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A New Method on Measure of Similarity between Interval-Valued Intuitionistic Fuzzy Sets for Pattern Recognition

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Abstract

Interval-valued intuitionistic fuzzy sets gained attention from researcher for their applications in various fields. In this paper a new cosine similarity measure for interval-valued intuitionistic fuzzy sets is proposed. Also the proposed and existing similarity measures are compared to show that the proposed similarity measure is more reasonable than some existing similarity measures. Finally, the proposed similarity measure is applied to pattern recognition.

Keywords: Interval-valued intuitionistic fuzzy sets; Similarity measures; Pattern recognition

Introduction

The theory of fuzzy sets, proposed by Zadeh [1], has gained successful applications in various fields. Measures of similarity between fuzzy sets have gained attention from researchers for their wide applications in real world. Similarity measures are very useful in some areas, such as pattern recognition, machine learning, decision making and market prediction etc. Many measures of similarity between fuzzy sets have been proposed [2-8].

Atanassov [9,10] presented intuitionistic fuzzy sets which are very effective to deal with vagueness. Gau and Buehere [11] researched vague sets. Bustince and Burillo [12] pointed out that the notion of vague sets is same as that of interval-valued intuitionistic fuzzy sets. Chen and Tan [13] proposed two similarity measures for measuring the degree of similarity between vague sets. De et al. [14] defined some operations on intuitionistic fuzzy sets. Szmidt and Kacprzyk [15] introduced the Hamming distance between intuitionistic fuzzy sets and proposed a similarity measure between intuitionistic fuzzy sets based on the distance. Li and Cheng [16] also proposed similarity measures of intuitionistic fuzzy sets and applied these similarity measures to pattern recognition. Liang and Shi [17] proposed several similarity measures to differentiate different intuitionistic fuzzy sets and discussed the relationships between these measures.

Furthermore, Mitchell [18] interpreted intuitionistic fuzzy sets as ensembles of ordered fuzzy sets from a statistical viewpoint to modify Dengfeng and Chuntian measures [16]. Hung and Yang [19] proposed another method to calculate the distance between intuitionistic fuzzy sets based on the Hausdorff distance and used it to propose several similarity measures between intuitionistic fuzzy sets. Liu [20] proposed some similarity measures between intuitionistic fuzzy sets and applied these measure methods in pattern recognition. Xu [21] proposed a method for deriving the correlation coefficients of intuitionistic fuzzy sets and also extended the developed method to the interval-valued intuitionistic fuzzy set theory. Hung and Yang [22] proposed similarity measures by inducing L_p metric. Xu [23] developed some similarity measures of intuitionistic fuzzy sets and define the notions of positive ideal intuitionistic fuzzy set and negative ideal intuitionistic fuzzy set. Finally, applied the similarity measures to multiple attribute decision making under intuitionistic fuzzy environment.

Xu et al. [24] defined the concepts of association matrix and equivalent association matrix, and introduce some methods for calculating the association coefficients of intuitionistic fuzzy sets and proposed a clustering algorithm for intuitionistic fuzzy sets. Lee [25] proposed a novel score function by taking into account the expectation of the hesitancy degree of interval-valued intuitionistic fuzzy sets also to identify the best alternative in muticriteria decision-making problems a multicriteria fuzzy decision making method is presented in which criterion values for alternatives are interval-valued intuitionistic fuzzy sets. Xia and Xu [26] proposed a series of similarity measures for intuitionistic fuzzy values based on the intuitionistic fuzzy operators. Also they have applied the similarity measures to aggregate intuitionistic fuzzy values and develop some aggregation operators, such as the intuitionistic fuzzy dependent averaging operator and the intuitionistic fuzzy dependent geometric operator. Xu [27] pointed out the the drawbacks of existing methods on measures of similarity between vague sets and proposed a new method on measures of similarity between vague sets.

Xu [28] introduced some relations and operations of interval-valued intuitionistic fuzzy numbers and define some types of matrices, including interval-valued intuitionistic fuzzy matrix, interval-valued intuitionistic fuzzy similarity matrix and interval-valued intuitionistic fuzzy equivalence matrix and also proposed a method based on distance measure for group decision making with interval-valued intuitionistic fuzzy matrices. Guha and Chakraborty [29] introduced a distance measure for intuitionistic fuzzy numbers and studied the metric properties of the proposed measure. Xu and Xia [30] proposed a variety of distance measures for hesitant fuzzy sets and based on which the corresponding similarity measures can be obtained.

