Journal of

# A Study of a Stellar Model with Kramer’s Opacity and Negligible Abundance of Heavy Elements 

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#### Abstract

This study deals with the investigated on a stellar model with Kramer's opacity and negligible abundance of heavy elements. This study contains the mechanism of energy production in a star and energy transports in stellar interior. In addition these studies solve radiative layers and connective layers equation and have drawn a curve in the U-V plane with the core solution for determining the value of constants. Finally this study determined the structure of a star of 2.5 solar masses in which this study chosen Keamer's opacity and the abundance of heavy elements are negligible.


Keywords: Stellar Model; Kramer’s Opacity; HR Diagram; Shooting Method; UV-plane.

## Introduction

Stars are hot, massive, and luminous celestial objects in plasma state. Observed masses of stars range from $0.01 M_{\Theta}$ to $100 M_{\Theta}$ where $M_{\ominus}$ is the solar mass and we have $M_{\ominus}=1.989 \times 10^{33}$ gram. Stars with masses greater than $100 M_{\Theta}$ may also exist. But their lifetime is very short, being of the order of $10^{6}$ years. Harm \& Schwarzschild [1] has shown that the maximum possible mass of a stable normal stars is 60 $M_{\ominus}$. There is a bit uncertainty in the minimum mass of a star. It is believed that stars with mass less than $0.01 M_{\Theta}$ can also exist. But they will never be main sequence stars. Because their interior will not be hot enough to burn hydrogen [2]. They will shrink directly to the while dwarf state.

The chemical composition of stellar matter is obviously very important, since it directly influences such basic properties as absorption of radiation or generation of energy by nuclear reaction. These reactions in turn alter the chemical composition, which represents a longlasting record of the nuclear history of the star.

The composition of stellar matter is extremely simple compared to that of terrestrial bodies. This is because of the high temperatures and pressures these are no chemical compounds in the stellar interior, and the atoms are for the most part completely ionized. It suffices then to count and keep track of the different types of nuclei [3].

The main constituent of a star is hydrogen. The composition of star is usually determined by the abundances $\mathrm{X}, \mathrm{Y}$ and Z of hydrogen, helium and other heavier elements in ionized form respectively such that

$$
\mathrm{X}+\mathrm{Y}+\mathrm{Z}=1
$$

This means that one gram of stellar material contains X gram of hydrogen, Y gram of helium and Z gram of heavy elements. For the sun, for example,

$$
\mathrm{X}=0.73, \mathrm{Y}=0.25, \mathrm{Z}=0.02 ;
$$

The only measurable quantity of star is its luminosity, which obviously depends on the physical conditions prevailing in its interior. The distribution of the thermodynamic variables such as pressure $(P)$ temperature ( $T$ ), and density $(\rho)$ inside a star determines its interior physical condition in other words, its structure. The basic problem in determining the structure of a star is to obtain a set of differential equa-
tions defining the structure with necessary boundary conditions and solve them for given mass $(M)$, radius $(R)$ and luminosity $(L)$ and also derive information about the chemical composition, the energy source and the transport of energy from the centre to the surface of the star.

## Methodology

The structure of a star is determined by the requirements of mass conservation, energy conservation, equilibrium of force, and by the mode of energy transport. On the other hand, the structure of the star depends on the chemical composition, which may vary in course of time either due to nuclear reaction in the deep interior, or due to mixing in convective layers of the star. If the star is taken to be non-magnetic, non-rotating, and spherically symmetric, all physical quantities are function of one single spatial variable (Lagrangian co-ordinate), and of time $t$. But it is convenient to use the radius $r$ directly as an independent variable.

If the chemical composition remain fixed in time, and on atmosphere is considered then the partial differential equation defining the stellar structure reduces to the following set of equations [4].

$$
\begin{array}{ll}
\frac{d P(r)}{d r}=-\frac{G M(r) \rho(r)}{r^{2}}, & \text { hydrostatic equation } \\
\frac{d M(r)}{d r}=4 \pi r^{2} \rho(r), & \text { conservation of mass } \\
\frac{d L(r)}{d r}=4 \pi r^{2} \rho(r) \varepsilon(r), & \text { conservation of energy } \\
\frac{d T}{d r}=-\frac{3}{4 a c} \frac{\kappa \rho}{T^{3}} \frac{L(r)}{4 \pi r^{2}}, \quad \text { radiative temperature gradient } \\
\frac{d T}{d r}=\left(1-\frac{1}{\gamma} \frac{T}{P} \frac{d P}{d T}, \quad\right. \text { convective temperature gradient } \tag{e}
\end{array}
$$

[^0]These equations pose the overall problem of the theory of stellar interior.

