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A Possibilistic Programming Approach for Vehicle Routing Problem with Fuzzy Fleet Capacity (FCVRP)

Seyed Mohammad Seyedhosseini[1]*, Shima Kabirian[1], Ahmad Makui[1] and Nooraddin Dabiri[2]

[1]*Department of Industrial Engineering, Iran University of Science and Technology, Narmak, Tehran, Iran*
[2]*Department of Industrial Engineering, Faculty of Engineering, Golestan University, Golestan, Iran*

Abstract

In this paper we present the vehicle routing problem with fuzzy fleet capacity (FCVRP) to find short routes with the minimal transportation cost. We propose a possibilistic 0-1 linear programming model to deal with such issues. To solve the proposed possibilistic optimization model, we apply an efficient possibilistic method proposed by Parra et al. [1] The FCVRP is NP-hard, which means that an efficient algorithm for solving the problem to optimality is unavailable. To deal with the problem efficiently, we develop a hybrid genetic algorithm (HGA). For the model verification, some small, medium and large-scale test problems are solved by HGA and the results are compared with obtained results from Lingo 9.0. Finally, we do a case study in the Kalleh Company in Amol and publish the results.

Keywords: Fuzzy mathematical programming; Vehicle routing problem; Fuzzy fleet capacity; Possibilistic programming; Hybrid genetic algorithm

Introduction

Transportation has an important role in various domains, such as enterprise, economic and service systems. By this way, researchers are interested in improving the routes, deleting the unnecessary travels and creating the replacement short routes. In addition, many problems, such as traveling salesman problem (TSP), vehicle routing problem (VRP) and the like, are developed by this approach. The Vehicle Routing Problem (VRP) is a widely studied combinatorial optimization problem that was introduced in 1959 by Dantzig and Ramser [2]. VRP is one of the most significant problems in distribution management. Its objective is to find the optimal routes for distributing various shipments [3], such as goods, mail and raw materials. The basic VRP consists of a number of geographically scattered customers, each requiring a specified weight (or volume) of goods to be delivered (or picked up). A fleet of identical vehicles dispatched from a single depot is used to deliver the goods required and once the delivery routes have been completed, the vehicles must return to the depot. Each vehicle can carry a limited weight and only one vehicle is allowed to visit each customer. It is assumed that all problem parameters, such as customer demands and travel times between customers are known with certainty. Solving the problem consists of finding a set of delivery routes which satisfy the above requirements at minimal total cost. In the literature the above described problem is called capacitated VRP (CVRP). In CVRP the total cost equals to the total distance or travel time [4]. Hundreds of papers in world literature have been devoted to this problem. But most of them assume that all information is deterministic, such as customer information, Vehicle information, state of roads information as well as dispatcher information and soon, and the proposed algorithm is only used to solve the deterministic VRP [5]. Actually, in some new systems, it is hard to describe the parameters of the vehicle routing problem as deterministic VRP because there exist much uncertain data such as customer demands, traveling time as well as the set of customers to be visited.

Erbao and Mingyong [5] considered a vehicle routing problem with fuzzy demand. Yanwen and Masatoshi [6] considered a vehicle routing problem where vehicles had finite capacities and demands of customers were uncertain. Yang and Yemei [7] proposed a novel real number encoding method of Particle Swarm Optimization (PSO) to solve the vehicle routing problem with fuzzy demands (FVRP). Erbao and Mingyong [8] considered the open vehicle routing problem with fuzzy demands (OVRPFD). Lian and Xiaoxia [9] considered the vehicle routing problem with fuzzy demands and established a fuzzy constrained programming mathematical model based on possibility theory. Changshi and Fuhua [10] proposed the vehicle routing problem with fuzzy demand at nodes. Tang et al. [11] proposed the vehicle routing problem with fuzzy time windows. They applied membership functions to characterize the service level issues associated with time window violation in a vehicle routing problem and proposed VRPFTW. Gupta et al. [12] considered multi objective fuzzy vehicle routing problem. Zheng and Liu [13] considered the vehicle routing problem in which the travel times are assumed to be fuzzy variables.

According to the Table 1, no studies that consider the capacity of fleets as fuzzy number have been done. According to statistics published by the freight companies, any vehicle regardless of its capacity can load 500 kg extra load. But with increasing loading, driver satisfaction also decreases. On the other hand, it is not convenient for driver, if he loads less than a certain amount of this. Thus, according to Figure 1, the capacity of the vehicle can be considered as fuzzy numbers with triangular membership function.

