

A Original Solution To The 4 Colours Theorem

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Abstract

Definition of theorem: On a politic map, the neighbour countries can not to take the same colour because they could seem the same country. When the frontier between countries is a point, we must not consider it as a frontier. This is possible with three and five colours, it is proved, but with four at this moment lack an evident proof, without pc.

There are infinite maps where the solution must to run, also inside of them it must to solve the relation between infinite and only four colours. For that I transform politic maps into maps in the plane graph, then I use their polygons to create other new polygons, with a particular centre.

Then the group of new polygons form a structure, which distribute all points in two independent substructures, the Centres and the Crowns. This particular centre is the common vertex of some polygons belonging to plane graph, and they form a new polygon. The points which surround the centre constitute a barrier that impede the direct relation between the centre and other points. I call crown to this barrier. Each polygon is linked with the previous, so it achieves shape of spiral. Also it achieves independence among centres, and the points of the same crown do not entwine, their relations are consecutives, two by two.

To specify the new structure group points in two substructures, the Centres and the Crowns, one colour goes to the centres, and three to the crowns.

On the crown there is a process of colours run on a finite number of points chosen by triangulations, which impose a Stopping Condition. The triangulation happens when two or more points with two different colours have a common neighbour, then this point must to take the third colour.

On each process after last triangulation happen always stopping condition, it mean that neighbour points without colours have two possibilities, and the rest three, which guarantee the resolution.

The global outcome is a Big Crown biggest after each process, whose internal points and their links do not influence on the following points.

There are two graphics files, Formation of structure and Plans, where I change the three colours by three shapes: triangle, circle and square. I recommend to see the two graphics files consecutively.

Keywords: Centre; Crown; Triangulation; Process; Basic polygon; Spiral shape; Polygon with external centre; Structure; Substructure; Chain; Block; Stopping condition; Big crown; Process of election

Conversion of the Politic Maps in Plane Graphs

Key: The final objective are the polygons of the plane graph. [1,2]

Move original map to the graph theory, it mean to convert countries in points, and their frontiers in links. These links do not cross among themselves. When the between two countries is a point, it is not considered like a frontier, then it is not a link. So way arise polygons with more than three sides in the plane graph.

Basic Polygon

Key: In the plane graph any point is common vertex of some number of polygons which surround it. In parallel on a politic map any country is surrounded for a number of countries, although their frontier can be a point. Most important matter on it is those countries impede it direct relations with another.

Key: Basic Polygon is based on an internal centre and a crown.

Each new polygon is formed from the choice of the central point. The points that que surround the centre belong to all polygons which the central point belongs too. They represent a physical barrier that I call crown. To visualize best the matter and to explain better the centre have a colour and the links between crown points another, for example blue and red respectively. So way it helps too to distinguish the two substructures on the below section [3].

Structure

Key: It represents all points and their links in two substructures. One represents the Centres, where each centre uses the same colour, and the other one represents the points of the crowns, they are the rest and use other three colours.

Formation depend on two ideas

1. Each centre of a new polygon is opposite to the immediate previous, and at least another previous centre. This opposition mean they meet from a distance of a point, which that the polygons share at least one point of their crown.

2. Spiral shape. From the first polygon, I create the next surrounding to the previous, and the centres are numbered in building order. If we join the centres with a line, then it appears a spiral. So it gets a compact structure, whose polygons with external centre are necessary.

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Polygons with External Centre

Because of structure appear hollows among the polygons, but really they are polygons too, have one or more polygons of the plane graph. And they join form another new polygon but with external centre. If we notice their links, we can check the definition of centre given 1st. section, the point which is common vertex of some polygons. In each hollow the centre is now in the crown, so it has an external centre. I mark these polygons on the graphic structure to pay attention in the next process of election, because all the triangulations affect to the external centre (explain it in the next section). Also those triangulations are linked, because if it happens one at that time happen all belongs same polygon, until to complete it.

As we can observe into the graphic structure, thanks to colour the centres in blue, link crowns in red, and polygons with external centre with a red dot, it gets distinction between substructures, centres and crowns (Figures 1-30).

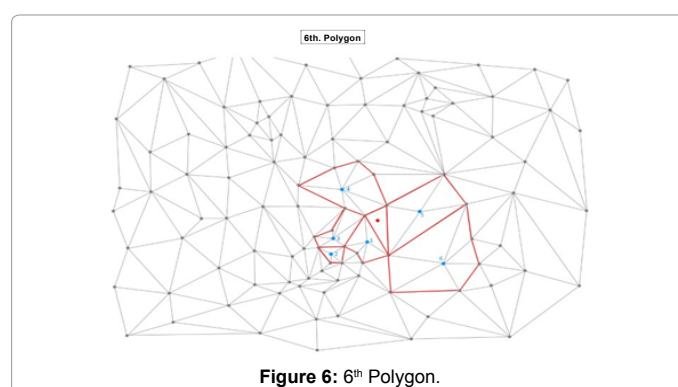
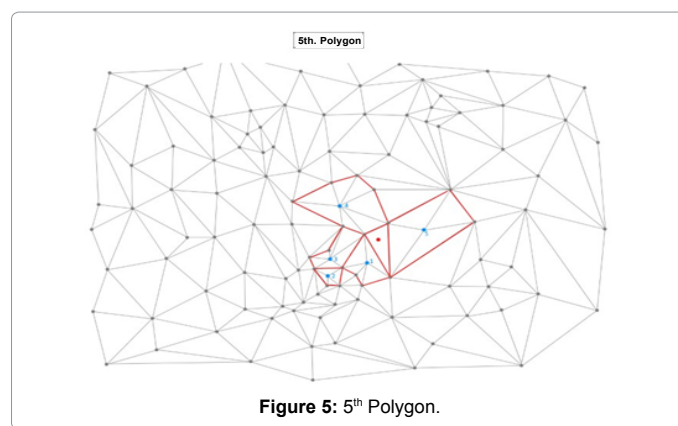
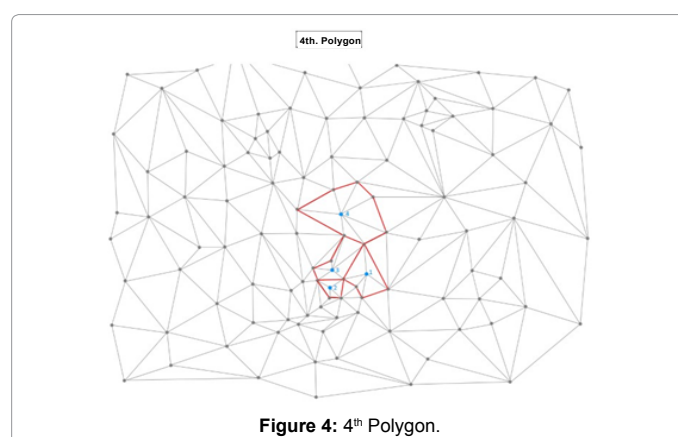
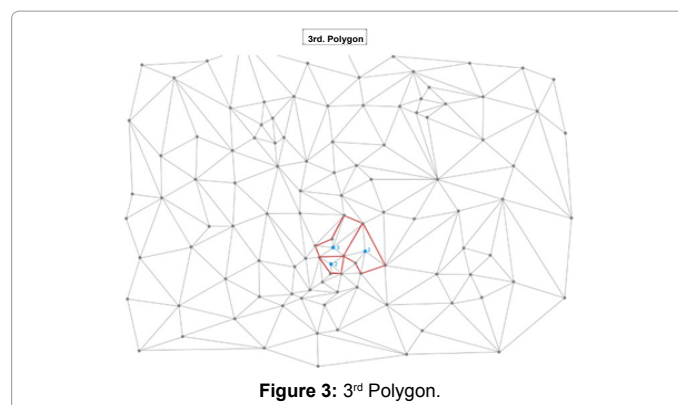
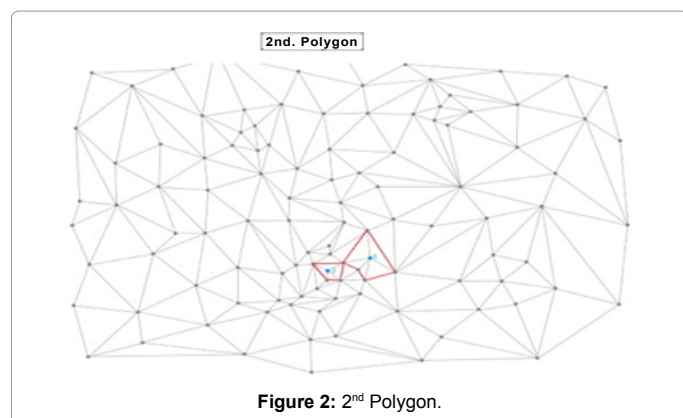
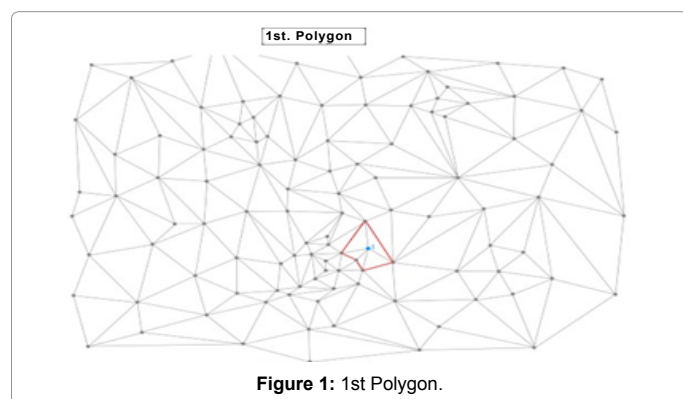
Process of Colours Election: Triangulation