In addition to the above differential equations which characterize general conditions, have three explicit relations which characterize more specifically the behavior of the interior of the star, the equation for the absorption coefficient, and the equation for the energy generation by nuclear processes, which represent by the following formal equations

$$
\begin{aligned}
& \mathrm{P}=\mathrm{P}(\rho, \mathrm{~T}, \mathrm{X}, \mathrm{Y}) \\
& \kappa=\kappa(\rho, \mathrm{T}, \mathrm{X}, \mathrm{Y}) \\
& \varepsilon=\varepsilon(\rho, \mathrm{T}, \mathrm{X}, \mathrm{Y})
\end{aligned}
$$

All these characteristic relations directly depend on the hydrogen abundance $X$ and the helium abundance $Y$. equations (a) to (e) must all be fulfilled in every layer of the star. This study has the following boundary conditions also.

Considering a sphere of infinitesimal radius $r$ at the centre, find that

$$
M(r)=\frac{4}{3} \pi r^{3} \rho \quad \text { and } \quad L(r)=\frac{4}{3} \rho r^{3} \rho \varepsilon=M(r) \varepsilon
$$

Since it may treat $\rho$ and $\varepsilon$ sensibly constant in the sphere, hence as $r \rightarrow 0$.

$$
M(r) \rightarrow 0 \text { and } L(r) \rightarrow 0
$$

(f)
for $\rho$ and $\varepsilon$ remain finite as $r \rightarrow 0$. It is clear that the condition $L(r)$ $=0$ at $r=0$.

$$
\text { is a consequence of the condition } \quad M(r)=0 \quad \text { at } \quad r=0
$$

This gives only one independent boundary condition at the center namely,

$$
\begin{equation*}
M(r)=0 \quad \text { at } \quad r=0 \tag{g}
\end{equation*}
$$

It is clear that
$M(r)=M \quad$ and $\quad L(r)=L$.
At the surface, i.e., at $\mathrm{r}=R$
In addition, this study can derive suitable conditions for pressure and temperature of a star at its surface. The surface temperatures of stars are in general of order of a few thousand degrees while their central temperatures are of order of a few million degrees, so that the surface temperatures may approximately be taken as zero. The mass of the atmosphere of a stare is just a minute fraction of its total mass; therefore we may take the pressure on its surface as approximately equal to zero. Thus we have two more condition at the surface, namely,

$$
\begin{equation*}
T=0, \quad P=0 \quad \text { at } r=R \tag{i}
\end{equation*}
$$

Which are referred to as the "zero boundary condition". For stars whose outermost layers are in radiative equilibrium, these conditions provided a good approximation to the actual boundary conditions.

Thus this study consist of four simultaneous, total, non-linear first order differential equations for four variables ( $P, M, L$, and $T$ ) all are the function of the fifth variable $r$. These five differential equations(equations(a), (b), (c), (d), and (e), together with the four boundary conditions above (equations (f) to (i) represent a typical, well define boundary value problem. According to Volgt-Rrussel theorem [5] if the pressure P , the opacity K and the rate of energy generation
$\varepsilon$ are function of the local values of density $\rho$, temperature $T$, and the chemical composition only, then the structure of a star is uniquely determined by its mass and chemical composition [6].

## The model star

This study consider a star of mass $2.5 M_{\Theta}$ with composition $X=0.90$, $Y=0.90$, and $Z=0.01$, in which ideal gas laws hold. Since for star of masses $\geq 2 M_{\ominus}$ the energy is principally due to $C N$ cycle, the energy generation law is taken as

$$
\begin{align*}
& \text { law is taken as }  \tag{1}\\
& \varepsilon_{C N}=\varepsilon_{0} \times \rho X X_{C N}\left(\frac{T}{10^{6}}\right)^{16}
\end{align*}
$$

$$
\text { Where } X_{C N}=\frac{z}{3} \text {, and } \varepsilon_{0}=10^{-17}
$$

For most main sequence stars opacity is caused by bound free and free-free transitions while for very hot stars it is due to electron scattering. For upper main sequence stars in the intermediate regime the opacity is likely to be mixed. Stellar models of mixed opacity have been calculated by Harm and Schwarzschild, Kushwaha, Morton and S.S. Huang [1, 7-9]. In high density and low temperature condition, the other two opacity sources collectively called Kramer's opacity, are dominant. In these calculations opacity has been taken due to Kramer's opacity combined by straight addition. However Reiz [10] proposed an expression for mixed opacity where free-free transition and electron scattering are of the same order. In our problem, we have chosen the Kramer's opacity which is given by

$$
\begin{equation*}
\kappa=\kappa_{0}\left(\frac{\rho}{T^{3.5}}\right) \tag{2}
\end{equation*}
$$

$$
\text { Where } \kappa_{0}=4.34 \times 10^{25} \times Z \times(1+X)
$$

The structure of the modal star is given by equations (28-32) together with (1), (2) and $P=\frac{\mathfrak{R}}{\mu} \rho T$, the of state for an ideal gas. Since the model star is likely to have a small convective core with a radiative envelope, in principle we have two solutions, one in the envelope and one in the core. These two solutions must match at the interface.