Possibility theory was proposed by Zadeh in 1978 and developed by Dubois and Prade in 1988. Since the 1980s, the possibility theory has become more and more important in the decision field and several methods have been developed to solve possibilistic programming problems [1].

Possibilistic Linear Programming (PLP) problem is a linear programming with imprecise coefficients restricted by possibilistic distribution. Possibilistic decision making models have provided an

***Corresponding author:** Seyed Mohammad Seyedhosseini, Department of Industrial Engineering, Iran University of Science and Technology, Narmak, Tehran, Iran, Tel: +98 9121261744; Fax: +98 21 77240482; E-mail: seyedhosseini@iust.ac.ir

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Fuzzy parameter	References	Category	Objective Function	Solving approach
Fuzzy demand	Erbao and Mingyong (2009)	VRPFD	Minimization of travel cost	Stochastic simulation and differential evolution algorithm
	Yanwen and Masatoshi (2010)	VRPFD	Minimization of travel cost	Two-stage possibilistic programming and ACSO
	Yang and Yemei (2010)	VRPFD	Minimization of travel cost	Fuzzy constrained programming and PSO algorithm
	Cao and Lai (2010)	OVRPFD	Minimization of travel cost	Fuzzy simulation and differential evolution algorithm
	Lian and Xiaoxia (2011)	VRPFD	Minimization of travel cost	Fuzzy simulation and differential evolution algorithm
	Changshi and Fuhua (2010)	VRPFD	Minimization of travel cost and the number of used vehicle	Possibilistic programming and improved Sweeping algorithm
Fuzzy time window	Tang et al. (2009)	VRPTW	Minimization of travel distance and maximization the service level of the suppliers to customers	Two-stage algorithm
	Gupta et al. (2010)	VRPTW	Minimization of fleet size, distance and waiting time and maximization of customer satisfaction grade	Fuzzy genetic algorithm
	Zheng and Liu (2006)	VRPTW	Minimization of travel distance	Fuzzy simulation and genetic algorithm

Table 1: The State-of-the art.

important aspect in handling practical decision making problems. Negoita et al. [14] were the first who formulated the possibilistic linear programming [15].

This paper proposes a possibilistic linear programming for vehicle routing problem with fuzzy capacity where the right hand coefficient, is a triangular fuzzy number.

Fuzzy CVRP Model Formulation

The CVRP can be formulated as follows. A customer is an entity that has a certain demand and therefore the presence of a vehicle, a unit that can move between customers and the depot. The fleet is defined as the total group of vehicles. Moving a vehicle between the depot and the customers come with a certain cost. A route is a sequence of visited customers by a certain vehicle, starting and ending at a depot. The goal of the vehicle routing problem is to serve all customers, minimizing the total cost of the routes of all vehicles. Let us consider that V= $\{v_0,v_1,v_2,\ldots,v_n\}$ is a set of n+1(n≥1) vertices. We distinguish the depot v_0 and exactly n customers $\{v_1,v_2,\ldots,v_n\}$. E= $\{(v_i,v_j) \mid 0\le i , j \le n , i\neq j\}$ is the set of |V|*(|V|-1) edges(arcs) between the vertices, called the roads. D= (d_{ij}) is a matrix, where $d_{ij}\ge 0$ is the distance corresponding to edge (v_i,v_j); d_{ii} is always equal to 0 and $d_{ij}=d_{ji}$. C is the capacity of vehicle that is a fuzzy number which have a triangular membership function. We assume that, if the vehicle travels from node i to node j, $x_{ij}=1$ and otherwise $x_{ij}=0$ and if the node i, visited with the vehicle k, $y_{ik}=1$ and otherwise $y_{ik}=0$.

The problem can be formulated as the following:

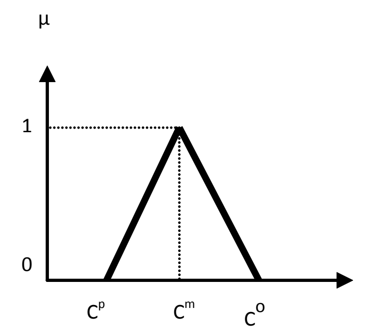

Figure 1: Membership functions of triangular fuzzy number.

