\beta = 1$$

Then the model becomes similar to stochastic version of von-Bertalanffy power law model and the results of the fit are given as,

$$\hat{r} \approx 0.0036,$$

$$std.error \approx 0.0004,$$

$$AIC \approx 205.63$$

But when the other existing model that are used to describe sigmoidal growth are fitted to the data, e.g., stochastic logistic model results are given in the following:

$$\hat{a} \approx 1,$$

$$std.error \approx 0.573,$$

$$\hat{b} \approx -3.3044,$$

$$std.error \approx 0.00004,$$

$$AIC \approx 280.5285$$

When this data set is fitted to stochastic Gompertz model, then we have;

$$\hat{a} \approx 2.175,$$

$$std.error \approx 1.37,$$

$$\hat{b} \approx -3.3044,$$

$$std.error \approx 0.0034,$$

$$AIC \approx 238.6262$$

Again from the described model of section 2, When one replaces α and β by

$$\alpha = 1$$

$$\beta = \frac{1}{2}$$

The model looks similar to stochastic Richard's model and the results of fit for this model are

$$\hat{r} \approx 0.0002,$$

$$std.error \approx 0.0001,$$

$$AIC \approx 225.05$$

$$\alpha = 1$$

$$\beta = \frac{2}{3}$$

$\hat{r} \approx 0.0002,$

$std.error \approx 0.0001,$

$AIC \approx 224.93$

$\alpha = 1$

$\hat{r} \approx 0.0002,$

$std.error \approx 0.0001,$

$AIC \approx 224.74$

$\alpha = 1$

$\beta = \frac{4}{5}$

The results of the fit for this case are,

$\hat{r} \approx 0.0003,$

$std.error \approx 0.0001,$

$AIC \approx 224.01$

The results of the fit for this case and when one replaces α and beta by this fashion the AIC's gets lower and lower [9]. So, from the above it is clear that except for stochastic logistic model and stochastic Gompertz model the other models fit comparatively better to the data set and among them the first model i.e, the model that seems to be close to the stochastic version of Von-Bertalanffy model has the best fit here [10]. Though it is stated a few times in this section that for different values of α and β the model that is described here looks similar to stochastic Von Bertalanffy model or stochastic Richard's model but if one observes the model closely, described in section 2, the model is actually a generalization of stochastic versions of Von Bertalanffy and Richard's model and also from the results of fit to different cases it is clear that the fit gets better and better for Richard's model but when β gets closer to 1 i.e, it becomes similar to Logistic model, surprisingly the fit gets worse as compared to the previous ones.

Conclusion

In this article, as it appears or already stated previously, it is clear that the model described here is actually a generalized version of Stochastic Von Bertalanffy power law model and Stochastic Richard's model and here in this case the model is applied to a real life data set and the results are presented in previous section and as it appears that stochastic versions of Von Bertanffy power law model and Richard's model performs better when they are compared to stochastic versions of Logistic model and Gompertz model. It is actually the growth pattern that allows us to consider such model to describe the growth or to fit the model to the data. It is not only this study but one can use this model to describe or understand the growth dynamics of other phenomena from different field according to the nature of the data and it's background as well. In this article the special cases of the described models are applied to the data set of South Korea but there are few more countries where no lockdown was imposed, so one can try to fit the model to the data sets of those countries as well and observe how it performs.

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