## Polytropic core solutions

In the convective core the non dimensional equations are

$$
\begin{align*}
& \frac{d p}{d x}=-\frac{p q}{t x^{2}}  \tag{3}\\
& \frac{d q}{d x}=\frac{p x^{2}}{t} \tag{4}
\end{align*}
$$

Therefore the solution of equation is

$$
\theta=1-\frac{1}{6} \mu^{2}+\frac{3 / 2}{120} \mu^{4}-\frac{3 / 2(8 \times 3 / 2-5)}{42 \times 360} \mu^{6}+\ldots \ldots .
$$

For small $\eta$ this is a rapidly converging series.

$$
\begin{equation*}
\theta=1-\frac{1}{6} \mu^{2}+\frac{3 / 2}{120} \mu^{4} \tag{6}
\end{equation*}
$$

Introducing Schwarzschild homology variables define by

$$
\begin{equation*}
\frac{d q}{d x}=\frac{5}{2} \frac{p}{t} \frac{d t}{d x} \text { or } p=E t \tag{5}
\end{equation*}
$$

$$
\begin{array}{r}
U=\frac{r}{M(r)} \frac{d M(r)}{d r}=\frac{x}{q} \frac{d q}{d x}=\frac{p x^{3}}{d t}=\frac{p x^{3}}{q t} \\
V=-\frac{r}{p} \frac{d p}{d r}=-\frac{x}{p} \frac{d p}{d x}=\frac{q}{t x} \\
n+1=\frac{T}{p} \frac{d p}{d t}=\frac{t}{p} \frac{d p}{d t}=\frac{q t^{8.5}}{C p^{2}} \tag{9}
\end{array}
$$

The advantage of these variables is that they are scale independent, multiplying $r$ and $M(r)$ by constant does not change U since they occur on both the numeration and denominator.

So as to good approximation

$$
\begin{equation*}
U=3-\frac{18}{50} v+\ldots \tag{10}
\end{equation*}
$$

This gives the core solution in the U-V plane.

## Envelope solution of the matching point

The envelope of the model star is in radiative equilibrium. Its structure is determined by equation (3) to (5). The equation (5) contains an unknown parameter C. This study thus have an one-parameter family of solutions, this studies aim is to determine the correct value of $C$ and obtain the envelope solution for the value of the parameter. In order to do this parameter this study have to solve the envelope solutions for different trial value of C and find which value of C the solution just matches the core solution at the interface. However the solution is not straightforward. Because of the existence of singularity at the surface, integration cannot be started right from the surface ( $x=1$ ). To avoid this difficulty this study looks for series expansion of the variables about the singular point.

The envelope solutions calculated numerically, however since the equations are singular at the surface, $p=t=0$, this study chosen the series expansion of the variables near the singular point in the following way.

$$
\begin{array}{r}
\text { Let } \frac{1}{x}-1=\xi, \\
\text { i.e, } x=\frac{1}{1+\xi} \\
o r, \frac{d p}{d \xi}=\frac{q p}{t} \\
o r, \frac{d t}{d \xi}=C \frac{p^{2}}{t^{8.5}} \\
o r, \frac{d q}{d \xi}=-\frac{p}{t(1+\xi)^{4}} \tag{14}
\end{array}
$$

Here the singular point is $\xi=0$ since $x=l$ i.e, $\xi=0$. Now the series expansion of the variables about $\xi=0$ can easily be done. By Fuchs theorem [6] a convergent development of the solution in a power series about the singular point having a finite number of terms is possible.

Therefore, taking

$$
\begin{align*}
& t=\xi^{u}\left(C_{0}+C_{1} \xi+\ldots . .+C_{n} \xi^{n}\right)  \tag{15}\\
& p=\xi^{v}\left(b_{0}+b_{1} \xi+\ldots . .+b_{n} \xi^{n}\right)  \tag{16}\\
& p=1+g_{1} \xi+g_{2} \xi^{2}+\ldots . .+g_{n} \xi^{n} \tag{17}
\end{align*}
$$

In equation (98) used the condition that

$$
q=1 \text { at } \xi=0
$$

Using p, q, and t in equation (12) have the follows,

$$
\begin{aligned}
c_{0} d_{0} v \xi^{v+u-1}+\left(c_{1} d_{0} v+\right. & \left.c_{0} d_{1} v+c_{0} d_{1}\right) \xi^{u+v}+\ldots . . . . \text { higher order terms of } \xi \\
& =d_{0} \xi^{v}+\left(d_{1}+d_{0} g_{1}\right) \xi^{v+1}+\ldots . . . \text { higher order terms of } \xi
\end{aligned}
$$

Since the two polynomials are equal,
$U+V-1=V, U+V=V+1$, etc.
And, $c_{0} d_{0} v=d_{0}$
From the equation (18) and (19), have the follows,

$$
\begin{equation*}
u=1 \text { and } c_{0} v=1 \tag{20}
\end{equation*}
$$

With $\mathrm{u}=1, t$ becomes

$$
t=\left(c_{0} \xi+c_{1} \xi^{2}+\ldots \ldots \ldots \ldots \ldots \ldots \ldots+c_{n} \xi^{n+1}\right)
$$