Key: At the end of each process exist a stopping condition. This mean all points which are linked with coloured points, have two possibilities of choice, and the rest keep their three possible colours.

Key: The number of points of each process can vary, but they are finite and finish by triangulations, which mean the stopping condition comes true.

See the Plans file. A process more a triangulation, more a deletion, require a Plan [4].

The Triangulation: Two or more points can be neighbour two by two or cannot be neighbour but if they have a common neighbour:



7th. Polygon

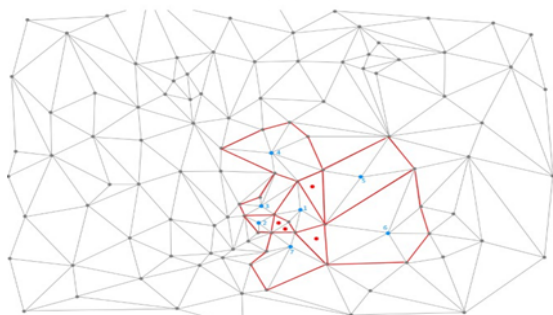


Figure 7: 7th Polygon.

11th. Polygon

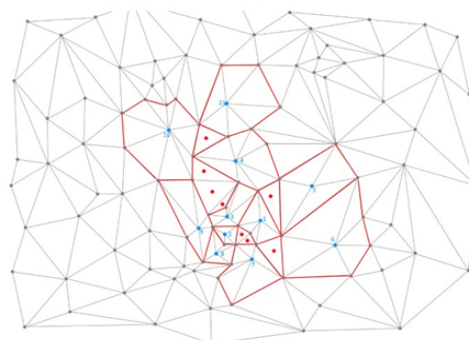


Figure 11: 11th Polygon.

8th. Polygon

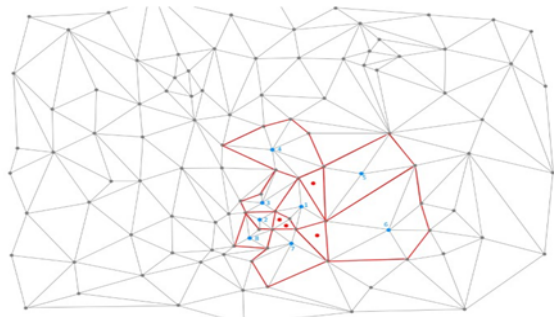


Figure 8: 8th Polygon.

12th. Polygon

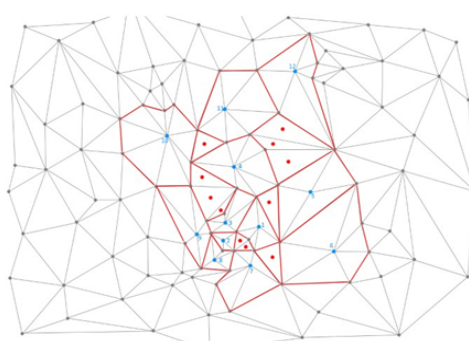


Figure 12: 12th Polygon.

9th. Polygon

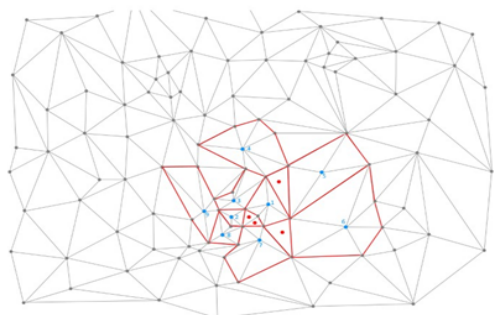


Figure 9: 9th Polygon.

13th. Polygon

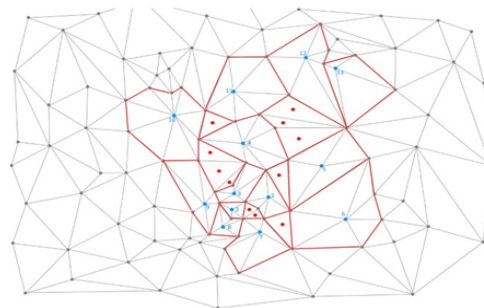


Figure 13: 13th Polygon.

10th. Polygon

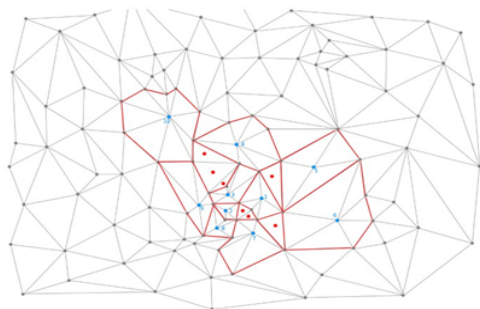


Figure 10: 10th Polygon.

14th. Polygon

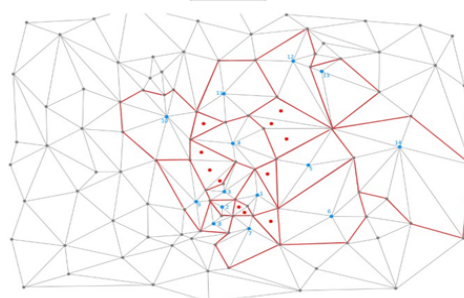


Figure 14: 14th Polygon.

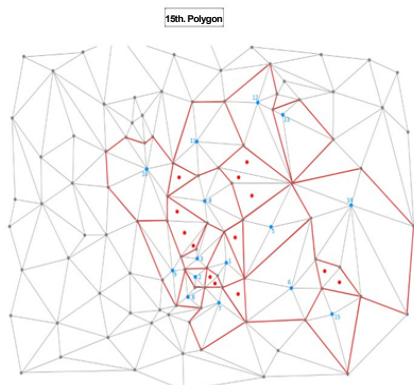


Figure 15: 15th Polygon.

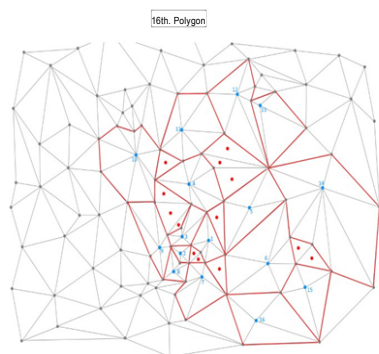


Figure 16: 16th Polygon.