$$\min \sum_{i=0}^{n}\sum_{j=0}^{n} d_{ij}x_{ij} \quad (1)$$

$$\sum_{i=0}^{n} x_{ij} = 1 \quad (j=1,..,n) \quad (2)$$

$$\sum_{i=0}^{n} x_{ij} = m \quad (j=0) \quad (3)$$

$$\sum_{j=0}^{n} x_{ij} = 1 \quad (i=1,\ldots,n) \quad (4)$$

$$\sum_{j=0}^{n} x_{ij} = m \quad (i=0) \quad (5) \qquad (1)$$

$$\sum_{i=o}^{n} x_{ip} - \sum_{j=0}^{n} x_{pj} = 0 \quad (p=0,\ldots,n) \quad (6)$$

$$\sum_{i=o}^{n} q_i y_{ik} \le \tilde{C} \quad (k=1,\ldots,m) \quad (7)$$

$$\sum_{i,j\in S} x_{ij} \le |S|-1 \quad (S\subset\{1,\ldots,n\}, |S|\ge 2) \quad (8)$$

$$x_{ij} \in \{0,1\} \quad (i=0,\ldots,n; j=0,\ldots,n) \quad (9)$$

$$y_{ik} \in \{0,1\} \quad (i=0,\ldots,n; j=0,\ldots,n) \quad (10)$$

Objective function (1) states that the total distance is to be minimized. Constraints (2) and (4) ensure that each demand node is served by exactly one vehicle and constraints (3) and (5) guarantee that each vehicle starts and ends at the distribution depot. Route continuity is represented by (6), i.e. if a vehicle enters in a demand node, it must exit from that node. Constraint (7) is the vehicle capacity constraints. Constraint (8) eliminates the sub-tour and finally, Constraints (9) and (10) define the nature of the decision variable.

The Proposed Solution Method

Several methods have been developed in the literature to deal with the possibilistic models involving the imprecise coefficients [1,16,17]. Here, we applying an efficient possibilistic method proposed by Parra et al. [1], to convert the proposed possibilistic 0-1 programming model into an equivalent auxiliary crisp model because of its several advantages as follows:

- This method is computationally efficient to solve fuzzy linear problems because it both preserves its linearity and do not increase the number of objective functions and inequality constraints.
- This method relies on the [18] general ranking method which can be applied to different kinds of membership functions such

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as triangular, trapezoidal and nonlinear ones in both symmetric and asymmetric forms.

- This method is based on the strong mathematical concepts such as expected interval and expected value of fuzzy numbers.

Assume that $\tilde{c}$ is a triangular fuzzy number, the following equation can be define as the membership function of $\tilde{c}$:

$$\mu_{\tilde{c}} = \begin{cases} f_c(x) = \dfrac{x - c^p}{c^m - c^p} & \text{if } c^p \le x \le c^m \\ 1 & \text{if } x = c^m \\ g_c(x) = \dfrac{c^o - x}{c^o - c^m} & \text{if } c^m \le x \le c^o \\ 0 & \text{if } x \le c^p \text{ or } x \ge c^o \end{cases} \quad (2)$$

According to (Jimenez, 1996) [18], the expected interval (EI) and expected value (EV) of triangular fuzzy number $\tilde{c}$ can be define as follow:

$$EI(\tilde{C}) = \left[E_1^c, E_2^c\right] = \left[\int_0^1 f_c^{-1}(x)dx, \int_0^1 g_c^{-1}(x)dx\right] = \left[\frac{1}{2}(c^p + c^m), \frac{1}{2}(c^m + c^o)\right] \quad (3)$$

$$EV(\tilde{C}) = \frac{E_1^C + E_2^C}{2} = \frac{c^p + 2c^m + c^o}{4} \quad (4)$$

It is noted that the same equations can be used for a trapezoidal fuzzy number. Moreover, according to the ranking method of Jimenez [18] for any pair of fuzzy numbers $\tilde{a}$ and $\tilde{b}$, the degree in which $\tilde{a}$ is bigger than $\tilde{b}$ is define as follows:

$$\mu_M(\tilde{a},\tilde{b}) = \begin{cases} 0 & \text{if } E_2^a - E_1^b \prec 0 \\ \dfrac{E_2^a - E_1^b}{E_2^a - E_1^b - (E_1^a - E_2^b)} & \text{if } 0 \in \left[E_1^a - E_2^b, E_2^a - E_1^b\right] \\ 1 & \text{if } E_1^a - E_2^b \succ 0 \end{cases} \quad (5)$$

When $\mu_M(\tilde{a},\tilde{b}) \ge \alpha$ it will be said that $\tilde{a}$ is bigger than or equal to, $\tilde{b}$ at least in degree α and it will be represented as

$$\tilde{a} \ge_\alpha \tilde{b} \quad (6)$$

Also, according to the definition of fuzzy equations in Parra et al. [1], for any pair of fuzzy numbers $\tilde{a}$ and $\tilde{b}$, it will be said that $\tilde{a}$ is indifferent (equal) to $\tilde{b}$ in degree of α if the following relationships hold simultaneously:

$$\tilde{a} \ge_{\frac{\alpha}{2}} \tilde{b}, \quad \tilde{a} \le_{\frac{\alpha}{2}} \tilde{b} \quad (7)$$