Now from equation (94), have the follows,
Or, $\quad\left[c_{0}^{9.5} \xi^{8.5}+\left(2 c_{1} c_{0}^{8.5}+8.5 c_{0}^{8.5} c_{1}\right) \xi^{9.5}+\ldots . . . . . . .\right.$. higher order terms of $\left.\xi\right]$

$$
=C\left[d_{0}^{2} \xi^{2 v}+\left(2 d_{1} d_{0}\right) \xi^{2 v+1}+\ldots \ldots . . . . . \text { higher order terms of } \xi\right]
$$

Again equating the powers and coefficients has the follows,

$$
\begin{align*}
& 2 v=8.5 \text { and } c_{0}^{9.5}=c d_{0}^{2}  \tag{21}\\
& \text { i.e, } v=4.25 \text { and } d_{0}=\frac{c_{0}^{4.75}}{C^{0.5}}
\end{align*}
$$

So, from equations (20) and (21) have the follows,

$$
\begin{aligned}
& d_{0}=\frac{1}{C^{0.5}}\left(\frac{1}{4.25}\right)^{4.75} \\
& u=1, v=4.25, c_{0}=\frac{1}{4.25}
\end{aligned}
$$

Therefore, in the first approximation have about $\xi=0$, i.e., $x=1$

$$
p \approx \xi^{v} d_{0}
$$

$$
=\left(\frac{1}{4.25}\right)^{4.75} \frac{1}{c^{0.5}} \xi^{4.75}
$$

$$
\begin{equation*}
=\left(\frac{1}{4.25}\right)^{4.75} \frac{1}{c^{0.5}}\left(\frac{1}{x}-1\right)^{4.75} \tag{22}
\end{equation*}
$$

$$
t \approx=\xi^{u} c_{0}=\frac{1}{4.25} \xi
$$

$$
=\frac{1}{4.25}\left(\frac{1}{x}-1\right) q \approx 1
$$

These relations determine the values of the parameters at any point near the surface. With these values as the boundary values the envelope equations can easily be solved numerically by given of $C$. $C$ is an unknown constant whose value for a start of given mass depends on its luminosity and radius. As is evident form equation (29) $C$ is very small. For solar type stars $C$ is of the order of $10^{-6}$. This study shall treat $C$ as a free parameter and consider of values of close to $10^{-6}$.

This study take a point $x=0.99$ very near to the surface. Then from equation (22) the values of the parameters that point are found to be.

$$
\begin{aligned}
& p_{0}=3.5136 \times 10^{-9} \\
& t_{0}=2.3767 \times 10^{-3} \\
& q_{0}=1
\end{aligned}
$$



Figure 1: The core solution and the envelope solutions with different values of C in the $\mathrm{U}-\mathrm{V}$ plane.

| $\mathbf{x}=\mathbf{r} / \mathbf{R}$ | $\mathbf{p}$ | $\mathbf{t}$ | $\mathbf{q}=\mathbf{M r} / \mathbf{M}$ | Lnp | Lr/L |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1.000 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $1.00 \mathrm{E}+00$ |  | $1.00 \mathrm{E}+00$ |
| 0.990 | $3.51 \mathrm{E}-09$ | $2.38 \mathrm{E}-03$ | $1.00 \mathrm{E}+00$ | $-1.43 \mathrm{E}+01$ | $1.00 \mathrm{E}+00$ |
| 0.970 | $2.32 \mathrm{E}-07$ | $6.46 \mathrm{E}-03$ | $1.00 \mathrm{E}+00$ | $-1.11 \mathrm{E}+01$ | $1.00 \mathrm{E}+00$ |
| 0.960 | $8.36 \mathrm{E}-07$ | $8.70 \mathrm{E}-03$ | $1.00 \mathrm{E}+00$ | $-1.02 \mathrm{E}+01$ | $1.00 \mathrm{E}+00$ |
| 0.940 | $5.46 \mathrm{E}-06$ | $1.35 \mathrm{E}-02$ | $1.00 \mathrm{E}+00$ | $-8.73 \mathrm{E}+00$ | $1.00 \mathrm{E}+00$ |
| 0.920 | $2.15 \mathrm{E}-05$ | $1.86 \mathrm{E}-02$ | $1.00 \mathrm{E}+00$ | $-7.67 \mathrm{E}+00$ | $1.00 \mathrm{E}+00$ |
| 0.900 | $6.40 \mathrm{E}-05$ | $2.40 \mathrm{E}-02$ | $1.00 \mathrm{E}+00$ | $-6.84 \mathrm{E}+00$ | $1.00 \mathrm{E}+00$ |
| 0.890 | $1.03 \mathrm{E}-04$ | $2.68 \mathrm{E}-02$ | $1.00 \mathrm{E}+00$ | $-6.48 \mathrm{E}+00$ | $1.00 \mathrm{E}+00$ |
| 0.870 | $2.38 \mathrm{E}-04$ | $3.27 \mathrm{E}-02$ | $1.00 \mathrm{E}+00$ | $-5.84 \mathrm{E}+00$ | $1.00 \mathrm{E}+00$ |
| 0.850 | $4.96 \mathrm{E}-04$ | $3.88 \mathrm{E}-02$ | $1.00 \mathrm{E}+00$ | $-5.27 \mathrm{E}+00$ | $1.00 \mathrm{E}+00$ |
| 0.830 | $9.57 \mathrm{E}-04$ | $4.53 \mathrm{E}-02$ | $9.99 \mathrm{E}-01$ | $-4.77 \mathrm{E}+00$ | $1.00 \mathrm{E}+00$ |
| 0.820 | $1.30 \mathrm{E}-03$ | $4.87 \mathrm{E}-02$ | $9.99 \mathrm{E}-01$ | $-4.54 \mathrm{E}+00$ | $1.00 \mathrm{E}+00$ |

