Journal of
Applied & Computational Mathematics
ISSN: 2168-9679

Ojha, J Appl Computat Math 2014, 3:1
DOI: 10.4172/2168-9679.1000150

Research Article Open Access

A Solid Transportation Problem with Partial Non-Linear Transportation Cost

Anupam Ojha*, **Shyamal Kumar Mondal and Manoranjan Maiti**

Department of Applied Mathematics with Oceanology and Computer Programming, Vidyasagar University, Midnapore - 721102, India

Abstract

This paper deals with a capacitated, multi-objective and solid transportation problem with imprecise nature of resources, demands, capacity of conveyance and cost. Here transportation cost is inversely varying with the quantity-to be transported from source to destinations in addition with a fixed per unit cost and a small vehicle cost. The transportation problem has been formulated as a constrained fuzzy non-linear programming problem. Next it is transformed into an equivalent crisp multi-objective problem using fuzzy interval approximation and then solved by Interactive Fuzzy Programming Technique (IFPT) and Generalized Reduced Gradient (GRG) method. An illustrative numerical example is demonstrated to find the optimal solution of the proposed model.

Keywords: Transportation models; Fuzzy numbers; Interval approximation; Interactive fuzzy programming technique

Introduction

The classical transportation problem (Hitchcock transportation problem) is one of the sub-classes of non-linear programming problem in which all the constraints are of equality type. In many industrial problem, a homogeneous product is delivered from an origin to a destination by means of different modes of transport called conveyances, such as trucks, cargo flights, goods trains, ships, etc. In general, the real life problems are modeled with multi-objective functions which are measured in different respects and they are non-commensurable and conflicting in nature. Furthermore, it is frequently difficult for the decision maker to combine the objective functions in one overall utility function. In a Solid Transportation Problem (STP) more than one objective is normally considered. In many practical situations, it is realistic to assume that the amount which can be sent on any particular route is restricted by the capacity of that route. Moreover, Appa [1] discussed about the different variations in transportation problem.

It is often difficult to estimate the accurate values of transportation cost, delivery time, quantity of goods delivered, demands, availabilities, the capacity of different modes of trans-port between origins to destinations, etc. Depending upon different aspects, these fluctuate due to uncertainty in judgment, lack of evidence, in sufficient information, etc. i.e., it is not possible to get relevant precise data, which are assumed by several researchers Shell [2]. So, a transportation model become more realistic if these parameters are assumed to be flexible/imprecise in nature i.e., uncertain in non-stochastic sense and may be represented by fuzzy numbers. (Here, for the first time a solid transportation problem is considered with fuzzy parameters like, Shell [2], Jimenez and Verdegay [3].

Based on Das et al. [4], the interval number transportation problems were converted into deterministic multi-objective problems. Grzegorzewski [5] approximated the fuzzy number to its nearest interval. Omar and Samir [6] and Chanas and Kuchta [7] discussed the solution algorithm for solving the transportation problem in fuzz environment. Sakawa and Yano [8] proposed an interactive fuzzy decision making method using linear and non-linear membership functions to solve the multi-objective linear programming problem. Gao and Liu [7,9] presented two-phase fuzzy algorithms. Shaocheng [10] discussed about the interval number linear programming. Verma et al. [11], Bit at al. [12,13], Jimenez and Verdegay [14], Li and Lai [15] and Waiel [16] presented the fuzzy compromise programming approach to multi-objective transportation problem.

In this paper, a capacitated-multi-objective, solid transportation problem is formulated in fuzzy environment with non-linear varying transportation charge and an extra cost for transporting the amount to an interior place through small vehicles (like rickshaw, auto etc.). In the non-linear varying transportation charge, one part is linearly and another nonlinearly proportional to the transported amount. Here, the non-linear cost increases with the increase of transported amount but the rate of increase decreases. The fuzzy quantities and parameters are replaced by equivalent nearest interval numbers and thus a fuzzy multi-objective, capacitated and solid transportation problem is transformed to corresponding crisp multi-objective transportation problems. Membership function is formulated for each objective function from their individual minimum and maximum. These membership functions may be of different types and it may depend upon the decision maker's (DM) choice/past data, if available. The main contribution of the paper is the mathematical formation of the above mentioned innovative transportation problem in fuzzy environment and its solution. Here for the first time, here man-machine interaction has been introduced in the transportation system. In this way, multi-objective transportation problem is solved using IFPT through GRG. This model is illustrated with an example. In particular, results of some specific transportation models are presented.