Now, we consider the following constraint with fuzzy parameters:

$$\tilde{a}_i x \ge \tilde{b}_i \quad i=1,\dots,m \quad (8)$$

As mentioned by Jimenez et al. [19], a decision vector $x \in R^n$ is feasible in degree if $\min_{i=1,\dots,m}\left\{\mu_M(\tilde{a}_i x, \tilde{b}_i)\right\} = \alpha$, according to (5) and (6), the equation $\tilde{a}_i x \ge \tilde{b}_i$ is equivalent to the following ones, respectively:

$$\frac{E_2^{a_i x} - E_1^{b_i}}{E_2^{a_i x} - E_1^{a_i x} + E_2^{b_i} - E_1^{b_i})} \ge \alpha \quad i=1,\dots,m \quad (9)$$

This equation can be rewritten as follows:

$$\left[(1-\alpha)E_2^{a_i} + \alpha E_1^{a_i}\right] X \ge \alpha E_2^{b_i} + (1-\alpha)E_1^{b_i} \quad i=1,\dots,m \quad (10)$$

According to above descriptions, the FCVRP model can be formulated as follows:

$$\begin{aligned} &\min \sum_{i=0}^{n}\sum_{j=0}^{n} d_{ij}x_{ij} \\ &\sum_{i=0}^{n} x_{ij} = 1\left(j = 1,..,n\right) \\ &\sum_{i=0}^{n} x_{ij} = m\left(j = 0\right) \\ &\sum_{j=0}^{n} x_{ij} = 1\left(i = 1,\dots,n\right) \\ &\sum_{j=0}^{n} x_{ij} = m\left(i = 0\right) \\ &\sum_{i=o}^{n} x_{ip} - \sum_{j=0}^{n} x_{pj} = 0\left(p = 0,\dots,n\right) \\ &\sum_{i=o}^{n} q_i y_{ik} \le \alpha\left(\frac{c^p + c^m}{2}\right) + (1-\alpha)\left(\frac{c^o + c^m}{2}\right) \quad (k=1,\dots,m) \\ &\sum_{i,j \in S} x_{ij} \le |S| - 1\left(S \subset \{1,\dots,n\}, |S| \ge 2\right) \\ &x_{ij} \in \{0,1\}\left(i = 0,\dots,n; j = 0,\dots,n\right) \\ &y_{ik} \in \{0,1\}\left(i = 0,\dots,n; j = 0,\dots,n\right) \end{aligned} \quad (11)$$

Hybrid Genetic Algorithm

As it is an NP-hard problem, the instances with a large number of customers and vehicles cannot be solved in optimality within reasonable time. Therefore, the metaheuristics algorithms used to solve these problems [20]. These algorithms can solve these problems

Figure 2: Swap Mutation.