Table 1: Radiative structure of the model star $\mathrm{M}=\mathbf{2 . 5 , X = 0 . 9 0 , \mathrm { Y } = 0 . 0 9 , \mathrm { Z } =}$ 0.01(solar Unit).

| $x=r / R$ | $p$ | $t$ | $q=M r / M$ | $L n \rho$ | $L r / L$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1.000 | $0.00 \mathrm{E}+00$ | $0.00 \mathrm{E}+00$ | $1.00 \mathrm{E}+00$ |  | $1.00 \mathrm{E}+00$ |
| 0.990 | $3.51 \mathrm{E}-09$ | $2.38 \mathrm{E}-03$ | $1.00 \mathrm{E}+00$ | $-1.43 \mathrm{E}+01$ | $1.00 \mathrm{E}+00$ |
| 0.950 | $2.33 \mathrm{E}-06$ | $1.11 \mathrm{E}-02$ | $1.00 \mathrm{E}+00$ | $-9.38 \mathrm{E}+00$ | $1.00 \mathrm{E}+00$ |
| 0.900 | $6.40 \mathrm{E}-05$ | $2.40 \mathrm{E}-02$ | $1.00 \mathrm{E}+00$ | $-6.84 \mathrm{E}+00$ | $1.00 \mathrm{E}+00$ |
| 0.850 | $4.96 \mathrm{E}-04$ | $3.88 \mathrm{E}-02$ | $1.00 \mathrm{E}+00$ | $-5.27 \mathrm{E}+00$ | $1.00 \mathrm{E}+00$ |
| 0.986 | $1.09 \mathrm{E}-08$ | $3.17 \mathrm{E}-03$ | $1.00 \mathrm{E}+00$ | $-1.35 \mathrm{E}+01$ | $1.00 \mathrm{E}+00$ |
| 0.985 | $1.40 \mathrm{E}-08$ | $3.36 \mathrm{E}-03$ | $1.00 \mathrm{E}+00$ | $-1.33 \mathrm{E}+01$ | $1.00 \mathrm{E}+00$ |
| 0.800 | $2.30 \mathrm{E}-03$ | $5.56 \mathrm{E}-02$ | $9.99 \mathrm{E}-01$ | $-4.10 \mathrm{E}+00$ | $1.00 \mathrm{E}+00$ |
| 0.750 | $8.07 \mathrm{E}-03$ | $7.48 \mathrm{E}-02$ | $9.97 \mathrm{E}-01$ | $-3.14 \mathrm{E}+00$ | $1.00 \mathrm{E}+00$ |
| 0.700 | $2.40 \mathrm{E}-02$ | $9.66 \mathrm{E}-02$ | $9.92 \mathrm{E}-01$ | $-2.31 \mathrm{E}+00$ | $1.00 \mathrm{E}+00$ |
| 0.600 | $1.59 \mathrm{E}-01$ | $1.51 \mathrm{E}-01$ | $9.70 \mathrm{E}-01$ | $-8.64 \mathrm{E}-01$ | $1.00 \mathrm{E}+00$ |
| 0.550 | $3.72 \mathrm{E}-01$ | $1.85 \mathrm{E}-01$ | $9.46 \mathrm{E}-01$ | $-2.14 \mathrm{E}-01$ | $1.00 \mathrm{E}+00$ |
| 0.450 | $1.83 \mathrm{E}+00$ | $2.71 \mathrm{E}-01$ | $8.51 \mathrm{E}-01$ | $9.95 \mathrm{E}-01$ | $1.00 \mathrm{E}+00$ |
| 0.400 | $3.86 \mathrm{E}+00$ | $3.26 \mathrm{E}-01$ | $7.71 \mathrm{E}-01$ | $1.56 \mathrm{E}+00$ | $1.00 \mathrm{E}+00$ |
| 0.350 | $7.82 \mathrm{E}+00$ | $3.91 \mathrm{E}-01$ | $6.63 \mathrm{E}-01$ | $2.08 \mathrm{E}+00$ | $1.00 \mathrm{E}+00$ |
| 0.250 | $2.71 \mathrm{E}+01$ | $5.50 \mathrm{E}-01$ | $3.77 \mathrm{E}-01$ | $2.98 \mathrm{E}+00$ | $1.00 \mathrm{E}+00$ |

Table 2: The complete structure of the model star for $M=2.5, X=0.90, Y=0.09, Z$ $=0.01$ (Solar Unit).