Pattern recognition has been one of the fastest growing area

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during the last two decades because of its usefulness and fascination. In pattern recognition, on the basis of the knowledge of known pattern our aim is to classify the unknown pattern. Because of the complex and uncertain nature of the problems, the problem of pattern recognition is given in the form of interval-valued intuitionistic fuzzy sets. Atanassov and Gargov [31] introduced interval-valued intuitionistic fuzzy sets. Clearly the interval-valued intuitionistic fuzzy sets are extensions of the intuitionistic fuzzy sets.

In this paper a new cosine similarity measure for interval-valued intuitionistic fuzzy sets is proposed. Also the proposed and existing similarity measures are compared to show that the proposed similarity measure is more reasonable than some existing similarity measures. The proposed similarity measure is applied to pattern recognition.

This paper is organized as follow: In Section 4, some basic definitions interval-valued intuitionistic fuzzy numbers are presented. In Section 5, cosine similarity measure for interval-valued intuitionistic fuzzy numbers is proposed. Results of the proposed similarity measure and existing similarity measures are compared in Section 6. In Section 7, the proposed similarity measure is applied to deal with the problem related to medical diagnosis. Final section is conclusions.

Basic Definition

Definition (i)

Let D [0,1] be the set of all closed subintervals of the interval [0, 1] [31]. Let X($\neq \varphi$) be a given set. An interval valued intuitionistic fuzzy set in X is an expression given by $\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)\rangle \mid x \in X\}$, where $\mu_{\tilde{A}}(x) : X \to D[0,1]$, $\nu_{\tilde{A}}(x) : X \to D[0,1]$, with the condition $0 < sup_X \mu_{\tilde{A}}(x) + sup_X \nu_{\tilde{A}}(x) \le 1$.

The intervals $\mu_{\sim A}(x)$ and $\nu_{\sim A}(x)$ denote, respectively, the degree of belongingness and the nonbelongingness of the element x to the set A. Thus for each x, $x \in X, \mu_{\tilde{A}}(x)$ and $\nu_{\tilde{A}}(x)$ are closed intervals whose lower and upper end points are, respectively, denoted by $\mu_{\tilde{A}L}(x), \mu_{\tilde{A}U}(x)$ and $\nu_{\tilde{A}L}(x), \nu_{\tilde{A}U}(x)$.

We can denote $\varphi_{\tilde{V}}(x_i) = |1 - \dfrac{\nu_{\tilde{A}}(x_i)}{2} - \dfrac{(1-\mu_{\tilde{B}}(x_i))}{2}|$ where $0 < \mu_{\tilde{A}L}(x) + \nu_{\tilde{A}U}(x) \le 1$, $\mu_{\tilde{A}L}(x) \ge 0, \nu_{\tilde{A}L}(x) \ge 0$.

For each element x we can compute the unknown degree (hesitancy degree) of an intuitionistic fuzzy interval of $x \in X$ in A defined as follows:

$$\pi_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x) - \nu_{\tilde{A}}(x) = [1 - \mu_{\tilde{A}U}(x) - \nu_{\tilde{A}U}(x), 1 - \mu_{\tilde{A}L}(x) - \nu_{\tilde{A}L}(x)].$$

Definition (ii)

Let $\tilde{A}$ and $\tilde{B}$ be two intuitionistic fuzzy sets in the universe of discourse X={x_1, $x_{2,...,}$ x_n}. The correlation coefficient of $\tilde{A}$ and $\tilde{B}$ is given by Gerstenkorn and Manko [32]:

$$k(\tilde{A}, \tilde{B}) = \frac{C(\tilde{A}, \tilde{B})}{\sqrt{T(\tilde{A}).T(\tilde{B})}} \tag{1}$$

where the correlation of two intuitionistic fuzzy sets $\tilde{A}$ and $\tilde{B}$ is given by

$$C(\tilde{A}, \tilde{B}) = \sum_{i=1}^{n} (\mu_{\tilde{A}}(x_i)\mu_{\tilde{B}}(x_i) + \nu_{\tilde{A}}(x_i)\nu_{\tilde{B}}(x_i))$$ and the informational intuitionistic energies of two intuitionistic fuzzy sets $\tilde{A}$ and $\tilde{B}$ are given by $T(\tilde{A}) = \sum_{i=1}^{n} (\mu_{\tilde{A}}^2(x_i) + \nu_{\tilde{A}}^2(x_i))$ and $T(B) = \sum_{i=1}^{n} (\mu_{\tilde{B}}^2(x_i) + \nu_{\tilde{B}}^2(x_i))$, respectively.