Fuzzy Number and its Interval Approximation

Definition 1: Fuzzy Number [17]:

***Corresponding author:** Anupam Ojha, Department of Applied Mathematics with Oceanology and Computer Programming, Vidyasagar University, Midnapore–721102, India, Tel: 3222 276 554; E-mail: ojhaanupam@yahoo.co.in

Received August 17, 2013; **Accepted** January 02, 2014; **Published** January 09, 2014

Citation: Ojha A, Mondal SK, Maiti M (2014) A Solid Transportation Problem with Partial Non-Linear Transportation Cost. J Appl Computat Math 3: 150. doi:10.4172/2168-9679.1000150

Citation: Ojha A, Mondal SK, Maiti M (2014) A Solid Transportation Problem with Partial Non-Linear Transportation Cost. J Appl Computat Math 3: 150. doi:10.4172/2168-9679.1000150

If X is a collection of objects denoted generically by x then a fuzzy set Ã in X is a set of ordered pairs:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \varepsilon X\}$$

$u_{\tilde{A}}(x))$ is called the membership function of x in $\widetilde{A}$ which maps X to the membership space [0,1].

(i) A is normal, i.e.the supremum of

(ii) A is fuzzy convex, i.e. $\mu_A(x)$ is 1 $\forall x \in S$ and

(iii) A is fuzzy convex, i.e.

$$\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \mu_A(x_1) \wedge \mu_A(x_2) \forall x_1, x_2 \in R, \ \ for \ \lambda \in [0,1]$$

Definition 2: α-Cut of a fuzzy number [17] (Figure 1):

A α-cut of a fuzzy number A is defined as crisp set

$A_\alpha = \{x : \mu\,\tilde{}\,(x) \geq \alpha, x \in X\}$ where $\alpha \in [0, 1]$

Definition 3: Interval Approximation [17]:

A_α is a non-empty bounded closed interval contained in X and it can be denoted by $A_\alpha=[AL(\alpha), AR(\alpha)]$. $A_L(\alpha)$ and $A_R(\alpha)$ are the lower and upper bounds of the closed intervalrespectively. A fuzzy number A with α_1, α_2-cut $A_{\alpha 1}=[A_L(\alpha_1), A_R(\alpha_1)]$,

$A_{\alpha 2}=[A_L(\alpha_2), A_R(\alpha_2)]$ and if $\alpha_2 \geq \alpha_1$, then $A_L(\alpha_2) \geq A_L(\alpha_1)$ and $A_R(\alpha_1) \geq A_R(\alpha_2)$.

Definition 4: Nearest interval approximation of a fuzzy number [18]:

Here a fuzzy number is approximated by a corresponding crisp interval. Suppose A and B, two fuzzy numbers with α-cuts are $[A_L(\alpha), A_R(\alpha)]$ and $[B_L(\alpha), BR(\alpha)]$ respectively. Then the distance between $\widetilde{A}$ and $\widetilde{B}$ is

$$d(\tilde{A},\tilde{B}) = \sqrt{\int_0^1 (A_L(\alpha) - B_L(\alpha))^2 d\alpha + \int_0^1 (A_R(\alpha) - B_R(\alpha))^2 d\alpha}$$

Given A is a fuzzy number. We find a closed interval $C_d(\widetilde{A})$ which is nearest to $\widetilde{A}$ with respect to metric d. It can be done since each interval is also a fuzzy number with constant α-cut for all α € [0, 1]. Hence $(C_d(A))\alpha=[C_L, C_R]$. Now we have to minimize

$$d(\tilde{A}, C_d(\tilde{A})) = \sqrt{\int_0^1 (A_L(\alpha) - C_L(\alpha))^2 d\alpha + \int_0^1 (A_R(\alpha) - C_R(\alpha))^2 d\alpha}$$

with respect to C_L and C_R. In order to minimize d ($\widetilde{A}$, $C_d(\widetilde{A})$), it is sufficient to minimize the function $D(C_L,C_R)=d^2(\widetilde{A}, Cd(\widetilde{A}))$. The first partial derivatives are

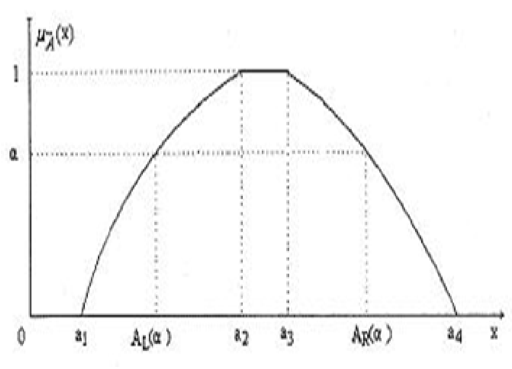

Figure 1: α-cut of a general fuzzy number.