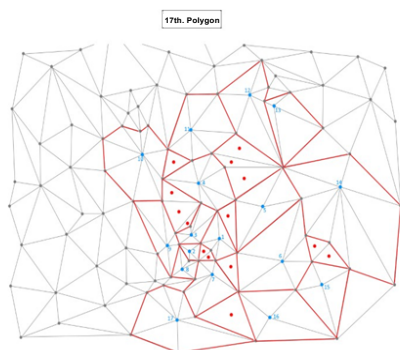


Figure 17: 17th Polygon.

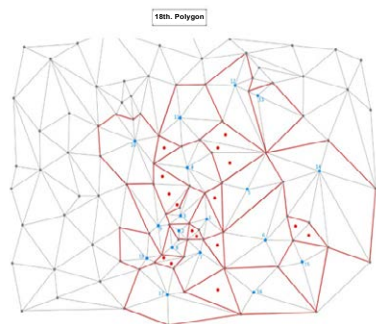


Figure 18: 18th Polygon.

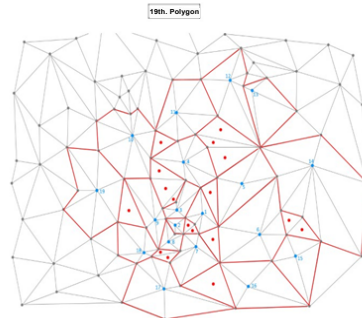


Figure 19: 19th Polygon.

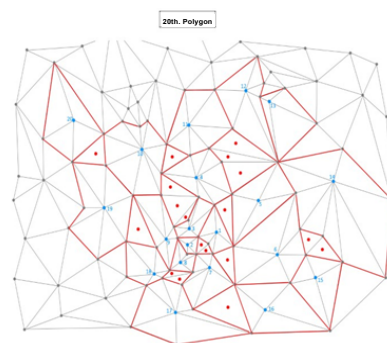


Figure 20: 20th Polygon.

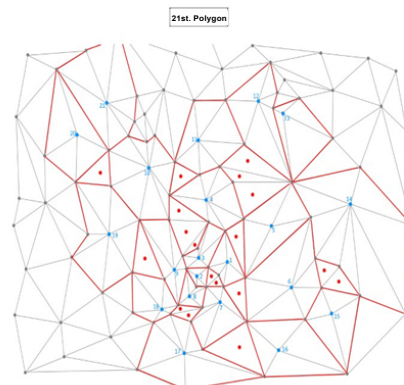


Figure 21: 21st Polygon.

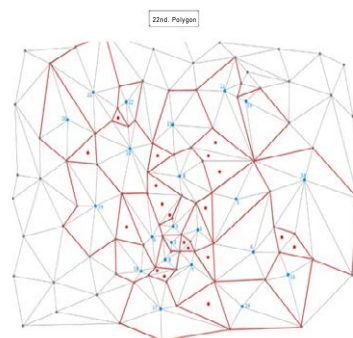


Figure 22: 22nd Polygon.

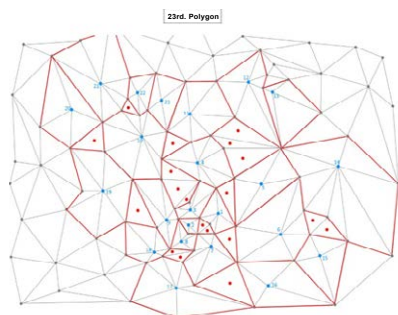


Figure 23: 23rd Polygon.

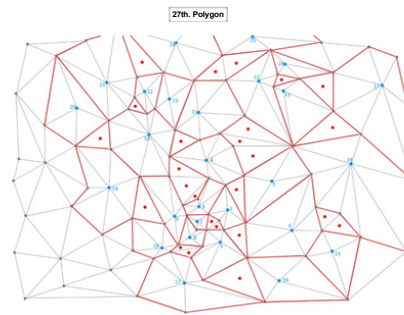


Figure 27: 27th Polygon.

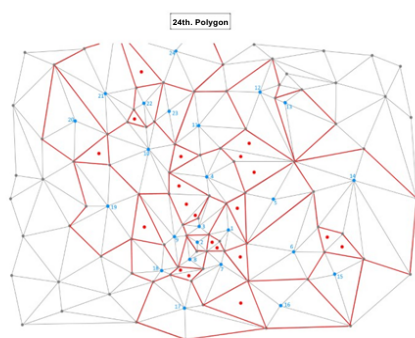


Figure 24: 24th Polygon.

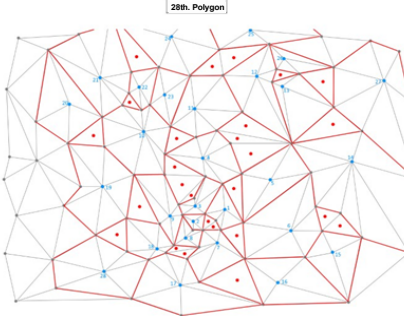


Figure 28: 28th Polygon.

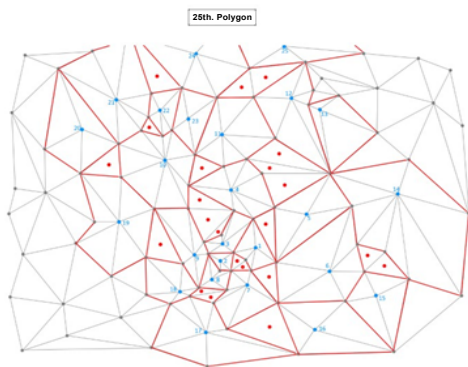


Figure 25: 25th Polygon.

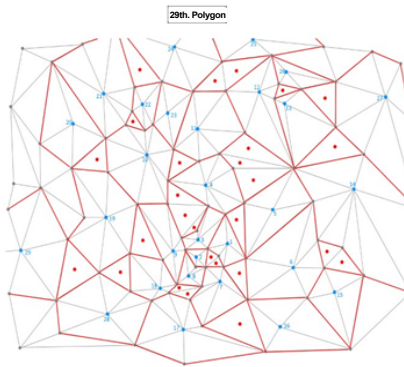


Figure 29: 29th Polygon.

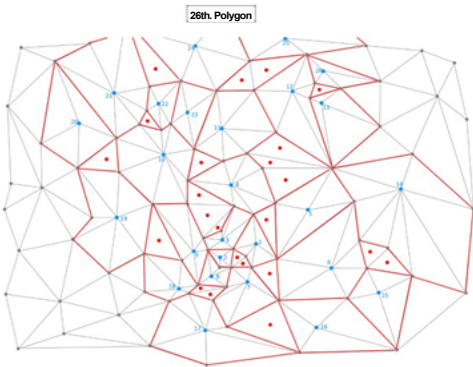


Figure 26: 26th Polygon.

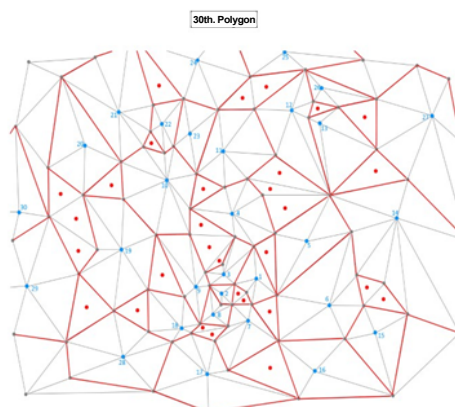


Figure 30: 30th Polygon.

A. If they are it, and have a common neighbour, force it to take the third colour, and they use the other two colours.

B. If they are not it, and have different colours, when they have a common neighbour force it to take the third colour too.

Stopping condition happens always. Because any point linked even only with a coloured point has two possibilities, therefore it belongs to the next process. If it has one possible colour it must be coloured in the current process, because it belongs to a triangulation.