Taking these values as the boundary values have integrated the equations for the radiative envelop numerically inwards up to where

$$
n+1=\frac{t}{p} \frac{d p}{d t}=2.5
$$

Appropriate for convection, by the fourth order Runge-Kutta
method for a number of trial values of $C$ ( $\mathrm{C}^{++}$Program). Some of these calculations, namely for $C=1.20 \times 10^{-6}, C=1.56 \times 10^{-6}, C=9.46 \times 10^{-7}$, $C=2.50 \times 10^{-7}, C=8.50 \times 10^{-7}$ etc. Together with the convective track, equation (10), are drawn in the (U-V) plane (Figure 1) at the junction between the convective core and the radiative envelope both $(\mathrm{U}, \mathrm{V})$ and their derivatives must be continuous. So the curve for the correct radiative solution must touch the convective curve at the interface. Form Fig I it is found that this happens for $C=9.46 \times 10^{-7}$. Therefore this is the correct value of C for our model star. For this value of C the matching point is at $x_{f}=0.168$. The radiative solution for the envelop $0.168 \leq x<$ 1 for $C=9.46 \times 10^{-7}$ is given in Table 1 .

## The complete solution

Form the Table 1 for matching point this study finds that $p_{f}=57.506, q_{f}=0.14665, t_{f}=0.70834, U_{f}=2.6253, V_{f}=1.2323$

Also $l_{f}=1$, at $x=x_{f}$, since all the energy is produced in the core.
With these values as this studies boundary conditions have to solve the core equations, namely equations (3), (4) and (5) inwards numerically. In order to do these studies need the correct value of $D$. This can be done by integrating the energy equation.

## Total luminosity,

$$
\begin{align*}
& L=\int_{0}^{x} 4 \pi r^{2} \rho(r) \varepsilon d r \\
& =D \int_{0}^{x f} x^{2} p^{2} t^{14} d x \\
& =D \int_{0}^{x} x^{2} p^{2} t^{14} d x=1 \\
& \text { or, } \frac{1}{D}=\int_{0}^{x f} x^{2} p^{2} t^{14} d x \tag{23}
\end{align*}
$$

From polytropic variables have,

$$
\begin{aligned}
& p=p_{c} \theta^{5 / 2}, t=t_{c} \theta \\
& x=\left(\frac{5}{2} \frac{t_{c}^{2}}{p_{c}^{2}}\right)^{1 / 2} \eta
\end{aligned}
$$

Using these in equation (24) gives,

$$
\begin{align*}
& \frac{1}{D}=\int_{0}^{x /} x^{2} p^{2} t^{14} d x \\
& =\int_{0}^{x f} p^{2} \theta^{5} t_{c}^{14} \theta^{14} \frac{t_{c}^{2}}{p_{c}} \eta^{2}\left(\frac{5 t_{c}^{2}}{2 p_{c}}\right)^{\frac{1}{2}} \\
& =\left(\frac{5}{2}\right)^{15} p_{c}^{0.5} t_{c}^{17} \int_{0}^{n f} \eta^{2} \theta^{19} d \eta \tag{24}
\end{align*}
$$

From equation (6) follows

$$
\theta=\left(1-\frac{1}{6} \eta^{2}+\frac{1}{80} \eta^{4}-\ldots \ldots \ldots . .\right)
$$

Using $\theta(\eta)$ in the equation (24) gives,

$$
\begin{equation*}
\frac{1}{D}=\left(\frac{5}{2}\right)^{15} p_{c}^{0.5} t_{c}^{17} \int_{0}^{n f} \eta^{2}\left(1-\frac{1}{6} \eta^{2}+\frac{1}{80} \eta 4+\ldots \ldots\right)^{19} d \eta \tag{25}
\end{equation*}
$$

Since $p$ and $t$ are continuous at $x_{f}$ have,

$$
\begin{aligned}
& p_{f}=p_{c} \theta_{f}^{5 / 2} \\
& \text { and }, t_{f}=t_{c} \theta_{f}
\end{aligned}
$$

And also have,

$$
\begin{aligned}
& U_{f}=3-\frac{3}{10} \eta_{f}^{2}+\ldots \ldots \ldots \\
& \text { and }, V_{f}=\frac{{ }^{5} \eta_{f}^{2}}{6}\left(1+\frac{\eta_{f}^{2}}{60} \ldots \ldots \ldots\right)
\end{aligned}
$$