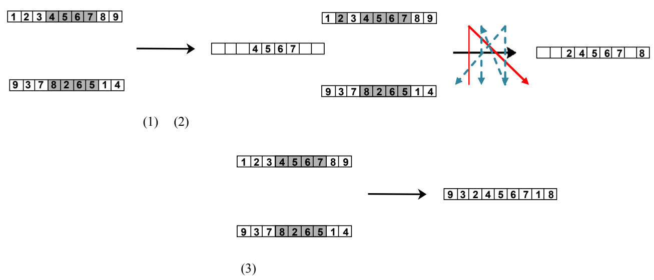

Figure 3: PMX Crossover

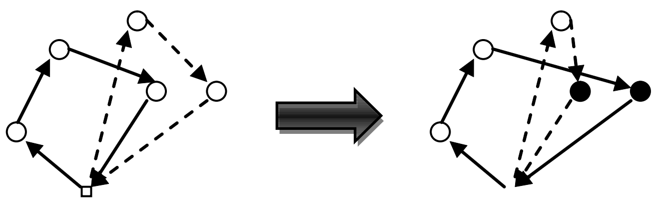

Figure 4: 2-opt operator

Test code	No. of customers	No. of vehicles	α -level	Lingo 9.0		HGA		Gap (%)	Saving (%)
				Best solution	Time Run(Sec)	Best solution	Time Run(Sec)		
E-n13-k4	13	4	1	277	280	277	8	0	
			0.8	277	280	277	8	0	
			0.6	268	280	268	8	0	
			0.5	247	280	247	8	0	
			0.4	240	280	240	8	0	2.9
			0.2	237	280	237	8	0	4.1
			0	230	280	230	8	0	6.9
P-n16-k8	16	8	1	Inf	-	Inf	-	-	
			0.8	Inf	-	Inf	-	-	
			0.6	461.32	409	460.83	18	0.1	
			0.5	450	409	450.45	18	0.1	
			0.4	451.34	409	451.875	18	0.1	0.4
			0.2	440.37	409	440.33	18	0	2.2
			0	428.56	409	428.63	18	0	4.7
P-n19-k2	19	2	1	Inf	-	Inf	-	-	
			0.8	Inf	-	Inf	-	-	
			0.6	223.39	339	231.48	19	0.1	
			0.5	212	339	212.31	19	0.1	
			0.4	196.65	339	196.65	19	0	7.3
			0.2	196.65	339	196.65	19	0	7.3
			0	196.65	339	196.65	19	0	7.3
P-n20-k2	20	2	1	Inf	-	Inf	-	-	
			0.8	Inf	-	Inf	-	-	
			0.6	218.31	370	222.22	22	1.8	
			0.5	216	370	217.39	22	0.6	
			0.4	209.12	370	212.31	22	1.4	1.7
			0.2	209.12	370	212.31	22	1.4	1.7
			0	208.20	370	208.33	22	0	3.6
P-n21-k2	21	2	1	Inf	-	Inf	-	-	
			0.8	212	563	215.98	26	1.8	
			0.6	215	563	215.05	26	0	
			0.5	211	563	212.77	26	0.8	
			0.4	208.29	563	208.33	26	0	1.3
			0.2	208.29	563	208.33	26	0	1.3
			0	204	563	204.1	26	0	3.3
P-n22-k2	22	2	1	Inf	-	Inf	-	-	
			0.8	Inf	-	Inf	-	-	
			0.6	222.22	806	222.62	28	0.2	
			0.5	217	806	217.82	28	0.4	
			0.4	212.31	806	210.53	28	0.1	2.6
			0.2	212.31	806	210.53	28	0.1	2.6
			0	207.13	806	207.81	28	0	3.8
E-n22-k4	22	4	1	Inf	-	Inf	-	-	
			0.8	Inf	-	Inf	-	-	
			0.6	394.75	10843	400	32	1	
			0.5	375	10843	375	32	0	
			0.4	369	10843	370.37	32	0.4	
			0.2	360.54	10843	363.64	32	0.9	
			0	355	10843	357.14	32	0.6	
E-n23-k3	23	3	1	Inf	-	Inf	-	-	-
			0.8	569.89	-	573.39	38	0.6	-
			0.6	568.57	946	568.57	38	0	
			0.5	569	946	569	38	0	
			0.4	564.08	946	567.85	38	0.6	0.8
			0.2	564.08	946	567.85	38	0.6	0.8
			0	563.81	946	563.81	38	0	1

Table 2: Comparison of the performance of the proposed HGA for small and medium scale of problem.