The process starts completing the corresponding crown, according to the number of his centre. It must be the first incomplete crown following order number of the centres. Process can continue with triangulations if it provokes some.

The algorithm of the process has three parts, two essential and one do not:

1. One previous reading. To know how many points belong to the process. It goes from initial crown and continue for the triangulations, when the points provoke them.
2. For it is necessary to pay attention to the triangulations kind B, because they can vary the number of points to colour. As they are not neighbour they can have the same colour, then do not provoke the triangulation. However the stopping condition run always.
3. One reading above the points with two possibilities that provoke previous reading. If these points are linked two by two, they cannot take the same two possibilities. Now can choose one of the possible blocks of colour.
4. The Deletion entail to eliminate after each election, the coloured points and their links when they do not affect directly to the following process. When a crown is not complete either belongs to the deletion, although some point do not affect to other.

The effects of election are picked up and projected from the points that form the Big Crown (Figures 31-58).

Proof

Key: Exist a Big Crown which grows in each process until takes up whole map.

Key: The process points constitute a block of little chains of coloured points, whose interior have not any point without colour.

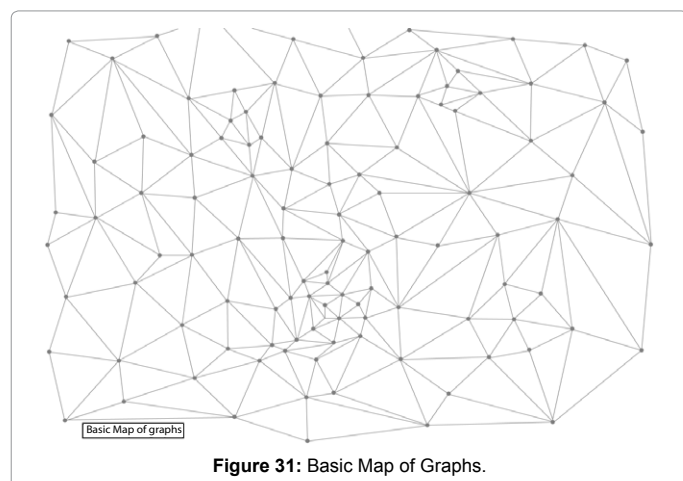


Figure 31: Basic Map of Graphs.

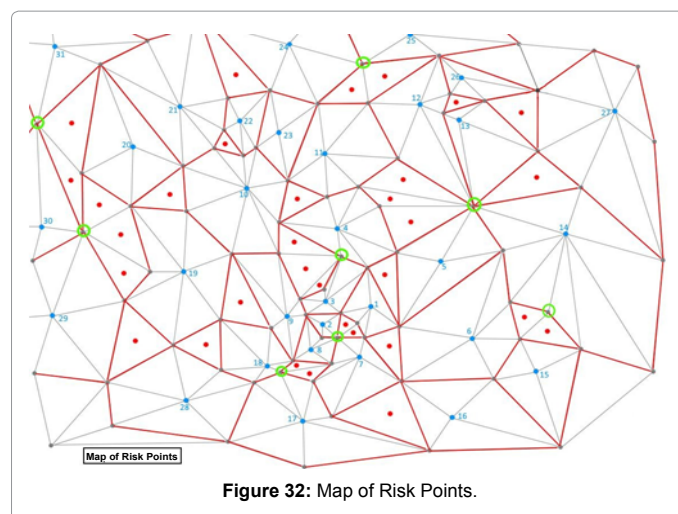


Figure 32: Map of Risk Points.

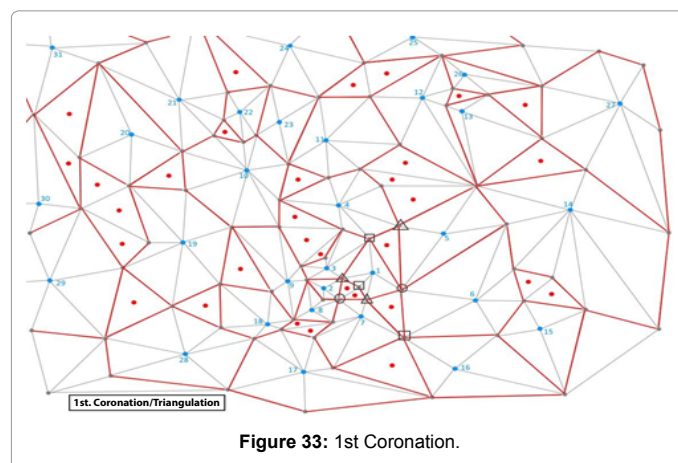


Figure 33: 1st Coronation.