$$\frac{\delta D(C_L, C_R)}{\delta C_L} = -2\int_0^1 A_L(\alpha) d\alpha + 2C_L \ and \ \frac{\delta D(C_L, C_R)}{\delta C_R} = -2\int_0^1 A_R(\alpha) d\alpha + 2C_R$$

Solving, $\dfrac{\delta D(C_L, C_R)}{\delta C_L} = 0 \ and \ \dfrac{\delta D(C_L, C_R)}{\delta C_R} = 0$

we get, $C_L^* = \int_0^1 (A_L(\alpha) d\alpha$ and $C_R^* = \int_0^1 (A_R(\alpha) d\alpha$

Again since, $\dfrac{\delta D^2(C_L^*, C_R^*)}{\delta C_L^2} = 2 > 0, \dfrac{\delta D^2(C_L^*, C_R^*)}{\delta C_R^2} = 2 > 0$

and $\dfrac{\delta D^2(C_L^*, C_R^*)}{\delta C_L^2} . \dfrac{\delta D^2(C_L^*, C_R^*)}{\delta C_R^2} - \left(\dfrac{\delta D^2(C_L^*, C_R^*)}{\delta C_L . \delta C_R}\right)^2 = 4 > 0$

So $D(C_L, C_R)$ i.e. d($\widetilde{A}$, Cd($\widetilde{A}$)) is global minimum. Therefore the interval

$C_d(\tilde{A}) = \left[\int_0^1 A_L(\alpha) d\alpha, \int_0^1 A_R(\alpha) d\alpha\right]$ is nearest interval approximation of fuzzy number $\widetilde{A}$ with respect to metric d.

Let $\widetilde{A}=(a_1, a_2, a_3, a_4)$ be a fuzzy number. The α-level interval of A is defined as $(\widetilde{A})\alpha = [AL(\alpha), AR(\alpha)]$.

When $\widetilde{A}$ is a trapezoidal fuzzy numbers then $A_L(\alpha)=a_1+\alpha(a_2-a_1)$ and $A_R(\alpha)=a_3-\alpha(a_4-a_3)$, $0<\alpha \leq 1$.

By nearest interval approximation method lower and upper limits of the interval are respectively

$$C_L = \int_0^1 A_L d\alpha = \int_0^1 [a_1 + \alpha(a_2 - a_1)] d\alpha] = \frac{1}{2}(a_2 + a_1)$$

and $C_R = \int_0^1 A_R d\alpha = \int_0^1 [a_3 - \alpha(a_4 - a_3)] d\alpha] = \frac{1}{2}(a_3 + a_4)$

Therefore, the nearest crisp interval number considering $\widetilde{A}$ as a trapezoidal fuzzy number is $[(a_1+a_2)/2, (a_3+a_4)/2]$.

Interval Analysis

We consider an interval $A=[a_L, a_R]=\{a:a_L \leq a \leq a_R, a \in R\}$, where a_L and a_R are the left and right limits of A respectively. The interval A is also denoted by its centre and width as:

$$A = \langle a_C, a_W \rangle = \{a : a_C - a_W \leq a \leq a_C + a_W, a \in R\}$$

Where $a_C=(a_R+a_L)/2$ and $a_W=(a_R-a_L)/2$ are the center and half-width of A respectively.

Definition 5 [18]**:** The order relation, $\leq_{RC}$ between $A=[a_L, a_R]$ and $B=[b_L, b_R]$ is defined as

$$A \leq_{RC} B \ iff \ a_R \leq b_R \ and \ a_C \leq b_{C,}$$
$$A <_{RC} B \ iff \ A \leq_{RC} B \ and \ A \neq B.$$

The order relation $\leq_{RC}$ represents the decision maker's preference for the alternative with the lower minimum value, i.e., if $A \leq_{RC} B$, then A is preferred to B for the minimization problem.

Theorem: The order relation $\leq_{RC}$ satisfied transitive law.

Proof: Let $A=[a_L, a_R]$, $B=[b_L, b_R]$ and $C=[c_L, c_R]$ such that

$$A \leq_{RC} B \,\&\, B \leq_{RC} C.$$
$$Now\, A \leq_{RC} B \,\&\, B \leq_{RC} C.$$
$$\Rightarrow a_R \leq b_R \,\&\, b_R \leq c_{R;} \qquad and \Rightarrow a_C \leq b_C \,\&\, b_C \leq c_C;$$
$$\Rightarrow a_R \leq b_R \leq c_R; \qquad and \Rightarrow a_C \leq b_C \leq c_C$$
$$and \Rightarrow 2a_C \leq 2b_C \leq 2c_C$$