Now from the value of $U f$ and $V f$ get the equations,

$$
\eta_{f}=1.20167
$$

And hence get from the above equations,
$\theta f=0.78539$
Therefore,

$$
\begin{aligned}
& p_{f=p_{c} \theta_{f}^{5 / 2}} \\
& \text { or, } p_{c}=\frac{p_{f}}{\theta_{f}^{5 / 2}}=\frac{57.506}{(0.78539) 5 / 2}=105.19723
\end{aligned}
$$

And, $t_{f}=t_{c} \theta_{f}$

$$
\text { or, } t_{c}=\frac{t_{f}}{\theta_{f}}=\frac{0.70834}{0.78539}=0.9019
$$

Now using the value of $p_{c}, t_{c}$ and $\eta_{f}$ in the equation (25) and evaluating the integration using Simpson's one third rules (Using $\mathrm{C}^{++}$ program) we have, $D=1.875173$

Using this $D$ in equation (5) have integrated the core equation again by the fourth order Runge-Kutta method from the interface downward up to $x=0.001$. The envelope solution and the core solution together give the complete internal structure of star. This study are however yet to find the luminosity and the radius of the star.

From equation (29) gives,
$C=\frac{3 k_{0}\left(\frac{R}{\mu G}\right)^{7.5} L R^{0.5}}{256 \pi^{3} a c M^{55}}$

Or, $L R^{0.5}=\frac{256 \pi^{3} a c M^{55}}{3 k_{0}\left(\frac{R}{\mu G}\right)^{7.5}}$
From equation (30) gives,

$$
\begin{equation*}
D=\frac{\varepsilon_{0} X X_{C N}\left(\frac{\mu G}{R}\right)^{16} M^{18}}{4 \times 19^{96} \pi L R^{19}} \tag{27}
\end{equation*}
$$

Or, $L R^{19}=\frac{\varepsilon_{0} X X_{C N}\left(\frac{\mu G}{R}\right)^{16} M^{18}}{4 \times 10^{96} \pi D}$
Eliminating L from equation (26) and equation (27) gives,

$$
\begin{align*}
& R^{18.5}=\frac{\varepsilon_{0} X X_{C N}\left(\frac{\mu G}{R}\right)^{16} M^{18}}{4 \pi D} \times \frac{3 k_{0}\left(\frac{R}{\mu G}\right)^{7.5}}{256 \pi^{3} C a c M^{5.5}}  \tag{28}\\
= & \frac{3 k_{0} \varepsilon_{0} X X_{C N}\left(\frac{\mu G}{R}\right)^{8.5} M^{12.5}}{1024 \times 10^{96} \pi^{4} C D a c}
\end{align*}
$$

Using all the values of the constants and parameters in equation (28), (Using $\mathrm{C}^{++}$program for solving the value of Luminosity and Radius using the values of constants), this study have, $R=1.4999 R_{\Theta}$ and
using this value of $R$ in equation (27) (Using $\mathrm{C}^{++}$program for solving the value of Luminosity and Radius using the values of constants) have the value of $L$, that is, $L=6.55 L_{\Theta}$. From the matching point inward integration have done for the convective solution (Using $\mathrm{C}^{++}$program for solving the value of D using the values of constants and boundary values) of the structure.

## Effect of variation of mass and chemical composition

If the chemical compositions remain same then there is an effect of varying mass on the other physical quantities, luminosity $L$, effective temperature $T_{\text {eff }}$, and radius $R$

These studies have

$$
\begin{equation*}
C=\frac{3 \kappa_{0}}{256 \pi^{3} a c}\left(\frac{\Re}{\mu G}\right)^{7.5} \frac{L R^{0.5}}{M^{5.5}} \tag{29}
\end{equation*}
$$

And

$$
D=\frac{\varepsilon_{0} X X_{C N}}{4 \times 10^{96} \pi}\left(\frac{\mu G}{\Re}\right)^{16} \frac{M^{18}}{L R^{19}}(30)
$$

From equation (29) thus have logarithmic measures

$$
\begin{equation*}
\log L+0.5 \log R-5.5 \log M+\log (\text { constant })=0 \tag{31}
\end{equation*}
$$

And from equation (30) thus have logarithmic measures

$$
\begin{equation*}
\log L+19 \log R-18 \log M+\log (\text { constant })=0 \tag{32}
\end{equation*}
$$

Differentiating equation (31) gives

$$
\begin{equation*}
\frac{\partial L}{L}+0.5 \frac{\partial R}{R}-5.5 \frac{\partial M}{M}=0 \tag{33}
\end{equation*}
$$

Again differentiating equation (32

$$
\begin{equation*}
\frac{\partial L}{L}+19 \frac{\partial R}{R}-18 \frac{\partial M}{M}=0 \tag{34}
\end{equation*}
$$

Eliminating $\frac{\partial L}{L}$ from equations (33) and (34) gives

$$
\begin{equation*}
\frac{\partial R}{R}=0.676 \frac{\partial M}{M} \tag{35}
\end{equation*}
$$