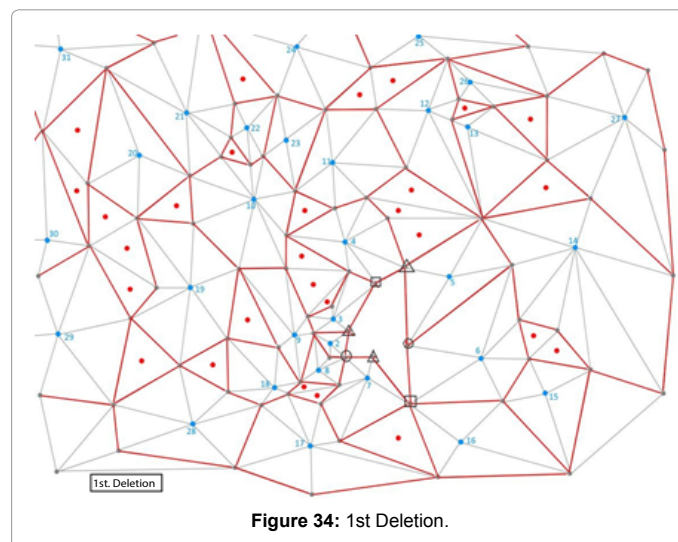
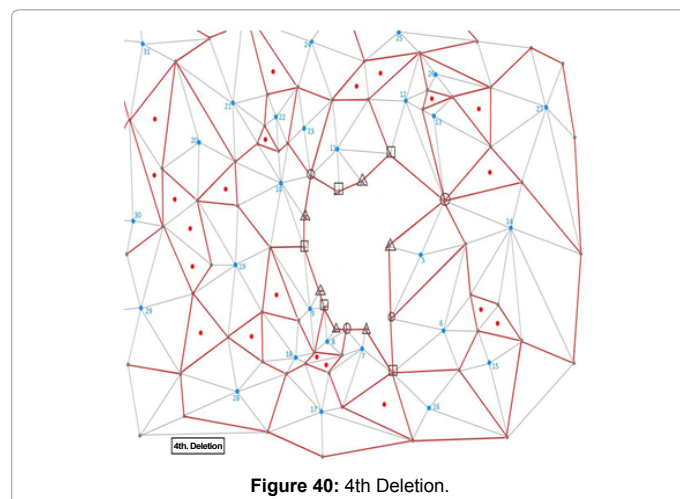
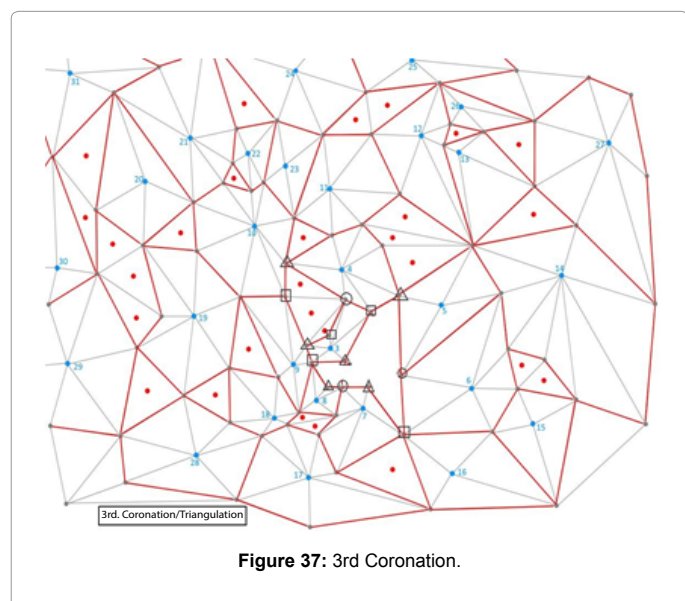
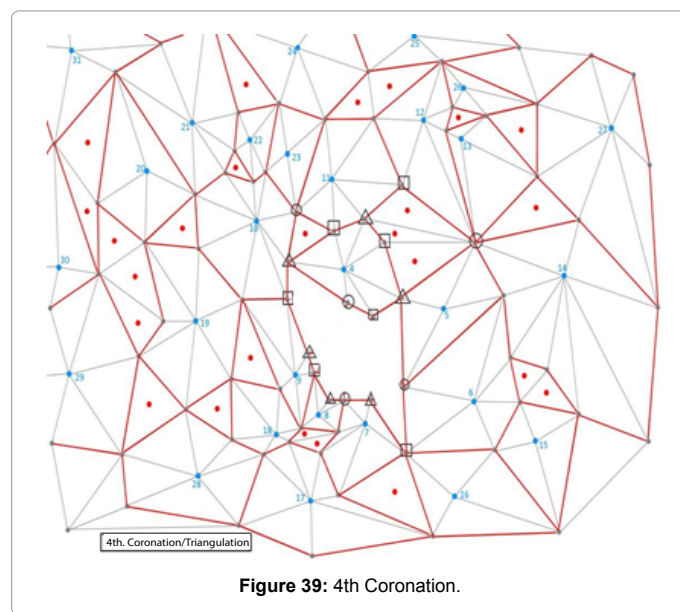
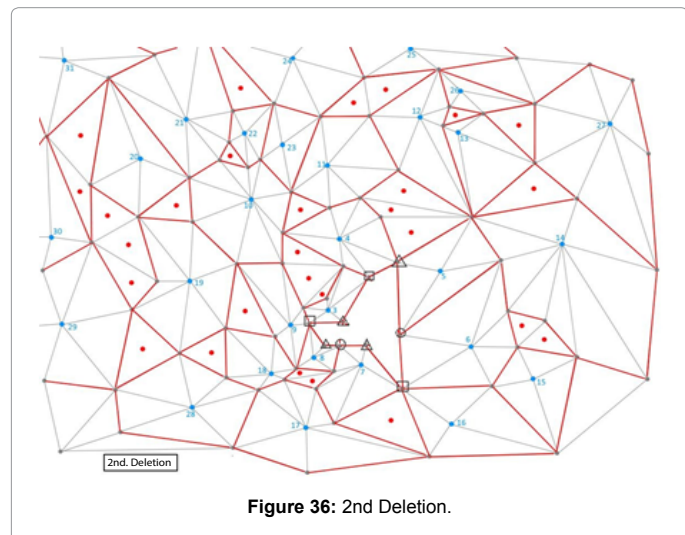
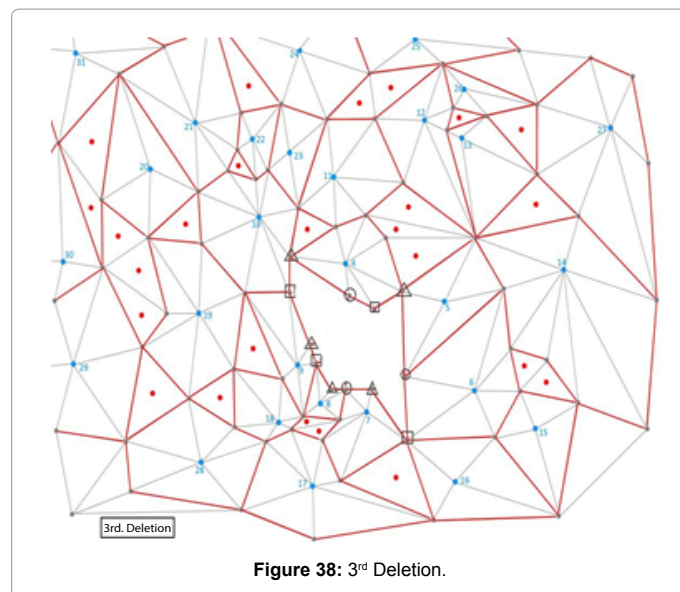
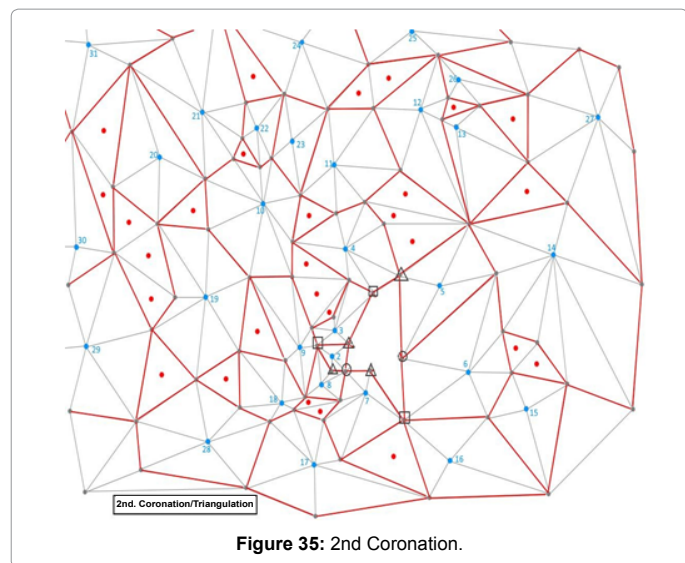
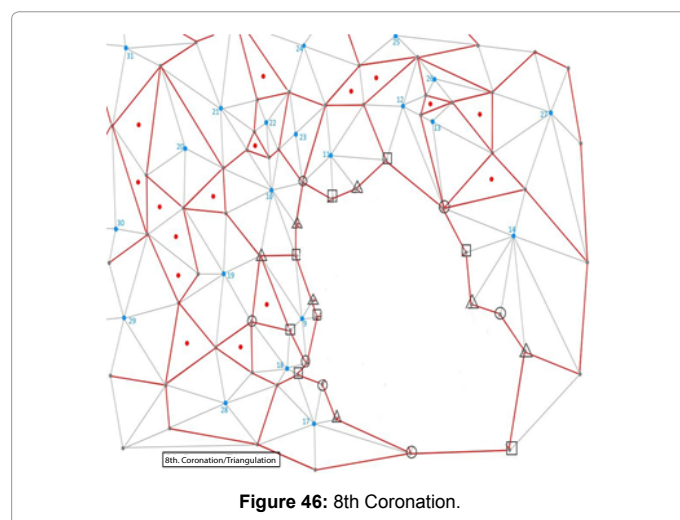
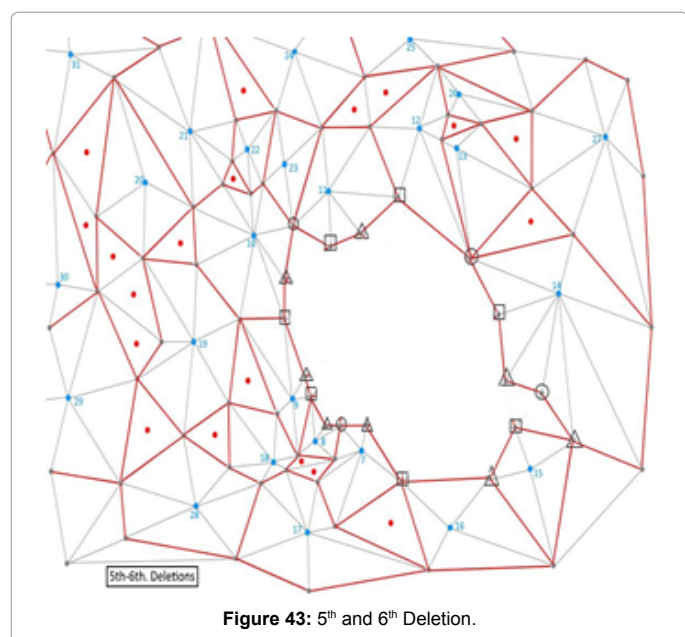
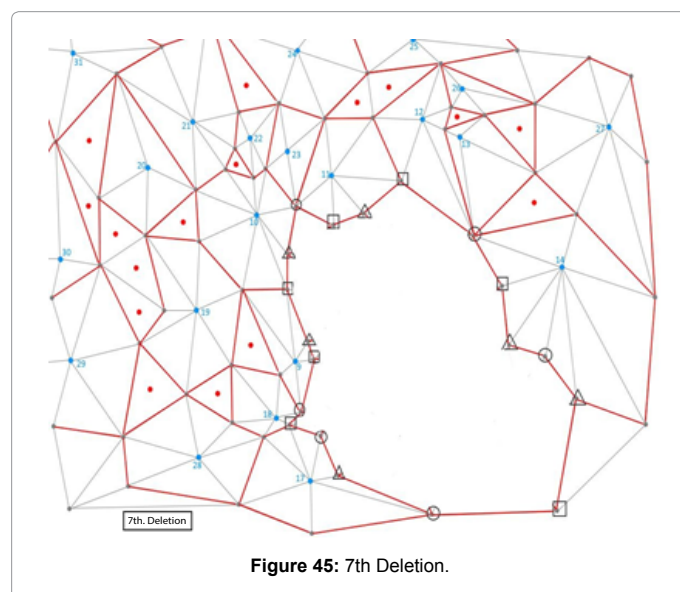
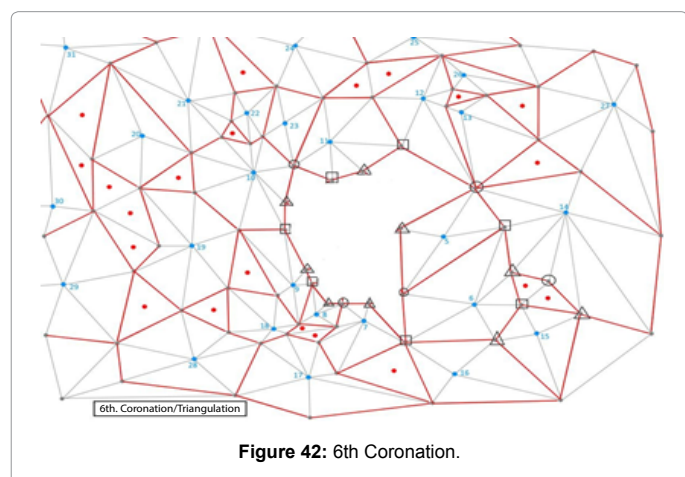
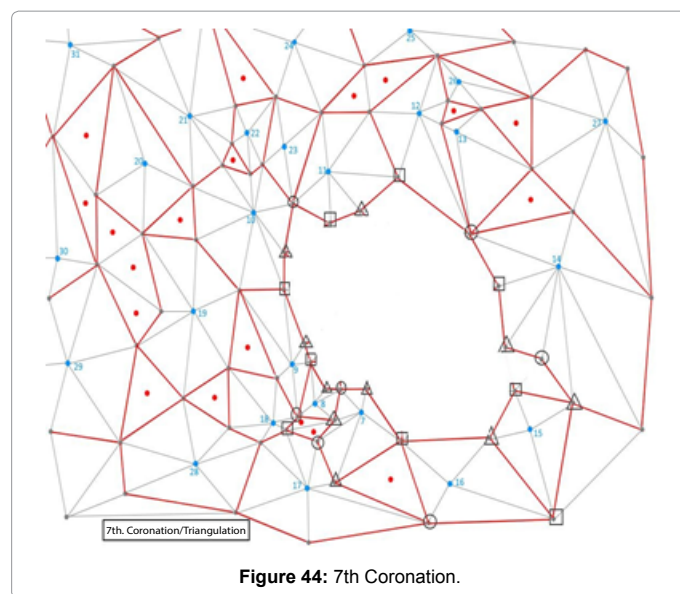
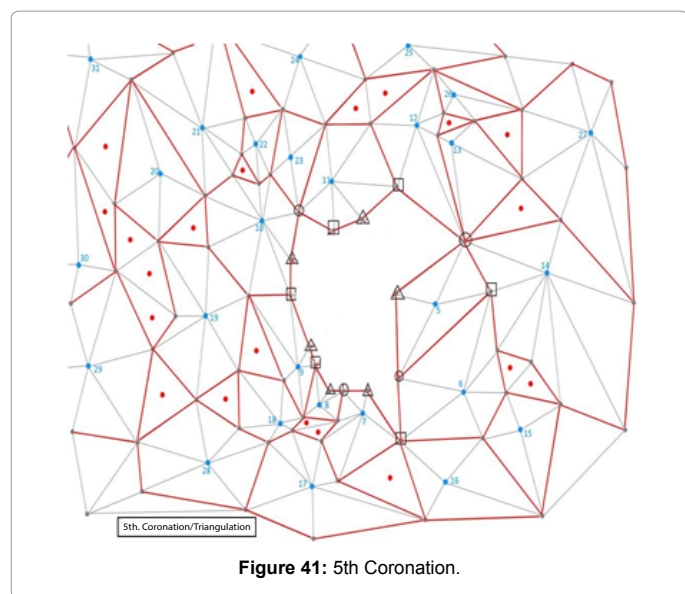