Then subtracting we have $a_L \le b_L \le c_L$ and

$A \le_{RC} B$ & $B \le_{RC} C \Rightarrow A \le_{RC} C$

$f\ A \ne_{RC} B\ and\ B \ne_{RC} C\ this\ implies\ A \ne_{RC} C$

If possible $A =_{RC} C \Rightarrow a_L = c_L\ and\ a_R = c_R$

$$Now\, a_R \le b_R \le c_R \quad (Since\ A \le_{RC} B, B \le_{RC} C\ \&\ A \le_{RC} C)$$
$$\Rightarrow a_R = b_R = c_R$$

Moreover $a_L=c_L$ and $a_R=c_R$

$$\Rightarrow a_C = \frac{a_L + a_R}{2} = \frac{c_L + c_R}{2}$$
$$\Rightarrow a_C = b_C = c_C \quad (Since\, a_C \le b_C \le c_C \quad A \le_{RC} B\ \&\ B \le_{RC} C)$$

which is the contradiction. Therefore $A \ne_{RC} B\ and\ B \ne_{RC} C \Rightarrow A \ne_{RC} C$

Thus the theorem proved.

Formulation of the STP in Fuzzy Environments

The solid transportation problem (STP) stated by Shell [2] is a generalization of classical transportation problem. Here, we consider m origins(or sources) O_i (i=1,2,....m) , n desti-nations(i.e. demands) D_j (j=1,2,....n) and K conveyances E_k (k=1,2,....K). Let $\tilde{a}_i$ be the fuzzy amount of a homogeneous product available at i-th origin, $\tilde{b}_j$ be the fuzzy demand at j-th destination and $\tilde{e}_k$ represents the fuzzy amount of product which can be carried by k-th conveyance. The fuzzy penalty $\tilde{C}^p_{ijk}$ is associated with transportation cost (for p=1), distance, time etc, (for p=2,3..P) of one unit of a product to transport from i-th source to j-th destination by means of the k-th conveyance for p-th criterions. $\tilde{H}_{ijk}$ be the inversely varying cost for different quantity to transport from i-th source to j-th destination by means of the k-th conveyance. The penalty may represent transportation cost, delivery time, quantity of goods delivered etc. The vehicle carrying cost F (x_{ijk}) for the quantity x_{ijk} from i-th source O_i to j-th destination D_j via k-th conveyance is defined as:

$$F(x_{ijk}) = \begin{cases} m.v\ i\,f\ m.v_c = x_{ijk} \\ (m+1).v\ Otherwise \end{cases}$$

$m=[x_{ijk}/v_c]$, v_c=vechicle capacity and v=vehicle cost.

Let r_{ijk} be the capacity restrictions on route (i, j) by means of k-th conveyance. Therefore, the fuzzy-capacitated -constrained multi-criteria solid transportation problem represented as:

$$Min\, Z^1(X) = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}[[\tilde{C}^1_{ijk} + \tilde{H}_{ijk} / (x_{ijk})^{\gamma ijk} + F(x_{ijk})] \tag{1}$$

$$Min\, Z^p(X) = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}\tilde{C}^p_{ijk}\, x_{ijk} \qquad p = 2,3,...P. \tag{2}$$

$0 <\gamma_{ijk}< 1$

subject to the constraints

$$\sum_{j=1}^{n}\sum_{k=1}^{k} x_{ijk} = \tilde{a}_i \quad i = 1,2,3,...,m \tag{3}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{k} x_{ijk} = \tilde{b}_j \quad j = 1,2,3,...,n \tag{4}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} = \tilde{e}_k \quad k = 1,2,3,...,K \tag{5}$$

$$and \quad 0 \le x_{ijk} \le r_{ijk} \quad for\, all\, i,j,k. \tag{6}$$

Where $\tilde{a}_i, \tilde{b}_j, \tilde{e}_k, \tilde{C}^P_{ijk}, and\ \tilde{H}_{ijk}$ are fuzzy numbers which may be represented by triangular, trapezoidal or other fuzzy numbers.

Reduced Crisp Model

Following ($4) fuzzy numbers $\tilde{a}_i, \tilde{b}_j, \tilde{e}_k, \tilde{C}^P_{ijk}, and\ \tilde{H}_{ijk}$ are approximated to

$[a^L_i, a^R_i],[b^L_j, b^R_j],[e^L_k, e^R_k],[C^{Lp}_{ijk}, C^{Rp}_{ijk}]\, and\, [H^L_{ijk}, H^R_{ijk}]$ respectively.