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Test code	No. of customers/ vehicles	α -level	Lingo 9.0		HGA		The best solution ever found	Gap (%)	Saving
			Lower Bound	Time Run(Sec)	Best solution	Time Run(Sec)			
P-n23-k9	23/9	1	Inf	-	Inf	-	-	-	
		0.8	Inf	-	Inf	-	-	-	
		0.6	Inf	-	Inf	-	-	-	
		0.5	416.95	1800	534.76	47	529	1.08	
		0.4	416.95	1800	510.64	47	-	-	3.5
		0.2	361.85	1800	510.64	47	-	-	3.5
		0	340.23	1800	504.49	47	-	-	4.7
B-n31-k5	31/5	1	498.54	1800	709.22	-	-	-	
		0.8	498.51	1800	692.04	-	-	-	
		0.6	504.14	1800	692.04	82	-	-	
		0.5	522.35	1800	689.65	82	672	2.6	
		0.4	503.16	1800	632.27	82	-	-	5.9
		0.2	484.17	1800	617.28	82	-	-	8.1
		0	422.88	1800	4613.5	82	-	-	8.7
A-n33-k6	33/6	1	Inf	1800	Inf	-	-	-	
		0.8	570.52	1800	813	-	-	-	
		0.6	572.07	1800	769	112	-	-	
		0.5	593.16	1800	751.88	112	742	1.3	
		0.4	570.95	1800	735.294	112	-	-	1
		0.2	569.11	1800	724.64	112	-	-	2.4
		0	535.65	1800	680.27	112	-	-	8.3
A-n37-k6	37/6	1	615.13	1800	1008	172	-	-	
		0.8	610.59	1800	980.39	172	-	-	
		0.6	612	1800	972.76	172	-	-	
		0.5	613.63	1800	949.67	172	949	0	
		0.4	568.13	1800	906.32	172	-	-	4.5
		0.2	563.76	1800	865.59	172	-	-	8.7
		0	569.68	1800	857.63	172	-	-	9.6
A-n38-k5	38/5	1	Inf	1800	Inf	-	-	-	
		0.8	Inf	1800	Inf	-	-	-	
		0.6	524.84	1800	833.33	144	-	-	
		0.5	525.20	1800	769.23	144	730	5.37	
		0.4	530.96	1800	751.88	144	-	-	-
		0.2	526.33	1800	740.74	144	-	-	-
		0	524.91	1800	740.74	144	-	-	-
A-n44-k6	44/6	1	Inf	1800	Inf	-	-	-	
		0.8	Inf	1800	Inf	-	-	-	
		0.6	864	1800	1023.57	223	-	-	
		0.5	751.63	1800	980.39	223	934	5	
		0.4	728.53	1800	915.45	223	-	-	1.9
		0.2	712.14	1800	909.91	223	-	-	2.7
		0	712.14	1800	909.91	223	-	-	2.7
B-n50-k7	50/7	1	672.83	1800	909.91	-	-	-	
		0.8	653.85	1800	866.29	334	-	-	
		0.6	625.14	1800	833.33	334	-	-	
		0.5	598.16	1800	813.33	334	741	9.7	
		0.4	538.75	1800	741.28	334	-	-	3.6
		0.2	512.24	1800	704.22	334	-	-	4.9
		0	473.46	1800	689.65	334	-	-	6.9
P-n50-k10	50/10	1	Inf	-	Inf	-	-	-	
		0.8	Inf	-	Inf	-	-	-	
		0.6	673.48	1800	787.40	350	-	-	
		0.5	640.65	1800	769.23	350	696	11	
		0.4	555.73	1800	680.27	350	-	-	2.2
		0.2	541.51	1800	671.14	350	-	-	3.5
		0	546.13	1800	666.67	350	-	-	4.2

Table 3: Comparison of the performance of the proposed HGA for large-scale of problem.

with a good solution in reasonable time. The different criteria used for classification metaheuristics algorithms, such as solution-based and population-based, inspiration of nature and without the inspiration of nature and Genetic algorithms like ant colony, bee colony and inspired by nature and will begin work on an initial population of solutions [21].

Genetic algorithm (GA) developed by John Holland in the 1960s, is a stochastic optimization technique. Similar to other artificial intelligence heuristics like SA (Simulated Annealing) and TS (Tabu search), GA can avoid getting trapped in a local optimum by the aid of one of the genetic operations called mutation.

The idea of genetic algorithm based on evolution in nature. GA starts with an initial set of random solutions, called population. Each solution in the population is called a chromosome, which represents a point in the search space. The chromosomes evolve through successive iterations, called generations. During each generation, the chromosomes are evaluated using some measures of fitness. The fitter the chromosomes, the higher the probabilities of being selected to perform the genetic operations, including crossover and mutation. In the crossover phase, the GA attempts to exchange portions of two parents, that is, two chromosomes in the population to generate an offspring. The crossover operation speeds up the process to reach better solutions. In the mutation phase, the mutation operation maintains the diversity in the population to avoid being trapped in a local optimum. A new generation is formed by selecting some parents and some offspring according to their fitness values, and by rejecting others to keep the population size constant. After the predetermined number of generations is performed, the algorithm converges to the best chromosome, which hopefully represents the optimal solution or may be a near-optimal solution of the problem. The mutation and crossover operation used in this algorithm is shown respectively in Figure 2 and Figure 3.

Zafari et al. [22] in "A hybrid genetic algorithm for solving the vehicle routing problem" selected 110 test problems from (http://branchandcut.org/VRP/data/) and solved them with HGA. In these test problems, the range of the number of customer was 12 to 149. The comparison of the best solution of Lingo and the proposed algorithm showed that the HGA algorithm obtained the best solution of lingo exactly in 85 cases from 110 test problems. Extensive computational tests on standard instances from the literature confirmed the effectiveness of the presented approach. So to solve the proposed FCVRP model, the HGA algorithm is applied.