The correlation coefficient of two intuitionistic fuzzy sets A and B satisfies the following properties [32]:

(i) $0 \le k(\tilde{A}, \tilde{B}) \le 1$

(ii) $k(\tilde{A}, \tilde{B}) = k(\tilde{B}, \tilde{A})$

(iii) $k(\tilde{A}, \tilde{B}) = 1$ if $A = \tilde{B}$

Cosine similarity measures are defined as the inner product of two vectors divided by the product of their lengths. This is nothing but the cosine of the angle between the vector representations of the two fuzzy sets.

Assume that $\tilde{A} = \mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2), ..., \mu_{\tilde{A}}(x_n)$ and $\tilde{B} = \mu_{\tilde{B}}(x_1), \mu_{\tilde{B}}(x_2), ..., \mu_{\tilde{B}}(x_n)$ are two fuzzy sets in the universe of discourse $X = \{x_1, x_2, ... x_n\}, x_i \in X$. A cosine similarity measure based on Bhattacharya's distance [33] between $\mu_{\tilde{A}}(x_i)$ and $\mu_{\tilde{B}}(x_i)$ can be defined as follows [25]:

$$C_F(\tilde{A}, \tilde{B}) = \frac{\sum_{i=1}^{n} \mu_{\tilde{A}}(x_i)\mu_{\tilde{B}}(x_i)}{\sqrt{\sum_{i=1}^{n} \mu_{\tilde{A}}^2(x_i)}\sqrt{\sum_{i=1}^{n} \mu_{\tilde{B}}^2(x_i)}} \tag{2}$$

The cosine of the angle between the vectors is within the values between 0 and 1.

Cosine Similarity Measure for Interval-Valued Intuitionistic Fuzzy Sets

The existing cosine similarity measure is defined as the inner product of these two vectors divided by the product of their lengths. Cosine similarity measure is the cosine of the angle between the vector representations of the two fuzzy sets. The cosine similarity is a classic measure used in information retrieval and is the most widely reported measure of vector similarity [34]. However, the existing cosine similarity measures do not deal with cosine similarity measures between interval-valued intuitionistic fuzzy sets. Therefore to over this limitation in this section, a new cosine similarity measure between interval-valued intuitionistic fuzzy sets is proposed based on the concept of the cosine similarity measure for fuzzy sets.

Let $\tilde{A}$ be an interval-valued intuitionistic fuzzy sets in a universe of discourse X = x, the interval-valued intuitionistic fuzzy sets is characterized by the interval of degree of membership $[\mu_{\tilde{A}L}(x), \mu_{\tilde{A}U}(x)]$ and the interval degree of non-membership , $[\nu_{\tilde{A}L}(x), \nu_{\tilde{A}U}(x)]$ which can be considered as a vector representation with the two elements. Therefore, a cosine similarity measure for interval-valued intuitionistic fuzzy sets is proposed in an analogous manner to the cosine similarity measure based on Bhattacharya's distance [33].

Assume that there are two interval-valued intuitionistic fuzzy sets $\tilde{A}$ and $\tilde{B}$ in X = x_1, x_2,…,x_n. Based on the extension of the cosine measure for fuzzy sets, a cosine similarity measure between interval-valued intuitionistic fuzzy sets $\tilde{A}$ and $\tilde{B}$ is proposed as follows:

$$C_S(\tilde{A},\tilde{B}) = \frac{1}{n}\sum_{i=1}^{n}\frac{(\mu_{\tilde{A}L}(x_i)+\mu_{\tilde{A}U}(x_i))(\mu_{\tilde{B}L}(x_i)+\mu_{\tilde{B}U}(x_i))+(\nu_{\tilde{A}L}(x_i)+\nu_{\tilde{A}U}(x_i))(\nu_{\tilde{B}L}(x_i)+\nu_{\tilde{B}U}(x_i))}{\sqrt{(\mu_{\tilde{A}L}(x_i)+\mu_{\tilde{A}U}(x_i))^2+(\nu_{\tilde{A}L}(x_i)+\nu_{\tilde{A}U}(x_i))^2}\sqrt{(\mu_{\tilde{B}L}(x_i)+\mu_{\tilde{B}U}(x_i))^2+(\nu_{\tilde{B}L}(x_i)+\nu_{\tilde{B}U}(x_i))^2}} \quad (3)$$

Proposition (i)

Let $\tilde{A}$ and $\tilde{B}$ be interval-valued intuitionistic fuzzy sets then

(i) $0 \le C_S(\tilde{A},\tilde{B}) \le 1$

(ii) $C_S(\tilde{A},\tilde{B}) = C_S(\tilde{B},\tilde{A})$

(iii) $C_S(\tilde{A},\tilde{B}) = 1$ if $\tilde{A} = \tilde{B}$ i.e.