Then the earlier transportation model takes the following form:

$$Minimize\ \{Z^p_C(X), Z^p_R(X)\} \quad p = 1,2,3,...P \tag{7}$$

$$where\ Z^1_C(X) = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}((C^{C1}_{ijk} + H^C_{ijk} / (x_{ijk})^{\gamma}_{ijk})x_{ijk} + F(x_{ijk})) \tag{8}$$

$$Z^1_R(X) = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K}((C^{R1}_{ijk} + H^R_{ijk} / (x_{ijk})^{\gamma}_{ijk})x_{ijk} + F(x_{ijk})) \tag{9}$$

$$Z^p_C(X) = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} C^{Cp}_{ijk}\, x_{ijk} \qquad p = 2,3,...P \tag{10}$$

$$Z^p_R(X) = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} C^{Rp}_{ijk}\, x_{ijk} \qquad p = 2,3,...P \tag{11}$$

$$0 < \gamma_{ijk} < 1. \tag{12}$$

subject to the constraints

$$\sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk} \in [a^L_i, a^R_i] \tag{13}$$

$$\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk} \in [b^L_j, b^R_j] \tag{14}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \in [e^L_k, e^R_k] \tag{15}$$

$$0 \le x_{ijk} \le r_{ijk}. \tag{16}$$

The constraints (13),(14),(15) and (16) can be written in the following form:

$$a^L_i \le \sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk} \le a^R_i \tag{17}$$

$$b^L_i \le \sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk} \le b^R_j \tag{18}$$

$$e^L_k \le \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \le e^R_k \tag{19}$$

$$0 \le x_{ijk} \le r_{ijk}. \tag{20}$$

Solution Procedure: Interactive Fuzzy Programming Technique

Step-1: Optimize (minimized & maximized) each objective function individually at a time ignoring the others. Let, x^p_R and x^p_C

x	Z_R^1	Z_C^1	Z_R^2	Z_C^2	–	–	Z_C^P
x_R^1	$Z_R^1(x_R^1)$	$Z_C^1(x_R^1)$	$Z_R^2(x_R^1)$	$Z_C^2(x_R^1)$	–	–	$Z_C^P(x_R^1)$
x_C^1	$Z_R^1(x_C^1)$	$Z_C^1(x_C^1)$	$Z_R^2(x_C1)$	$Z_C^2(x_C^1)$	–	–	$Z_C^P(x_C^1)$
x_R^2	$Z_R^1(x_R^2)$	$Z_C^1(x_R^2)$	$Z_R^2(x_R^2)$	$Z_C^2(x_R^2)$	–	–	$Z_C^P(x_R^2)$
x_C^2	$Z_R^1(x_C^2)$	$Z_C^1(x_C^2)$	$Z_R^2(x_C^2)$	$Z_C^2(x_C^2)$	–	–	$Z_C^P(x_C^2)$
–	–	–	–	–	–		–
–	–	–	–	–	–		–
x_R^P	$Z_R^1(x_R^P)$	$Z_C^1(x_R^P)$	$Z_R^2(x_R^P)$	$Z_C^2(x_R^P)$	–	–	$Z_C^P(x_R^P)$
x_C^P	$Z_R^1(x_C^P)$	$Z_C^1(x_C^P)$	$Z_R^2(x_C^P)$	$Z_C^2(x_C^P)$	–	–	$Z_C^P(x_C^P)$

Table 1: Pay-off Matrix for the objective functions.

be the solutions for the objective functions Z_R^p and Z_C^p (p=1,2,3,...,P) respectively [8].

Step-2: Determine the corresponding values for every objective at each solution derived and form a pay-off (Table 1).

Also consider (U_R^i, L_R^i) and (U_C^i, L_C^i) be the upper and lower bounds of (Z_R^i, Z_C^i) functions (i=1,2,3,....,P) from Table 1 respectively.

Step-3: Let μ_R^i and μ_C^i be the membership functions of Z_R^i and Z_C^i (i=1,2,3,.....,P) respectively. The decision maker can select the above membership functions in a subjective manner from among the following four types of functions: linear, exponential, hyperbolic and hyperbolic inverse functions.

Different types of membership functions: Let Z_U and Z_L be the upper and lower bounds of the function Z(x) respectively. For the function Z(x), the corresponding membership functions are defined as follows:

(i) (Type-I) Linear membership function:

$$\mu_Z(x)=[Z_U-Z(x)]/[Z_U-Z_L] \quad (21)$$

(ii) (Type-II) Exponential membership function:

$$\mu_Z(x)=a[1-\exp\{-b([Z_U-Z(x)]/[Z_U-Z_L])\}] \quad (22)$$

$$b=\log\left(\frac{a}{a-1}\right) \; and \; a>1$$

(iii) (Type-III) Hyperbolic membership function:

$$\mu_Z(x)=\tanh\{c.\frac{Z_U-Z(x)}{Z_U-Z_L}\}| \quad (23)$$

where c=tanh^{-1}1.

