

# A Numerical Scheme for a Weakly Coupled System of Singularly Perturbed Delay Differential Equations

Vivek Kumar\*

Department of Mathematics, Visvesvaraya National Institute of Technology, Nagpur, India

## Editorial

In this paper, an adaptive mesh selection strategy is presented for solving a weakly coupled system of singularly perturbed delay differential equations of convection-diffusion type using second order central finite difference scheme. Layer adaptive mesh is generated via an entropy production operator. The small print of the situation and width of the physical phenomenon isn't required within the proposed method unlike the favoured layer adaptive meshes mainly by Bakhvalov and Shishkin. The tactic is independent of perturbation parameter and provides us an oscillation-free solution. The applicability of the proposed method is illustrated by means of three examples.

Singular perturbations, now is a maturing mathematical subject with a reasonably long history and a robust promise for continued important applications throughout science and engineering. For numerical treatment of singular perturbation problems (SPPs) one may refer the books, few research articles. If the longer term state of the system depends of past states, the governing differential equations contain delay arguments and a subclass of those equations contains singularly perturbed delay differential equations. Singularly perturbed delay differential equations arise within the mathematical modelling of biological sciences, applied math, and a number of other branches of engineering like the modeling of biological oscillators, immune reaction, dynamics of networks of two identical amplifiers, mathematical ecology, population dynamics, etc. The numerical treatment of singularly perturbed delay differential equations with large delays are often with little or no literature is out there for numerical treatment of system of singularly perturbed delay differential equations.

It is noticed that the quality central difference scheme on a consistent mesh when applied to unravel singularly perturbed differential equations would cause oscillatory solution thanks to boundary or interior layers. To eliminate these oscillations while retaining the order of accuracy, one needs a fine mesh at the layer regions. This will be done via adaptive mesh strategy. Thus the utilization of adaptive mesh refinement techniques is nowadays a typical component in numerical computation. In fact, the primary to use layer adapted meshes. The widely used for numerical approximation of SPPs thanks to their

simplicity, the main drawback of Shishkin mesh is that the requirement of a priori information of the situation of the layer regions. To beat this drawback, we proposed an adaptive mesh selection strategy using the concept of entropy function for solving a coupled system of singularly perturbed delay differential equations. The tactic is independent of perturbation parameter  $\epsilon$  and provides us oscillation free solution.

The paper is organized as follows: In Section 2, the matter under study is stated and a few analytical results were derived. In Section 3, we introduce an adaptive mesh and that we use a classical finite difference scheme to unravel the system of singularly perturbed delay equation of convection-diffusion type. In Section 4, we offer the error analysis for the proposed method. In Section 5, three numerical examples are solved to corroborate the applicability and efficiency of the proposed method according with the theoretical estimates. Finally the paper ends with Section 6 where we offer brief conclusion. To generate the adaptive mesh, we followed the steps of Kumar and Srinivasan.<sup>30</sup> We now defines an auxiliary equation, that is, the entropy production equation by multiplying with an appropriate test function. From the idea of scalar hyperbolic conservation laws, we all know that for scalar conservation laws, it's an appropriate entropy variable and, therefore,  $2u(x)$  may be a suitable multiplying test function.

We solved the system of Equations with the boundary conditions by Gauss elimination method with partial pivoting. The presence of singular perturbation parameter  $\epsilon$  results in occurrence of untamed oscillations within the numerical solution. So as to avoid such oscillations, an outsized number of mesh points are required within the layer regions, if one restricts to uniform meshes for an arbitrary small  $\epsilon$ . to beat this, we generate an adaptive mesh using entropy function, which can introduce more mesh points inside the oscillatory parts of the answer. The adaptive mesh generation algorithm is already given within the above subsection.

To demonstrate the applicability of the proposed numerical approach, we feature out the numerical experiments for 3 test problems. We start with a user chosen uniform mesh and use the entropy functions to locate the position to introduce the new mesh point as defined within the above mention algorithm for a given  $M$ , allow us to assume that one requires seeking out the numerical solution on  $M_g$  number of adapted mesh points to get oscillation-free solution.

\*Address for Correspondence: Vivek Kumar, Department of Mathematics, Visvesvaraya National Institute of Technology, Nagpur, India, E-mail: vivek.k@gmail.com

Copyright: © 2021 Kumar V. This is an open-access article distributed under the terms of the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited.

Received 01 September 2021; Accepted 08 September 2021; Published 15 September 2021

How to cite this article: Vivek Kumar. "A Numerical Scheme for a Weakly Coupled System of Singularly Perturbed Delay Differential Equations." *J Appl Computat Math* 9(2021):488.