$\mu_{\tilde{A}L}(x_i) = \mu_{\tilde{A}U}(x_i) = \mu_{\tilde{B}L}(x_i) = \mu_{\tilde{B}U}(x_i)$ and

$\nu_{\tilde{A}L}(x_i) = \nu_{\tilde{A}U}(x_i) = \nu_{\tilde{B}L}(x_i) = \nu_{\tilde{B}U}(x_i)$ for i = 1,2,…,n.

Proof:(i) It is obvious that the proposition is true according to the cosine value.

(ii) It is obvious that the proposition is true.

(iii) When $\tilde{A} = \tilde{B}$, there are

$\mu_{\tilde{A}L}(x_i) = \mu_{\tilde{A}U}(x_i) = \mu_{\tilde{B}L}(x_i) = \mu_{\tilde{B}U}(x_i)$ and

$\nu_{\tilde{A}L}(x_i) = \nu_{\tilde{A}U}(x_i) = \nu_{\tilde{B}L}(x_i) = \nu_{\tilde{B}U}(x_i)$ for i = 1,2,…,n So there is $C_S(\tilde{A},\tilde{B}) = 1$.

Proposition (ii)

Let the distance measure of the angle as $d(\tilde{A},\tilde{B}) = arccos(C_S(\tilde{A},\tilde{B}))$. then it satisfies the following properties:

(i) $d(\tilde{A},\tilde{B}) \ge 0$ if $0 \le C_S(\tilde{A},\tilde{B}) \le 1$

(ii) $d(\tilde{A},\tilde{B}) = arccos(1) = 0$ if $C_S(\tilde{A},\tilde{B}) = 1$

(iii) $d(\tilde{A},\tilde{B}) = d(\tilde{B},\tilde{A})$ if $C_S(\tilde{A},\tilde{B}) = C_S(\tilde{B},\tilde{A})$

(iv) $d(\tilde{A},\tilde{C}) \le d(\tilde{A},\tilde{B}) + d(\tilde{B},\tilde{C})$ if $\tilde{A} \subseteq \tilde{B} \subseteq \tilde{C}$ for any interval-valued intuitionistic fuzzy sets $\tilde{C}$.

Proof: Obviously, $d(\tilde{A},\tilde{B})$ satisfies the (i)-(iii). In the following, $d(\tilde{A},\tilde{B})$ will be proved to satisfy the (iv).

For any $\tilde{C} = \{\langle x_i, \mu_{\tilde{C}}(x_i), \nu_{\tilde{C}}(x_i)\rangle \mid x_i \in X\}$, $\tilde{A} \subseteq \tilde{B} \subseteq \tilde{C}$, since Eq. (3) is the sum of terms, Let us consider the distance measures of the angle between vectors:

$d_i(\tilde{A}(x_i),\tilde{B}(x_i)) = arccos(C_S(\tilde{A}(x_i),\tilde{B}(x_i)))$,

$d_i(\tilde{B}(x_i),\tilde{C}(x_i)) = arccos(C_S(\tilde{B}(x_i),\tilde{C}(x_i)))$ and

$d_i(\tilde{A}(x_i),\tilde{C}(x_i)) = arccos(C_S(\tilde{A}(x_i),\tilde{C}(x_i)))(i = 1,2,...,n)$, where

$$C_S(\tilde{A},\tilde{B}) = \frac{1}{n}\sum_{i=1}^{n}\frac{(\mu_{\tilde{A}L}(x_i)+\mu_{\tilde{A}U}(x_i))(\mu_{\tilde{B}L}(x_i)+\mu_{\tilde{B}U}(x_i))+(\nu_{\tilde{A}L}(x_i)+\nu_{\tilde{A}U}(x_i))(\nu_{\tilde{B}L}(x_i)+\nu_{\tilde{B}U}(x_i))}{\sqrt{(\mu_{\tilde{A}L}(x_i)+\mu_{\tilde{A}U}(x_i))^2+(\nu_{\tilde{A}L}(x_i)+\nu_{\tilde{A}U}(x_i))^2}\sqrt{(\mu_{\tilde{B}L}(x_i)+\mu_{\tilde{B}U}(x_i))^2+(\nu_{\tilde{B}L}(x_i)+\nu_{\tilde{B}U}(x_i))^2}} \quad (4)$$