Again eliminating $\frac{\partial R}{R}$ from equations (33) and (34) gives

$$
\begin{equation*}
\frac{\partial L}{L}=5.162 \frac{\partial M}{M} \tag{36}
\end{equation*}
$$

From black body relationship gives

$$
\begin{equation*}
L=4 \pi \sigma R^{2} T_{e f f}^{4} \tag{37}
\end{equation*}
$$

Where $\sigma$ is Stefan-Boltzmann's constant.
Logarithmic differentiation of equation (37) gives
$\frac{\partial L}{L}-2 \frac{\partial R}{R}-4 \frac{\partial T_{\text {eff }}}{T_{\text {eff }}}=0$
Substituting the value of $\frac{\partial R}{R}$ and $\frac{\partial L}{L}$ from equations (35) and (36) into equation (37) gives

$$
\begin{equation*}
\frac{\partial T_{e f f}}{T_{e f f}}=0.952 \frac{\partial M}{M} \tag{39}
\end{equation*}
$$

From equations (35), (36), and (38) it is evident that as $M$ increases $R$ and $T_{e f f}$ increase slightly, but the increase in $L$ is quite sharp.

If mass is kept constant then there are some effect on the physical quantities, $L, R$, and $T_{e f f}$ for the variation of $\kappa_{0}, X$ and $Z$.

From the logarithmic differentiation of equation (29) gives

$$
\begin{equation*}
\frac{\partial L}{L}+0.5 \frac{\partial R}{R}+\frac{\partial \kappa_{0}}{\kappa_{0}}=0 \tag{40}
\end{equation*}
$$

Again from the logarithmic differentiation of equation (30) gives

$$
\begin{equation*}
\frac{\partial L}{L}+19 \frac{\partial R}{R}-\frac{\partial X}{X}-\frac{\partial Z}{Z}=0 \tag{41}
\end{equation*}
$$

Eliminating $\frac{\partial L}{L}$ from equations (40) and (41) gives

$$
\begin{equation*}
\frac{\partial R}{R}=0.054\left(\frac{\partial k_{0}}{k_{0}}+\frac{\partial X}{X}+\frac{\partial Z}{Z}\right) \tag{42}
\end{equation*}
$$

Again eliminating $\frac{\partial R}{R}$ from equations (40) and (41) gives

$$
\begin{equation*}
\frac{\partial L}{L}=-1.027 \frac{\partial k_{0}}{k_{0}}-0.027\left(\frac{\partial X}{X}+\frac{\partial Z}{Z}\right) \tag{43}
\end{equation*}
$$

Substituting the value of $\frac{\partial R}{R}$ and $\frac{\partial L}{L}$ from equations (42) and (43) into equation (38) gives

$$
\begin{equation*}
\frac{\partial T_{e f f}}{T_{e f f}}=-0.284 \frac{\partial k_{0}}{k_{0}}-0.034\left(\frac{\partial X}{X}+\frac{\partial Z}{Z}\right) \tag{44}
\end{equation*}
$$

But depends on $X$. therefore the change in $\kappa_{0}$ is in effect due to the change in $X$. Equations (42), (43) and (44) indicate that any increase in $X$ and $Z$ slightly increases $R$ but decrease $L$ and $T_{\text {eff }}$. This is expected because $R$ and $T_{e f f}$ vary inversely, the mass being constant.

## Discussion

For an increase in $M$ the position of the star in the HR diagram is slightly shifted to toward the upper end of the main sequence. If the mass is constant then a decrease in the hydrogen content of the star increases luminosity and effective temperature. But as time goes on in the main sequence lifetime of a star its hydrogen content gradually diminishes giving rise to the helium content. That means, as a main sequence star ages its position in the HR diagram slowly moves along the main sequence toward the hot end. The position of a main sequence star in the HR diagram is thus determined mainly by its mass and chemical composition.

## Conclusion

This study determine the structure of a 2.5 solar mass in which the abundance of elements has taken as $X=0.90, Y=0.09, Z=0.01$ and also assumed that the opacity is Kramer's opacity i.e, due to electron scattering. This study solved the equation of structure numerically by the fourth order Runge-Kutta method in which the step length has been taken as $h=0.001$. To determine the structure this study followed a simple fitting method devised by Cowling (1930). It is found that the mass and chemical compositions are prescribed then the distribution of the thermodynamic variables inside the star as well as its total luminosity, radius and effective temperature can be uniquely determined. It is interesting to observe that this study results obtained by simple fitting method do not vary significantly from the recent calculation of W. Brunish (e.g., Bohm-Vitense) by rigorous treatment of the problem. If the mass varies keeping the composition fixed, then all variables $L, T_{\text {eff }}$ and $R$ are found to vary.

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    Received February 13, 2012; Accepted April 24, 2012; Published April 28, 2012
    Citation: Chowdhury NM, Hossain MM (2012) A Study of a Stellar Model with Kramer's Opacity and Negligible Abundance of Heavy Elements. J Appl Computat Math 1:103. doi:10.4172/2168-9679.1000103

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