Figure 34: 1st Deletion.

Key: All choice have beginning and end in coloured points, which belong either to the same process or other.

Next I am going to define both idea, chain and block. Process points are selected by belong to the initial crown or by triangulation.





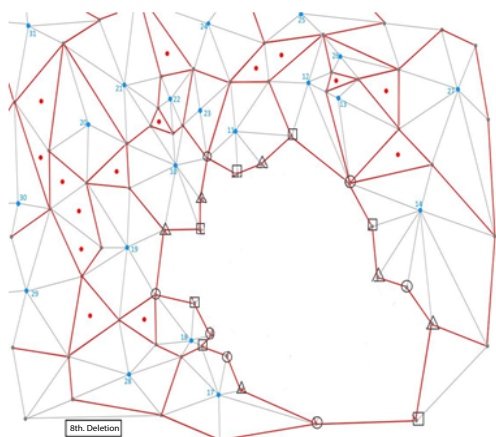


Figure 47: 8th Deletion.

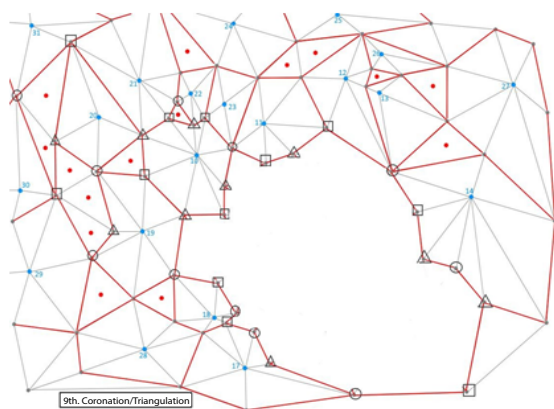


Figure 48: 9th Coronation.

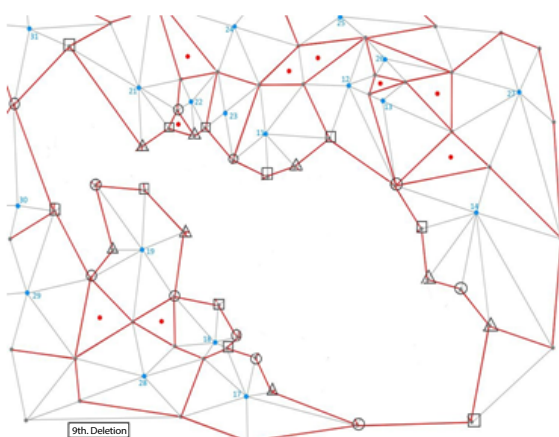


Figure 49: 9th Deletion.