$$C_S(\tilde{B},\tilde{C}) = \frac{1}{n}\sum_{i=1}^{n}\frac{(\mu_{\tilde{B}L}(x_i)+\mu_{\tilde{B}U}(x_i))(\mu_{\tilde{C}L}(x_i)+\mu_{\tilde{C}U}(x_i))+(\nu_{\tilde{B}L}(x_i)+\nu_{\tilde{B}U}(x_i))(\nu_{\tilde{C}L}(x_i)+\nu_{\tilde{C}U}(x_i))}{\sqrt{(\mu_{\tilde{B}L}(x_i)+\mu_{\tilde{B}U}(x_i))^2+(\nu_{\tilde{B}L}(x_i)+\nu_{\tilde{B}U}(x_i))^2}\sqrt{(\mu_{\tilde{C}L}(x_i)+\mu_{\tilde{C}U}(x_i))^2+(\nu_{\tilde{C}L}(x_i)+\nu_{\tilde{C}U}(x_i))^2}} \quad (5)$$

$$C_S(\tilde{A},\tilde{C}) = \frac{1}{n}\sum_{i=1}^{n}\frac{(\mu_{\tilde{A}L}(x_i)+\mu_{\tilde{A}U}(x_i))(\mu_{\tilde{C}L}(x_i)+\mu_{\tilde{C}U}(x_i))+(\nu_{\tilde{A}L}(x_i)+\nu_{\tilde{A}U}(x_i))(\nu_{\tilde{C}L}(x_i)+\nu_{\tilde{C}U}(x_i))}{\sqrt{(\mu_{\tilde{A}L}(x_i)+\mu_{\tilde{A}U}(x_i))^2+(\nu_{\tilde{A}L}(x_i)+\nu_{\tilde{A}U}(x_i))^2}\sqrt{(\mu_{\tilde{C}L}(x_i)+\mu_{\tilde{C}U}(x_i))^2+(\nu_{\tilde{C}L}(x_i)+\nu_{\tilde{C}U}(x_i))^2}} \quad (6)$$

For three vectors $\tilde{A}(xi) = \langle[\mu_{\tilde{A}L}(x_i),\mu_{\tilde{A}U}(x_i)],[\nu_{\tilde{A}L}(x_i),\nu_{\tilde{A}U}(x_i)]\rangle$, $\tilde{B}(xi) = \langle[\mu_{\tilde{B}L}(x_i),\mu_{\tilde{B}U}(x_i)],[\nu_{\tilde{B}L}(x_i),\nu_{\tilde{B}U}(x_i)]\rangle$, $\tilde{C}(xi) = \langle[\mu_{\tilde{C}L}(x_i),\mu_{\tilde{C}U}(x_i)],[\nu_{\tilde{C}L}(x_i),\nu_{\tilde{C}U}(x_i)]\rangle$, in a plane, if $\tilde{A}(x_i) \subseteq \tilde{B}(x_i) \subseteq \tilde{C}(x_i)$ (i = 1,2,…,n), then it is obvious that $d_i(\tilde{A}(x_i),\tilde{C}(x_i)) \le d_i(\tilde{A}(x_i),\tilde{B}(x_i)) + d_i(\tilde{B}(x_i),\tilde{C}(x_i))$. According to the triangle inequality. Combining the inequality with Eq. (3), we can obtain $d(\tilde{A}),\tilde{C}) \le d(\tilde{A},\tilde{B}) + d(\tilde{B},\tilde{C})$. Thus $d(\tilde{A},\tilde{B})$ satisfies the property (iv). So we have finished the proof.

Comparison of Similarity Measure Results

Let $\tilde{A}$ and $\tilde{B}$ be two intuitionistic fuzzy sets in the universe of discourse X = $x_1, x_2, ..., x_n$. For the comparisons between the cosine similarity and the existing similarity measures of interval valued intuitionistic fuzzy sets (Table 1) introduced in [35], they are listed as follows:

$$\text{(i)}\ S_C(\tilde{A},\tilde{B}) = 1 - \frac{\sum_{i=1}^{n}|S_{\tilde{A}}(x_i) - S_{\tilde{B}}(x_i)|)}{2n} \quad (7)$$

where $S_{\tilde{A}}(x_i) = \mu_{\tilde{A}}(x_i) - \nu_{\tilde{A}}(x_i)$ and $S_{\tilde{B}}(x_i) = \mu_{\tilde{B}}(x_i) - \nu_{\tilde{A}}(x_i)$

