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A Survey on Triangular Number and Factorial Number

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Abstract

Objectives: The paper aims to present a survey of both time-honored and contemporary studies on triangular number, factorial, relationship between the 2, and a few other numbers related to them.

Methods: The research is expository in nature. It focuses on expositions regarding the triangular number, its multiplicative analog – the factorial and other numbers associated with them.

Findings: Much had been studied about triangular numbers, factorials and other numbers involving sums of triangular numbers or sums of factorials. However, it seems that no-one had explored the properties of the sums of corresponding factorials and triangular numbers. Hence, explorations on these integers, called factoriangular numbers, were conducted. Series of experimental mathematics resulted to the characterization of factoriangular numbers on its parity, compositeness, number and sum of positive divisors and other minor characteristics. The sequence of factoriangular numbers may be a recurring sequence and it's a rational closed-form of exponential generating function. These numbers were also characterized on when a factoriangular number are often expressed as a sum of two triangular numbers and/or as a sum of two squares.

Application/ Improvement: The introduction of factoriangular number and expositions on this sort of number may be a novel contribution to the idea of numbers. Surveys, expositions and explorations on existing studies may still be a serious undertaking in number theory.

Keywords: Factorial • Factorial-like number • Factoriangular number • Polygonal number • Triangular number

Introduction

In the mathematical field, a way of beauty seems to be almost the sole useful drive for discovery and its imagination and not reasoning that seems to be the moving power for invention in mathematics. It's in number theory that a lot of the best mathematicians in history had tried their hand3 paving the way for mathematical experimentations, explorations and discoveries. Gauss once said that the idea of numbers is that the queen of mathematics and arithmetic is that the queen of science. The idea of numbers concerns the characteristics of integers and rational numbers beyond the standard arithmetic computations. Due to its unquestioned historical importance, this theory had occupied a central position within the world of both ancient and contemporary mathematics [1].

As far back as ancient Greece, mathematicians were studying number theory. The Pythagoreans were considerably curious about the somewhat mythical properties of integers [2]. They initiated the study of perfect numbers, deficient and abundant numbers, amicable numbers, polygonal numbers, and Pythagorean triples. Since then, almost every major civilization had produced number theorists who discovered new and interesting properties of numbers for nearly every century.

Until today, number theory has shown its irresistible appeal to professional, also as beginning, mathematicians. One reason for this lies within the basic nature of its problems [3]. Although many of the amount theory problems are extremely difficult to unravel and remain to be the foremost elusive unsolved problems in mathematics, they will be formulated in terms that are simple enough to arouse the interest and curiosity of even those without much mathematical training.

More than in any a part of mathematics, the methods of inquiry in number theory adhere to the scientific approach [4]. Those performing on the sector must rely to an outsized extent upon trial and error and their own curiosity, intuition and ingenuity. Rigorous mathematical proofs are often preceded by patient and time-consuming mathematical experimentation or experimental mathematics.

Experimental mathematics refers to an approach of studying mathematics where a field is often effectively studied using advanced computing technology like computer algebra systems. However, computing system alone isn't enough to unravel problems [5]; human intuition and insight still play an important role in successfully leading the mathematical explorer on the trail of discovery. And besides, number theorists within the past have done these mathematical experimentations only by hand analysis and computations.

Among the various works in number theory which will be done through experimental mathematics, exploring patterns in integer sequences is one among the foremost interesting and regularly conducted. It's quite difficult now to count the amount of studies on Fibonacci sequence, Lucas sequence, the Pell and associated Pell sequences, and other well-known sequences [6]. Classical number patterns just like the triangular and other polygonal and figurate numbers have also been studied from the traditional up to the fashionable times by mathematicians, professionals and amateurs alike, in almost every a part of the planet. The multiplicative analog of the triangular number, the factorial, has also a special place within the literature being very useful not only in number theory but also in other mathematical disciplines like combinatorial and mathematical analysis. Quite recently, sequences of integers generated by summing the digits were also being studied.

The current work aims to present a survey of both time-honored and contemporary studies on triangular number and other polygonal numbers, factorial and factorial-like numbers, and a few other related or associated numbers. It also includes some interesting results of recent studies conducted by the author regarding the sum of corresponding factorial and triangular number, which is known as factoriangular number [7]. The history of mathematics generally and therefore the history of number theory especially are inseparable. Number theory is one among the oldest fields in mathematics and most of the best mathematicians contributed for its development. Although it's probable that the traditional Greek mathematicians were largely indebted to the Babylonians and Egyptians for a core of data about the properties of natural numbers, the primary rudiments of an actual theory are generally credited to Pythagoras and his followers, the Pythagoreans.

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Survey on Triangular Number, Factorial and Related Numbers

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Triangular Number and Other Polygonal Numbers

An important subset of natural numbers in ancient Greece is that the set of polygonal numbers [9]. The name polygonal number was introduced by Heysicles to ask positive integers that are triangular, oblong, square, and therefore the like. These numbers are often considered because the ancient link between number theory and geometry. The characteristics of those polygonal numbers were studied by Pythagoras and therefore the Pythagoreans. They depicted these numbers as regular arrangements of dots in geometric patterns. The triangular numbers were represented as triangular array of dots, the oblong numbers as rectangular array of dots, and therefore the square numbers as square array of dots [10].

Contemporary Studies on Triangular Numbers

Triangular numbers, though having an easy definition, are amazingly rich in properties of varied kinds, starting from simple relationships between them and therefore the other polygonal numbers to very complex relationships involving partitions, modular forms and combinatorial properties. Many other important results on these numbers are discussed within the literature. A theorem of Fermat states that a positive integer are often expressed as a sum of at the most three triangular, four square, five pentagonal, or n n-gonal numbers. Gauss proved the triangular case. Euler left important results regarding this theorem, which were utilized by Lagrange to prove the case for squares and Jacobi also proved this independently. Cauchy showed the complete proof of Fermat's theorem. The expression of an integer as a sum of three triangular numbers are often wiped out quite a method and Dirichlet showed the way to derive such number of ways. The modular form theory also can be wont to calculate the representations of integers as sums of triangular numbers. The ways a positive integer are often expressed as a sum of two n-sided regular figurate numbers also can be generalized [11]. Furthermore, it had been shown that a generating function manipulation and a combinatorial argument are often used on the partitions of an integer into three triangular numbers and into three distinct triangular numbers, respectively.

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