A simple GA may not perform well in this situation. Therefore, the GA developed in this paper is hybridized with one heuristic to improve the solution further. The 2-opt local search heuristic is generally used to improve the solutions of the hard optimization problems. However, it increases the computational time because every two swaps are examined. If a new solution generated is better than the original one, or parent, in terms of quality, it will replace and become the parent. All two swaps are examined again until there is no further improvement in the parent [23]. The 2-opt exchange operation is shown in Figure 4, in which the edge (i, i + 1) and (j, j + 1) are replaced by edge (i, j) and (i +1, j +1), thus reversing the direction of customers between i + 1 and j [24].

Numerical Examples

In this section, the proposed algorithm is tested on three categories of VRP problems [25]. The first group includes problems that Lingo optimization software can reach optimum solution in a reasonable time. In the second category, the Lingo solution is optimized but the time to solve the problem is not appropriate and finally, the third category, which includes problems Lingo not able to solve them. It is important to note that this algorithm has only been tested 10 times for each problem and the best answer is shown. Then, to evaluate the quality of the solutions, the model is solved by the lingo 9 and the results of this have been compared with the results of the presented metaheuristics algorithm. For this purpose, the relative gap between the best solutions obtained from the Lingo (Z (Lingo)) with the best solutions obtained from the proposed HGA (Z (HGA)) is computed by: 100[Z (HGA)-Z (Lingo)]/Z (Lingo).

The comparison of Lingo with the proposed algorithm shows that our proposed algorithm can obtain approximately an optimal solution in less time than Lingo as shown in Table 2 and 3. The average gap between the optimal and the HGA solutions is 0.32% showing the efficiency of the proposed HGA. Furthermore, increasing the size of

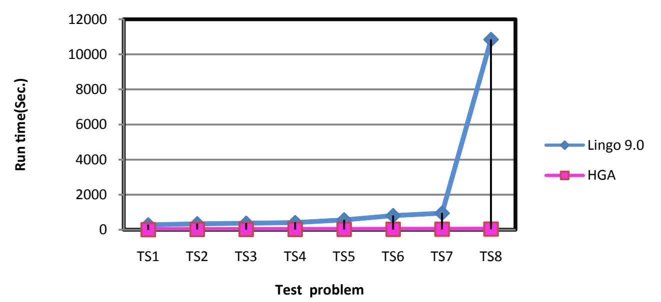

Figure 5: Comparison of the time spent in solving problem with Lingo 9.0 and HGA.

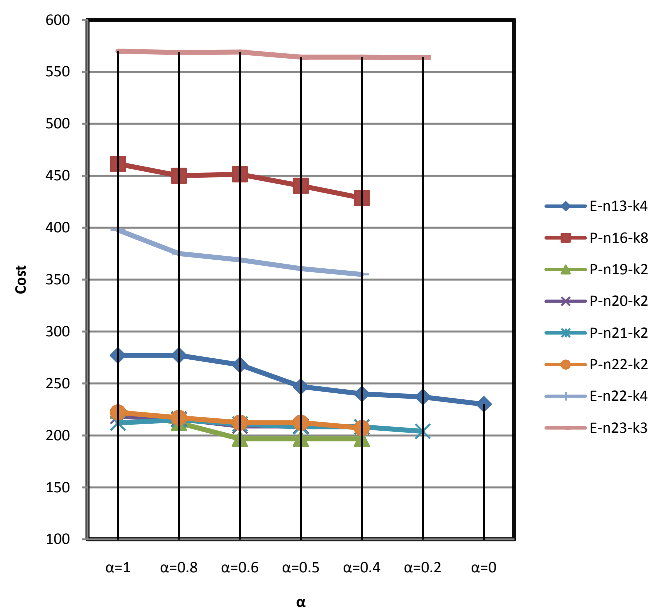

Figure 6: Saving in travel costs.

Name of vehicle	Number of vehicle	Capacity of vehicle(Ton)
Ten wheel drive Renault	5	8.5
Ten wheel Tkv	5	8.5

Table 4: Vehicles.

Citation: Seyedhosseini SM, Kabirian S, Makui A, Dabiri N (2013) A Possibilistic Programming Approach for Vehicle Routing Problem with Fuzzy Fleet Capacity (FCVRP). Ind Eng Manage 1: 101. doi:10.4172/2169-0316.1000101

Customer	Rasht	Esfahan	Shiraz	Karaj	Tehran	Sorkhrood	Tonekabon	Sari	Babol	Amol	Arak
Demand(kilo)	5635	7371	6102	7173	37949	918	882	2528	1245	1311	1486
Customer	Gorgan	Yazd	Mashhad	Kerman	Ghazvin	Booshehr	Ahvaz	Bandar-abbas	Tabriz	Hamedan	
Demand(kilo)	1290	1798	2993	2293	3607	4147	8521	4855	3900	1835	

Table 5: Demand of the customers.