$$\text{(ii)}\ S_H(\tilde{A},\tilde{B}) = 1 - \frac{\sum_{i=1}^{n}(|\mu_{\tilde{A}}(x_i) - \mu_{\tilde{B}}(x_i)| + |\nu_{\tilde{A}}(x_i) - \nu_{\tilde{B}}(x_i)|)}{2n} \quad (8)$$

$$\text{(iii)}\ S_L(\tilde{A},\tilde{B}) = 1 - \frac{\sum_{i=1}^{n}|S_{\tilde{A}}(x_i) - S_{\tilde{B}}(x_i)|)}{4n} - \frac{\sum_{i=1}^{n}(|\mu_{\tilde{A}}(x_i) - \mu_{\tilde{B}}(x_i)| - |\nu_{\tilde{A}}(x_i) - \nu_{\tilde{B}}(x_i)|)}{4n} \quad (9)$$

$$\text{(iv)}\ S_O(\tilde{A},\tilde{B}) = 1 - \sqrt{\frac{\sum_{i=1}^{n}(\mu_{\tilde{A}}(x_i) - \mu_{\tilde{B}}(x_i))^2 + (\nu_{\tilde{A}}(x_i) - \nu_{\tilde{B}}(x_i))^2}{2n}} \quad (10)$$

$$\text{(v)}\ S_{DC}(\tilde{A},\tilde{B}) = 1 - \sqrt[p]{\frac{\sum_{i=1}^{n}|\psi_{\tilde{A}}(x_i) - \psi_{\tilde{B}}(x_i)|^p}{n}} \quad (11)$$

	Set 1	Set 2	Set 3	Set 4	Set 5	Set 6
$\tilde{A}$	⟨0.3,0.3⟩	⟨0.3,0.4⟩	⟨1.3⟩	⟨0.5,0.5⟩	⟨[0.4,0.8],[0.2,0.6]⟩	⟨[0.4,0.6],[0.2,0.4]⟩
$\tilde{B}$	⟨0.4,0.4⟩	⟨0.4,0.3⟩	⟨0.0⟩	⟨0.0⟩	⟨[0.5,0.7],[0.3,0.6]⟩	⟨[0.5,0.9],[0.2,0.7]⟩
S_C	1	0.9	0.5	1	N.A.	N.A.
S_H	0.9	0.9	0.5	0.5	N.A.	N.A.
S_L	0.95	0.9	0.5	0.75	N.A.	N.A.
S_0	0.9	0.9	0.3	0.5	N.A.	N.A.
S_{DC}	1	0.9	0.5	1	N.A	N.A.
S_e^P	0.9	0.9	0.5	0.5	N.A.	N.A.
X_u	0.88	0.97	0.5	0.5	N.A.	N.A.
C_S	0.9	0.9	0.5	0.5	0.99	0.99

Table 1: Comparison of results of proposed and existing similarity measures.

where $\psi_{\tilde{A}}(x_i)=\dfrac{(\mu_{\tilde{A}}(x_i)+1-\nu_{\tilde{A}}(x_i))}{2}$, $\psi_{\tilde{B}}(x_i)=\dfrac{(\mu_{\tilde{B}}(x_i)+1-\nu_{\tilde{B}}(x_i))}{2}$;

(vi) $$S_e^p(\tilde{A},\tilde{B})=1-\sqrt[p]{\frac{\sum_{i=1}^{n}(\varphi_{\tilde{\mu}}(x_i)-\varphi_{\tilde{\nu}}(x_i))^p}{n}} \quad (12)$$

where $\varphi_{\tilde{\mu}}(x_i)=\dfrac{|\mu_{\tilde{A}}(x_i)-\mu_{\tilde{B}}(x_i)|}{2}$ and

$\varphi_{\tilde{\nu}}(x_i)=\left|1-\dfrac{\nu_{\tilde{A}}(x_i)}{2}-\dfrac{(1-\mu_{\tilde{B}}(x_i))}{2}\right|$ (vii) Xu et al. [24] proposed similarity measure, as concept of association coefficient, for intuitionistic fuzzy sets as follows:

$$X_u(\tilde{A},\tilde{B})=\frac{\sum_{i=1}^{n}(\mu_{\tilde{A}}(x_i).\mu_{\tilde{B}}(x_i))+(\nu_{\tilde{A}}(x_i).\nu_{\tilde{B}}(x_i))+(\pi_{\tilde{A}}(x_i).\pi_{\tilde{B}}(x_i))}{max(\sum_{i=1}^{n}(\mu_{\tilde{A}}^2(x_i)+\nu_{\tilde{A}}^2(x_i)+\pi_{\tilde{A}}^2(x_i)),\ \sum_{i=1}^{n}(\mu_{\tilde{B}}^2(x_i)+\nu_{\tilde{B}}^2(x_i)+\pi_{\tilde{B}}^2(x_i)))} \quad (13)$$