The last by logic have at least two links with coloured points, so they give idea of a route with beginning and end in these points. Also initial crown points complete a crown, that seem a circle, they are like a chain that finish the circle.

This is the idea I want to show, they are little chains because the triangulations are little two, three points, and the piece of crown too.

Conclusion

In conclusion the little chains surround a central point or nothing, the triangulation kind A is a triangle, without centre. As they are linked some over some, they constitute a block with all their points coloured. They seem like scales on a fish.

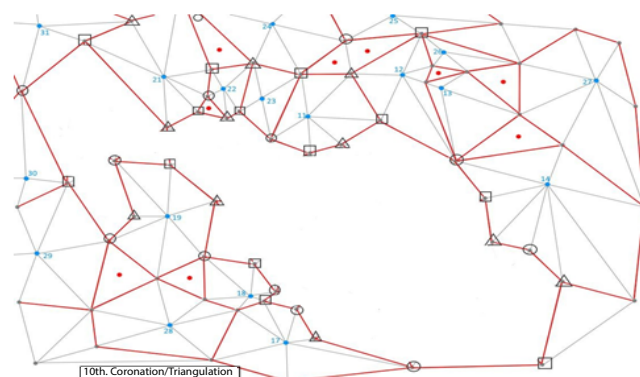


Figure 50: 10th Coronation.

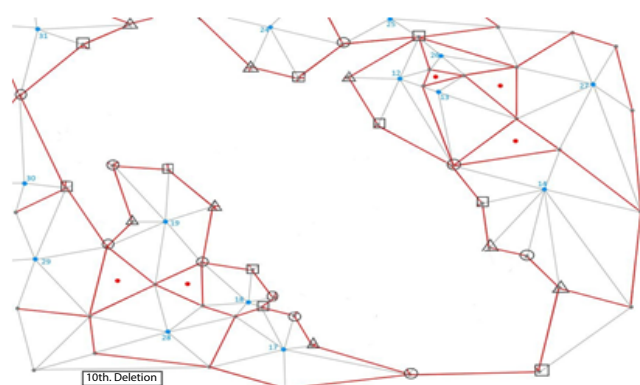


Figure 51: 10th Deletion.

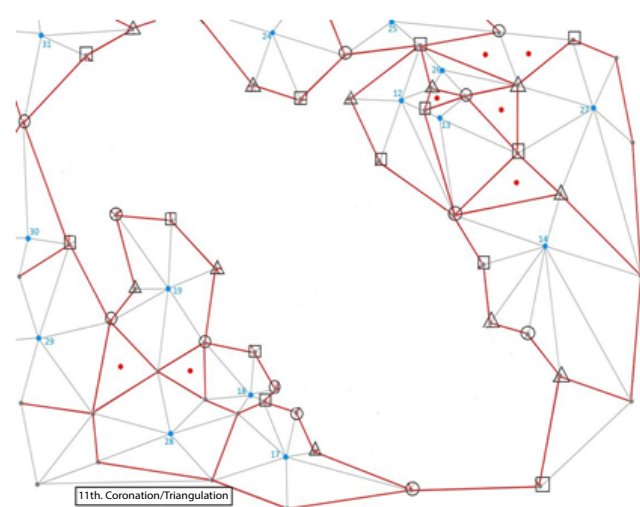
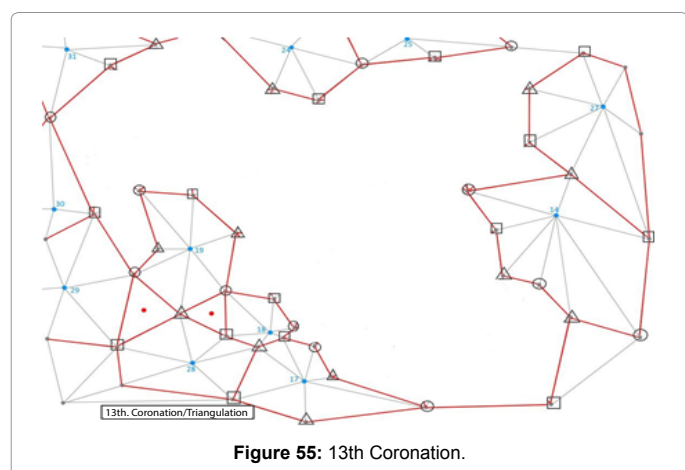
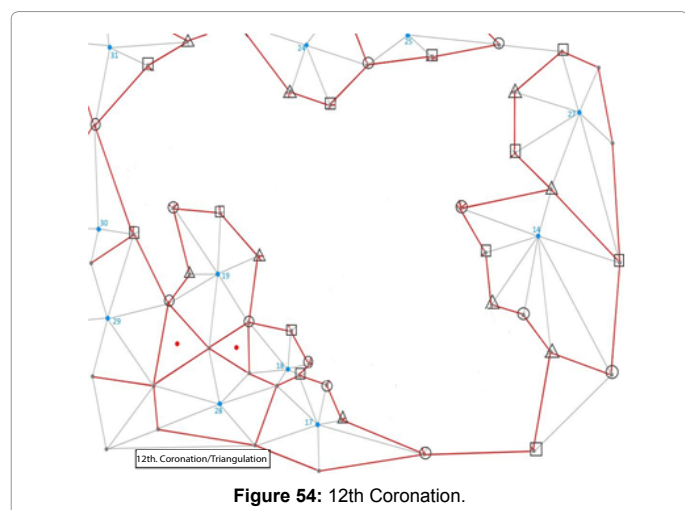
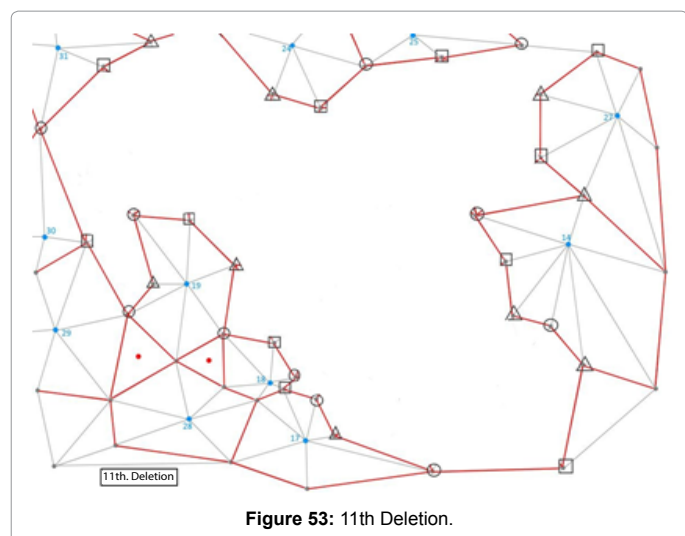


Figure 52: 11th Coronation.