No. of customers	No. of vehicles	α -level	Lingo 9.0		HGA	HGA		Saving (%)
			Lower Bound	Time Run(Sec)	Best solution	Best solution	Time Run(Sec)	
23	10	1	Inf	-	Inf	Inf	-	
		0.8	Inf	-	Inf	Inf	-	
		0.6	Inf	-	Inf	Inf	-	
		0.5	11217	1800	14471.78	14471.78	50	
		0.4	9381	1800	14285.71	14285.71	50	1.2
		0.2	7015	1800	14084.5	14084.5	50	2.7
		0	7015	1800	14084.5	14084.5	50	2.7

Table 6: Results of the case study.

the problem increases the solution time of Lingo exponentially while it does not tangible effect on the solution time of the proposed algorithm as shown in Figure 5. Problem solving results in small, medium and large sizes is shown respectively, in table 2 and 3.

According to table 2, the saving of transportation cost for small and medium scale of problem is computed by: 100[Z(HGA)-Z(Lingo$\alpha_{=0.5}$)]/Z(Lingo$\alpha_{=0.5}$) and according to table 3, the saving of transportation cost for large scale of problem is computed by:

100[Z(HGA)-Z$\alpha_{=0.5}$ (The best solution ever found)]/Z$\alpha_{=0.5}$ (The best solution ever found).

The comparison of best solution of both Lingo and HGA shows that with decreasing α, also the transportation cost decrease as shown in Figure 6. This means that with considering the acceptable extra loading, we have a saving in travel cost.

Case Study

In order to provide a better understanding of the model, the Kalleh Company's data of 2011 is used. The Company situated at Amol city and their production activities has started since 1983. Products of this company are divided into three categories: dairy products, sausages and salami, and prepared foods. For studying the proposed model in this company, we consider the customers and the vehicles that assigned to their prepared foods. The company, having 10 trucks, will serve 23 cities in the Iran. These data are shown separately in tables 4 and 5.

As we mention before, any vehicle regardless of its capacity can load 500 kg extra load. But with increasing loading, driver satisfaction also decreases. On the other hand, it is not convenient for driver, if he loads less than a certain amount of this. So the capacity of the vehicle can be considered as fuzzy numbers with triangular membership function.

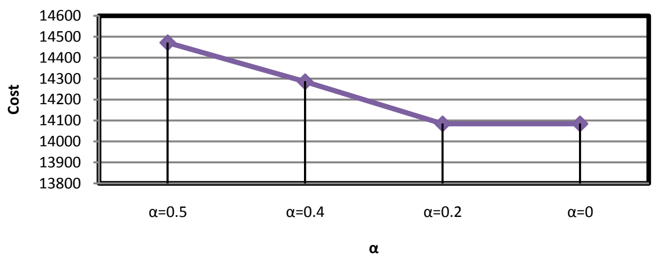

Figure 7: Saving in travel costs.

According to table 6, the comparison of best solution of both Lingo and HGA shows that with decreasing α, (increasing loading) also the transportation cost decrease. This means that with considering the acceptable extra loading, we have a saving in travel cost (Figure 7).

Conclusion

This paper has presented a vehicle routing problem with fuzzy fleet capacity (FCVRP) in which the right hand coefficient, is a triangular fuzzy number. According to statistics published by the freight companies, any vehicle regardless of its capacity can load 500 kg extra load. But with increasing loading, driver satisfaction also decreases. On the other hand, it is not convenient for driver, if he loads less than a certain amount of this. Thus, the capacity of the vehicle can be considered as fuzzy numbers with triangular membership function. We proposed a possibilistic 0-1 linear programming model to deal with this problem. To solve the proposed possibilistic optimization model, we apply an efficient possibilistic method proposed by Parra et al. [1]. In addition we developed a hybrid genetic algorithm (HGA) to find the short routes with the minimum travel cost. To verify the solution technique, 16 test problems have been solved by the Lingo 9.0 software and the related results obtained by the proposed hybrid genetic algorithm (HGA) have been very efficient approaching to the optimal solution. For small sizes, the average gap between the proposed HGA and Lingo solutions has been equal to 0.32 showing an acceptable result. The comparison of best solution of both Lingo and HGA shows that with decreasing α, (increasing loading) also the transportation cost decrease. This means that with considering the acceptable extra loading, we have a saving in travel costs.

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