(iv) (Type-IV) Hyperbolic inverse membership function:

$$\mu_Z(x)=e.\tanh^{-1}\{d.\frac{Z_U-Z(x)}{Z_U-Z_L}\} \quad (24)$$

where d= $\tanh(\frac{1}{e})$

Step-4: After determining the membership functions for each of the objective functions, we adopt the maximizing decision proposed by Bellman and Zadeh [19], the resulting problem to be solved is:

Max (min of all μ(x))

subject to x ∈ S where S is the feasible region of problem.

By introducing an auxiliary variable β, the above problem can be transformed into the following equivalent conventional problem.

$$\text{Maximize } \beta, \quad (25)$$

$$\mu_R^i \geq \beta, \quad i=1,2,3,...,P \quad (26)$$

$$\mu_C^i \geq \beta, \quad i=1,2,3,...,P \quad (27)$$

where 0 ≤ β ≤ 1 and x ∈ S (the set of all feasible solutions)

However, with the four types of membership functions given by (21),(22),(23) and (24), the resulting problem is a non-linear programming problem. The constraint like $\mu_z(x) \geq \beta$ takes the following form:

$$Z(x) \leq (1-\gamma).Z_U+\gamma.Z_L \quad (28)$$

Where $\gamma=\beta, \frac{1}{b}\ln\left(\frac{\alpha-\beta}{a}\right), \frac{1}{c}\tanh^{-1}(\beta) \; and \; \frac{1}{d}\tanh(\frac{\beta}{e})$ for Types -I, II, III and IV respectively. The solution of this final problem gives β=β* and the corresponding x=x* is the required optimum solution.

Numerical Experiment

To illustrated proposed model numerically, we a set of input data for two origins and destination, three different conveyance and criteria (Table 2).

$\tilde{a}_1=(31, 33, 35, 37)$, $\tilde{a}_2=(27, 29, 30, 32)$, $\tilde{b}_1=(25, 28, 32, 34)$,

$\tilde{b}_2=(31, 33, 34, 36)$, $\tilde{e}_1=(18, 20, 24, 26)$, $e_2=(23, 26, 27, 28)$,

$\tilde{e}_3=(21, 24, 25, 26)$,

v=12, V_c=5 and

Let the route capacities, r_{ijk}'s are

r_{111}=16; r_{121}=19; r_{211}=14; r_{221}=15; r_{112}=17; r_{122}=19; r_{212}=16; r_{222}=14;

$\tilde{C}_{ijk}^p$	p=1	p=2	p=3	$\tilde{H}_{ijk}$	
$\tilde{C}_{111}^p$	(1, 3, 5, 7)	(2, 5, 6,9)	(6, 9, 10,12)	$\tilde{H}_{111}$	(1, 2, 6, 7)
$\tilde{C}_{121}^p$	(3, 7, 9, 11)	(7, 9, 12,15)	(6, 9, 11,14)	$\tilde{H}_{121}$	(4, 5, 7, 12)
$\tilde{C}_{211}^p$	(3, 6, 9, 11)	(6, 9, 13,17)	(5, 9, 11,13)	$\tilde{H}_{121}$	(3, 7, 9, 12)
$\tilde{C}_{221}^p$	(2, 5, 7, 12)	(5, 8, 11,15)	(5, 7, 9, 12)	$\tilde{H}_{221}$	(5, 9, 11, 13)
$\tilde{C}_{112}^p$	(2, 3, 5, 9)	(1, 5, 9, 10)	(3, 7, 10,13)	$\tilde{H}_{112}$	(3, 5, 9, 12)
$\tilde{C}_{122}^p$	(4, 6, 8, 12)	(4, 7, 8, 14)	(7, 9, 11,15)	$\tilde{H}_{122}$	(6,10,12,15)
$\tilde{C}_{212}^p$	(3, 7, 9, 12)	(2, 7, 10,13)	(6, 9, 13,15)	$\tilde{H}_{212}$	(4, 6, 8, 10)
$\tilde{C}_{222}^p$	(5, 8, 10, 13)	(3, 8, 12, 16)	(4, 7, 9, 12)	$\tilde{H}_{222}$	(6, 9, 11, 14)
$\tilde{C}_{113}^p$	(4, 5, 7, 9)	(4, 8, 12, 14)	(1, 3, 5, 7)	$\tilde{H}_{113}$	(2, 4, 6, 10)
$\tilde{C}_{123}^p$	(5, 9, 12, 14)	(3, 7, 11, 13)	(3, 5, 8, 12)	$\tilde{H}_{123}$	(3, 7, 10, 13)
$\tilde{C}_{213}^p$	(6, 8, 13, 15)	(6, 9, 12, 15)	(2, 5, 9, 10)	$\tilde{H}_{213}$	(3, 5, 9, 13)
$\tilde{C}_{223}^p$	(3, 5, 9, 13)	(5, 9, 12, 15)	(5, 9, 13, 16)	$\tilde{H}_{223}$	(4, 6, 10, 12)

Table 2: Input data for transportation costs.