Application of Cosine Similarity Measure for Interval-Valued Intuitionistic Fuzzy Numbers to Pattern Recognition

In order to demonstrate the application of the proposed cosine measure for interval-valued intuitionistic fuzzy numbers to pattern recognition, we discuss the medical diagnosis problem as follow:

Let us consider a set of diagnosis $\tilde{A}=\{\tilde{A}_1$ (*Viral fever*), $\tilde{A}_2$ (*Malaria*), $\tilde{A}_3$ (*Typhoid*)$\}$ and a set of symptoms $S=\{x_1$(*Temprature*), x_2 (*Headache*), x_3 (*Cough*)$\}$. Suppose a patient, with respect to all the symptoms, can be represented by the following interval-valued intuitionistic fuzzy numbers:

$$\tilde{P}=\{\langle x_1,[0.6,0.8],[0.3,0.5]\rangle,\langle x_2,[0.3,0.7],[0.2,0.4]\rangle,\langle x_3,[0.6,0.8],[0.3,0.5]\rangle\}$$

And each diagnosis $\tilde{A}_i$ (i = 1,2,3) can also be represented by interval-valued intuitionistic fuzzy numbers with respect to all the symptoms as follows:

$$\tilde{A}_1=\{\langle x_1,[0.4,0.5],[0.3,0.4]\rangle,\langle x_2,[0.4,0.6],[0.2,0.4]\rangle,\langle x_3,[0.4,0.8],[0.1,0.2]\rangle\}$$

$$\tilde{A}_2=\{\langle x_1,[0.3,0.6],[0.3,0.4]\rangle,\langle x_2,[0.5,0.6],[0.3,0.4]\rangle,\langle x_3,[0.4,0.5],[0.1,0.3]\rangle\}$$

$$\tilde{A}_3=\{\langle x_1,[0.7,0.8],[0.1,0.2]\rangle,\langle x_2,[0.6,0.7],[0.1,0.3]\rangle,\langle x_3,[0.3,0.4],[0.1,0.2]\rangle\}$$

Our aim is to classify the pattern $\tilde{P}$ in one of the classes $\tilde{A}_1$, $\tilde{A}_2$ and $\tilde{A}_3$. According to the recognition principle of maximum degree of similarity between interval-valued intuitionistic fuzzy numbers, the process of diagnosis $\tilde{A}_k$ to patient $\tilde{P}$ is derived according to

$$k=arg\ Max\{C_S(\tilde{A}_i,\tilde{P})\}$$

From the previous formula (3), we can compute the cosine similarity measure between $\tilde{A}_i$ (I = 1,2,3) and $\tilde{P}$ as follows:

$$C_S(\tilde{A}_1,\tilde{P})=0.9312,\ C_S(\tilde{A}_2,\tilde{P})=0.9360 \text{ and } C_S(\tilde{A}_3,\tilde{P})=0.9125.$$

Then, we can assign the patient to diagnosis $\tilde{A}_2$ (*Malaria*) according to recognition of principal.

Conclusions

In this paper a new similarity measure for interval-valued intuitionistic fuzzy sets is proposed. The results of the proposed similarity measure and existing similarity measures are compared. Finally, the proposed similarity measure is applied to pattern recognition.

References

1. Zadeh LA (1965) Fuzzy sets. Inform Control 8: 338-353.
2. Chen SM (1994) A weighted fuzzy reasoning algorithm for medical diagnosis. Decis Support Syst 11: 37-43.
3. Chen SM Yeh MS, Hsiao PY (1995) A comparison of similarity measures of fuzzy values. Fuzzy Set Syst 72: 79-89.
4. Hyung LK, Song YS, Lee KM (1994) Similarity measures between fuzzy sets and between elements. Fuzzy Set Syst 62: 291-293.

5. Pappis CP, Karacapilidis NI (1993) A comparative assessment of measures of similarity of fuzzy values. Fuzzy Set Syst 56: 171-174.