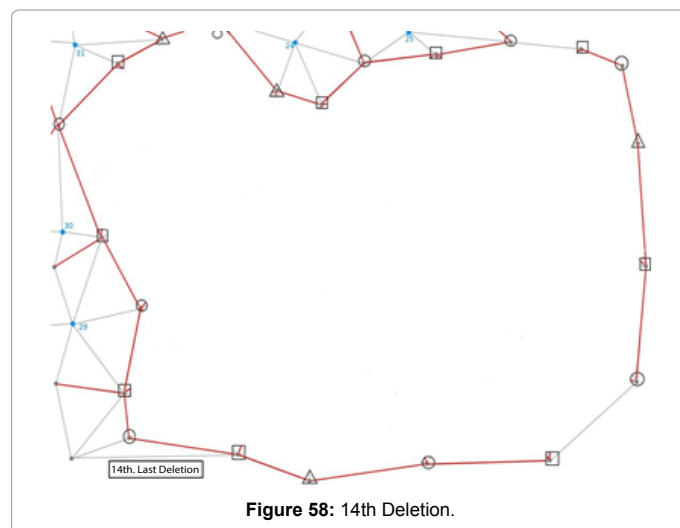
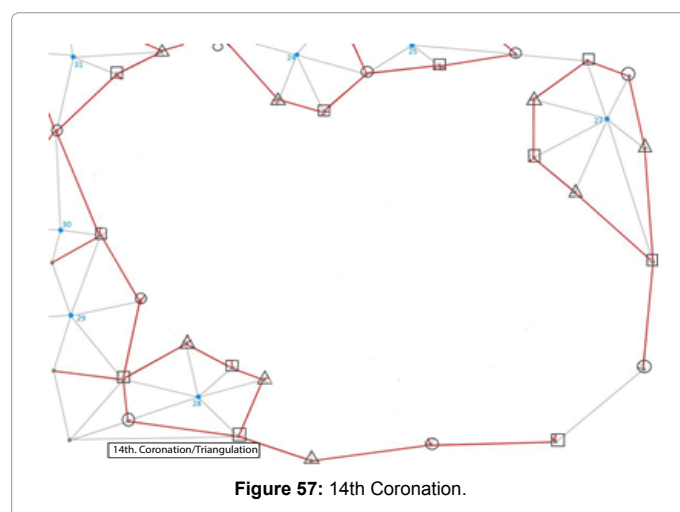
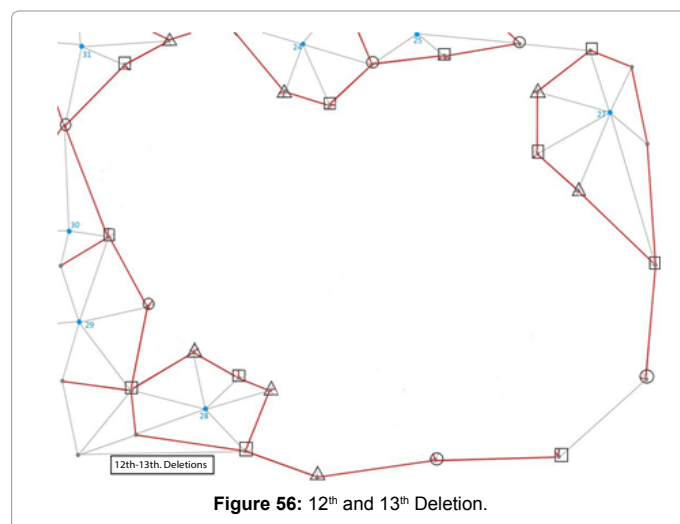


Only the chains that are down are linked with previous process, they have the points with two colours, and the chains stay over with three, without contact with previous process.

The proof has two parts, where is necessary to confirm two questions:

A. All colours elections that process can form, regarding points as a whole, are valid. There are two at least.

B. Structure is infallible.



A. I am going round chains block, from up to down, from points with three possibilities until two. Chains with only points with three possible colours:

- Do not link with previous process.
- They do not entwine because surrounded a central point or nothing.
- Any closed chain with three colours has enough, what is more all points in the chain can have three colours, then they generate all possible combinations.

The chains which have points with two, only can be linked two by two and if they have the same two colours, only can generate two combinations, because colours alternate. For example -A-B-A-B-, and -B-A-B-A-.

Before to election of a specific block of colours, we must mark points with two colours. If they have the two same colours, and they provoke a triangulation, this third point only has a colour as possibility. What is more when they are three or more points linked two by two, with the same two colours, and also provoke triangulations, then they provoke a mistake. But it can be avoided, rectifying the previous process. Here it is not necessary to rectify by the following reasons.

If in the beginning of a process we have this case, but it not generates problems, the initial chain has two way to be coloured. As all the next points have three colours, there are more than one end. Because of it, the neighboring points of the points with two colours have not got to take the same colour. They belong to the current process, and the points with two to the next. So that these last have the same two opportunities, the first ones must take the same final colour.

That is the only one final that cause the problem in the last analysis. We have found out the ones possible points with an opportunity. But there are more finals, and then more beginnings.

B. The structure is infallible because thanks to the basic polygons, the effect of multiple links is ordered and simplified.

The plane graph has a pattern. Each link between two points opposite two or more polygons, at least one on each side. If the polygon is a triangle they have a common neighbour, in another case do not. But they can have numerous opposite polygons.

When we observe the structure, here it does not happen because of the definition of basic polygon.

Each centre of each new polygon only use a polygon of each link of its crown, and the following new polygons too. The centres are the first key in this work.

As the plane graph is based on polygons, the structure can run on it, and the centres are in charge to achieve the independence among the new polygons (Figure 59).

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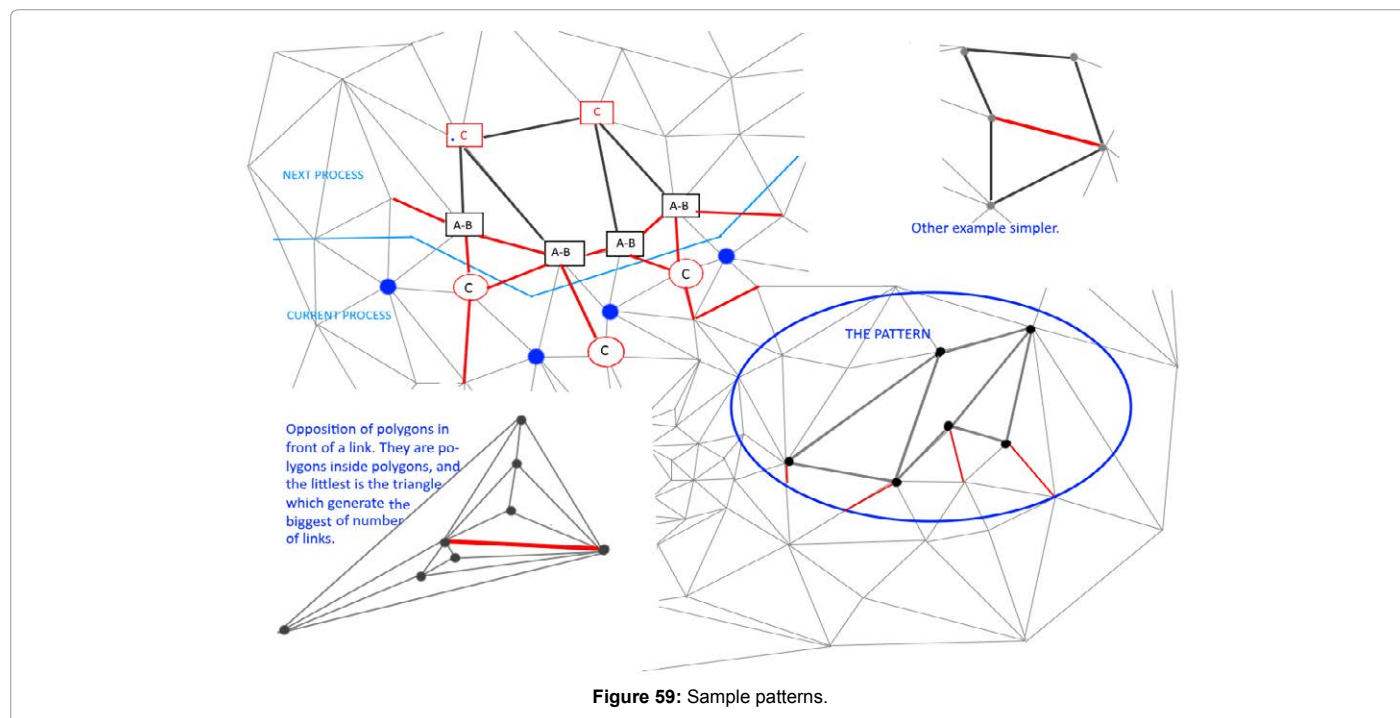


Figure 59: Sample patterns.