Obj.	Z_C^1	Z_R^1	Z_C^2	Z_R^2	Z_C^3	Z_R^3
Max.	817.548	1036.145	582.368	817.282	630.437	772.24
Min.	718.0256	936.0466	522	751	467.812	615.5

Table 3: Pay-off table.

$r_{113}=16$; $r_{123}=19$; $r_{213}=14$; $r_{223}=16$;

$\gamma_{111}=0.3$; $\gamma_{121}=0.7$; $\gamma_{211}=0.45$; $\gamma_{221}=0.75$;

$\gamma_{112}=0.25$; $\gamma_{122}=0.5$; $\gamma_{212}=0.25$; $\gamma_{222}=0.35$;

$\gamma_{113}=0.75$; $\gamma_{123}=0.8$; $\gamma_{213}=0.95$; $\gamma_{223}=0.85$;

We now approximate the fuzzy numbers to their nearest intervals, then form the objective functions Z_R^1, Z_C^1, Z_R^2, Z_C^2, Z_R^3, Z_C^3 find their individual solutions x_R^1, x_C^1, x_R^2, x_C^2, x_R^3, x_C^3 minimizing Z_R^1, Z_C^1, Z_R^2, Z_C^2, Z_R^3, and Z_C^3 separately.

The individual minimum and maximum of Z_R^1, Z_C^1, Z_R^2, Z_C^2, Z_R^3, Z_C^3 are shown is pay-off table in Table 2. Now solving the problem (25), with all membership functions as linear, we get Table 3.

Following IFPT, the auxiliary variable β is maximized and the optimal solutions are displayed in Tables 3 and 4 and corresponding decision variable are again if we take the membership functions μ^1_C, μ^2_C and μ^3_R as exponential and others linear, then the solution and the objective functions are shown in Tables 5 and 6, and corresponding decision variable are shown in Table 7.

Particular cases

Case I: Multi-objective and Solid Transportation Problem (i.e., Non-capacitated)

In this case all routes can be used to transport any amount of products. For this situation we remove the route restrictions (r_{ijk}) from the formulation of above problem. For the reduced problem we get the following optimal solution using all the linear membership functions and corresponding decision variable are displayed in Tables 8 and 9.

β*	Z_C^1*	Z_R^1*	Z_C^2*	Z_R^2*	Z_C^3*	Z_R^3*	Z_{av}
0.426653	775.086	988.212	556.612	789.001	561.052	693.894	727.31

Table 4: Optimal auxiliary variable.

Conv-1		Conv-2		Conv-3	
12.418	0	5.903	6.676	11	0
0	6.582	3.676	8.244	0	11.497

Table 5: The transported amounts.

β*	Z_C^1*	Z_R^1*	Z_C^2*	Z_R^2*	Z_C^3*	Z_R^3*	Z_{av}
0.25486E-06	804.934	1021.602	551.658	775.665	615.786	757.824	754.58

Table 6: Optimal auxiliary variable.

β*	Z_C^1*	Z_R^1*	Z_C^2*	Z_R^2*	Z_C^3*	Z_R^3*	Z_{av}
0.394097	766.902	976.196	570.267	793.062	550.606	698.779	725.97

Table 7: The transported amounts.

β*	Z_C^1*	Z_R^1*	Z_C^2*	Z_R^2*	Z_C^3*	Z_R^3*	Z_{av}
0.394097	766.902	976.196	570.267	793.062	550.606	698.779	725.97

Table 8: Optimal auxiliary variable.

Conv-1		Conv-2		Conv-3	
6.938	0	5.009	9.863	14.184	0.004
0	12.061	6.867	2.758	0	8.312

Table 9: The transported amounts.

β*	Z_C^1*	Z_R^1*	Z_C^2*	Z_R^2*	Z_C^3*	Z_R^3*	Z_{av}
0.020707	824.98	981.033	485.622	688.26	570.573	741.198	715.28

Table 10: Non-solid and capacitated for conveyance-2.

13.01	18.989
14.489	13.510
14.265	17.73
13	15

Table 11: The transported amounts.

β*	Z_C^1*	Z_R^1*	Z_C^2*	Z_R^2*	Z_C^3*	Z_R^3*	Z_{av}
0.463242	701.649	893.878	576.515	776.265	426.945	603.929	663.197
0.11801	770.265	956.16	488.064	678.495	619.398	790.023	717.067

Table 12: Non-solid and capacitated for conveyance-2.

3.245	28.754
24.254	3.745

Table 13: The transported amounts.