6. Santini S, Jain R (1999) Similarity measures. IEEE Trans on Pattern Anal Mach Intell 21: 871-883.

7. Wang WJ (1997) New similarity measures on fuzzy sets and on elements. Fuzzy Set Syst 85: 305-309.

8. Zwich R, Carlstein E, Budescu DV (1987) Measures of similarity among fuzzy sets: A comparative analysis. Int J Approx Reason 1: 221-242.

9. Atanassov K (1986) Intuitionistic fuzzy sets. Fuzzy Set Syst 20: 87-96.

10. Atanassov K (1989) More on intuitionistic fuzzy sets. Fuzzy Set Syst 33: 37-46.

11. Gau WL, Buehere DJ (1993) Vague sets. IEEE T Syst Man Cyb 23: 610-614.

12. Bustince H, Burillo P (1996) Vague sets are intuitionistic fuzzy sets. Fuzzy Set Syst 79: 403-405.

13. Chen SM, Tan JM (1994) Handling multi-criteria fuzzy decision-making problems based on vague sets. Fuzzy Set Syst 67: 163-172.

14. De SK, Biswas A, Roy AR (2000) Some operations on intuitionistic fuzzy sets. Fuzzy Set Syst 114: 477-484.

15. Szmidt E, Kacprzyk J (2000) Distances between intuitionistic fuzzy sets. Fuzzy Set Syst 114: 505-518.

16. Dengfeng L, Chuntian C (2002) New similarity measures of intuitionistic fuzzy sets and application to pattern recognition. Pattern Recogn Lett 23: 221-225.

17. Liang Z, Shi P (2003) Similarity measures on intuitionistic fuzzy sets. Pattern Recog Lett 24: 2687-2693.

18. Mitchell HB (2003) On the Dengfeng-Chuntian similarity measure and its application to pattern recognition. Pattern Recog Lett 24: 3101-3104.

19. Hung WL, Yang MS (2004) Similarity measures of intuitionistic fuzzy sets based on Hausdorff distance. Pattern Recog Lett 25: 1603-1611.

20. Liu HW (2005) New similarity measures between intuitionistic fuzzy sets and between elements. Math Comput Model 42: 61-70.

21. Xu Z (2006) On correlation measures of intuitionistic fuzzy sets. Lect Notes Comput Sc 4224: 16-24.

22. Hung WL, Yang MS (2007) Similarity measures of intuitionistic fuzzy sets based on L_p metric. Int J Approx Reason 46: 120-136.

23. Xu Z (2007) Some similarity measures of intuitionistic fuzzy sets and their applications to multiple attribute decision making, Fuzzy Optim Decis Making 6: 109-121.

24. Xu Z, Chen J, Wu J (2008) Clustering algorithm for intuitionistic fuzzy sets. Inform Sciences 178: 3775-3790.

25. Lee W (2009) A novel method for ranking interval-valued intuitionistic fuzzy numbers and its application to decision making. International Conference on Intelligent Human-Machine Systems and Cybernetics 282-285.

26. Xia M, Xu Z (2010) Some new similarity measures for intuitionistic fuzzy values and their application in group decision making. J Syst Sci Syst Engin 19: 430-452.

27. Xu FS (2009) A new method on measures of similarity between vague sets. World Congress on Computer Science and Information Engineering.

28. Xu Z (2010) A method based on distance measure for interval-valued intuitionistic fuzzy group decision making. Inform Sciences 180: 181-190.

29. Guha D, Chakraborty D (2010) A theoretical development of distance measure for intuitionistic fuzzy numbers. Internat J Math Math Sci.

30. Xu Z, Xia M (2011) Distance and similarity measures for hesitant fuzzy sets. Inform Sciences 181: 2128-2138.

31. Atanassov K, Gargov G (1989) Interval-valued intuitionistic fuzzy sets. Fuzzy Set Syst 31: 343-349.

32. Gerstenkorn T, Manko J (1991) Correlation of intuitionistic fuzzy sets, Fuzzy Set Syst 44: 39-43.

33. Bhattacharya A (1946) On a measure of divergence of two multinomial populations. Sankhya Ser A 7: 401-406.

34. Salton G, McGill MJ (1983) Introduction to modern information retrieval, McGraw-Hill Book Company, New York.

35. Li YH, Olson DL, Qin Z (2007) Similarity measures between intuitionistic fuzzy (vague) sets: a comparative analysis. Pattern Recog Lett 28: 278-285.

36. Ye J (2010) Fuzzy decision-making method based on the weighted correlation coefficient under intuitionistic fuzzy environment. Eur J Oper Res 205: 202-204.