β*	Z_C^1*	Z_R^1*	Z_C^2*	Z_R^2*	Z_C^3*	Z_R^3*	Z_{av}
0.460702	699.197	890.871	576.90	776.717	426.618	603.33	662.273

Table 14: For not solid and not capacitated for conv-3.

14.717	17.283
12.76	15.24

Table 15: The transported amounts.

Case II: Multi-objective and Capacitated Transportation Problem (i.e., Non-solid)

In this case only one conveyance is used for transportation with some route restrictions. Since every conveyance has some capacity, sometimes one conveyance cannot transport all products. Due to this reason we take individual conveyance for transportation with route restrictions and all membership functions as linear, the first conveyance is not suitable for transportation: Table 10 and corresponding decision variable are for conveyance-2 & conveyance- 3 shown in Tables 11 and 12 respectively.

Case III: Transportation Problem not being solid and capacitated

This case is the combination of above two cases (Case I and II). Taking all the membership functions as linear, we find the following optimal solution for the individual conveyance-2 displayed in Tables 12 and 13) or conveyance 3 in Table 14. But if we take only conveyance-1, then feasible solution does not exist, and corresponding decision variable are for conveyance-2 shown in Tables 12 and 15.

Conclusion

This paper proposes an interactive fuzzy programming approach to find the optimal com-promise solution for an innovative capacitated, multi-objective, solid transportation problem with fuzzy fixed charge, fuzzy partial varying transportation charge and vehicle cost. Here the transportation model is more realistic and flexible in nature. In the proposed problem the coefficient of objective function, resources, demands, and conveyances are taken as trapezoidal fuzzy number and approximated to corresponding nearest interval. Finally the solution procedures have been illustrated by an example. The present formulation and solution procedures can be applied to transportation problems with other and general fuzzy numbers.

References

1. Appa GM (1973) The Transportation problem and its variants. Operational Research Quarterly 24: 79-99.

2. Shell E Distribution of a product by several properties". Direclorale of ManagementAnalysis, Procs 2nd Symposium in Liner Programming, Washington DC 2: 615-642.
3. Grzegorzewski P (2002) Nearest interval approximation of a fuzzy number. Fuzzy Sets and Systems 130: 321-330.
4. Bit AK, Biswal MP, Alam SS (1993) Fuzzy programming approach to multi-objective solid transportation problem. Fuzzy Sets and Systems 57: 183-194.
5. Chanas S, Kuchta D (1996) A concept of the optimal solution of the transportation problem with fuzzy cost coefficients. Fuzzy Sets and System 82: 299-305.
6. Omar MS, Samir AA (2003) A Parametric study on transportation problem under fuzzy environment. The Journal of Fuzzy Mathematics 11: 115-124.
7. Chanas S, Kuchta D (1996) A concept of the optimal solution of the transportation problem with fuzzy cost coefficients. Fuzzy Sets and System 82: 299-305.
8. Sakawa M, Yano H (1989) An Interactive fuzzy satisfying method for generalized multi-objective linear programming problems with fuzzy parameters. Fuzzy Sets and Systems 30: 221-238.
9. Gao SP, Liu SY (2004) Two-phase fuzzy algorithms for multi-objective transportation problem. The Journal of Fuzzy Mathematics 12: 147-155.
10. Shaocheng T (1994) Interval number and fuzzy number linear programming. Fuzzy Sets and Systems, 66: 301-306.
11. Verma R, Biswal MP, Biswas A (1997) Fuzzy programming technique to solve multi-objective transportation problems with some non-linear membership functions. Fuzzy Sets and Systems 91: 37-43.
12. Bit AK (2005) Fuzzy programming with hyperbolic membership functions for multi-objective capacitated solid transportation problem. The Journal of Fuzzy Mathematics 13: 373-385.
13. Bit AK, Biswal MP, Alam SS (1993) Fuzzy programming approach to multi-objective solid transportation problem. Fuzzy Sets and Systems 57: 183-194.
14. Jimenez F, Verdegay JL (1998) Uncertain solid transportation problem. Fuzzy Sets and Systems 100: 45-57.
15. Li L, Lai KK (2000) A fuzzy approach to the multi-objective transportation problem. Computers and Operation Research 27: 43-57.
16. Waiel F, Abd El-Wahed (2001)A multi-objective transportation problem under fuzziness. Fuzzy Sets and Systems 117: 27-33.
17. Zader LA (1965)Fuzzy sets. Information and Control 8: 338-353.
18. Grzegorzewski P (2002) Nearest interval approximation of a fuzzy number. Fuzzy Sets and Systems 130: 321-330.
19. Bellman RE, Zadeh LA (1970) Decision-making in a fuzzy environment. Management science 17: 141-164.