Journal of Physical Mathematics
ISSN: 2090-0902
Zoabi, J Phys Math 2019, 10:2
Research Article
Open Access

# A New Approach for the Derivation of Lorentz Transformations and Its Implications on Understanding the Special Theory of Relativity

**Zoabi E***
*The Arab Academic College for Education, Haifa, Israel*

**Abstract**

In this article we introduce a new approach for obtaining Lorentz transformations. It is based on tracking light that is emitted from an object that reaches two stationary observers who are in motion relative to one other.

This new approach, based on known physical principles and not on an axiom, makes the Lorentz transformations more understandable. In addition, it reveals the meaning of the constancy of the speed of light as well as the meaning of the results of the Special Theory of Relativity.

The new approach also reveals that Lorentz transformations for an object found at a general point ($x'$,$y'$) at the coordinate system, cannot be achieved as a superposition of the transformations of the coordinates ($x'$,0) and (0,$y'$). In this case we need to take into account the time delay between cases.

Light aberration, the Doppler effect, time dilation and length contraction are considered within the new approach of obtaining Lorentz transformations.

**Keywords:** Lorentz transformation; Special relativity; Speed of light; Light aberration; Doppler effect

## 1-Introduction - Galilean and Lorentz Transformations

We know that Galilean transformations are mathematical relations that relate the coordinates of the location of an event that is measured by two different stationary observers, under the assumption that the time is the same for the two observers.

In Figure 1, two stationary frames are shown; frame S which is defined by the xyz coordinate system and frame $S'$ which is defined by the $x'$,$y'$,$z'$ coordinate system (for simplicity, the z and $z'$ axes are not shown). It is given that the x and $x'$ axes overlap and that the $y$,$y'$, and the $z$, $z'$, axes are parallel. It is given that frame $S'$ moves with constant velocity v relative to frame S along the x-axis, and at t=0 the two origins, $O$ and $O'$ of frames S and $S'$, respectively, coincide [1-4].

If there are two observers at $O$ and $O'$, and the observer at $O$ observes an event A that happens at time $t_A$ at location ($x_A$,$y_A$,$z_A$) as described in Figure 1, then according to Galilean transformations, the observer at $O'$ will observe the same event at time $t'_A$ at location $(x'_A, y'_A, z'_A)$ which is related to $t_A$ and $(x_A, y_A, z_A)$ by the following:

$$t'_A = t_A \tag{1}$$

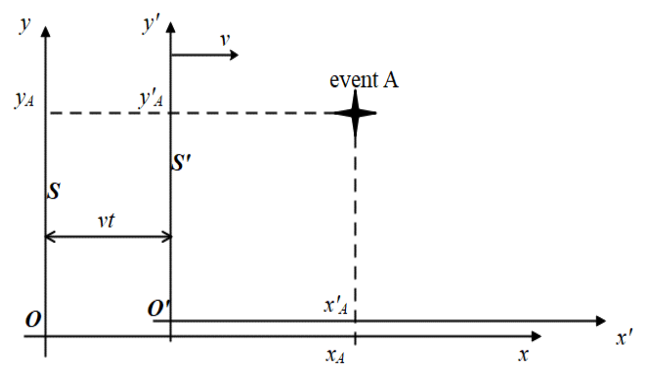


**Figure 1:** Two stationary frames S and S'. S' moves with constant velocity in the positive direction of the axis.

$$x'_A = x_A - vt_A \tag{2}$$

$$y'_A = y_A \tag{3}$$

$$z'_A = z_A \tag{4}$$

Galilean transformations do not take into account the role played by light in measurement, and the fact that light has finite speed. Rather, they are based on the intuitive assumption that light has infinite speed and, therefore, the same event happens at the same time relative to observers at O and $O'$. Therefore, the same event will be measured simultaneously by both observers and the relation $t_A = t'_A$ will be fulfilled.

The fact that the light has finite speed makes Galilean transformations valid only for systems that move relative to each other with velocities that are very low relative to speed of light (that is, for velocities at which the speed of light can be considered infinite) [5,6].

In order to take into account that the measurement of the location of an event made by the two observers found at $O$ and $O'$ occurs at different times as a result of the fact that light has finite speed, Albert Einstein followed the front of the light ray emitted from the origin of one frame (O, for example) toward an event found at ($x_A$,$y_A$,$z_A$). He found the transformation that relates these coordinates to the coordinates $(x'_A, y'_A, z'_A)$ measured by an observer at $O'$ taking into consideration that the light has finite speed. The fact that light has finite speed leads to the conclusion that the same event happens at different times relative to observers at O and $O'$. If we denote the time of the event at frame S by $t_A$ and the time of the event at frame $S'$ by $t'_A$, then $t_A \neq t'_A$.

***Corresponding author:** Zoabi E, The Arab College for Education, Haifa, Israel, Tel: +972 0508271756; E-mail: 012essam@gmail.com

**Received** June 22, 2019; **Accepted** June 29, 2019; **Published** July 06, 2019

**Citation:** Zoabi E (2019) A New Approach for the Derivation of Lorentz Transformations and Its Implications on Understanding the Special Theory of Relativity. J Phys Math 10: 301.

Einstein showed that the relation between measurements $(x_A, y_A, z_A, t_A)$ made by an observer at *O* and measurements $(x'_A, y'_A, z'_A, t'_A)$ made by an observer at *O′* are:

$$t'_A = \frac{t_A - vx/c^2}{\sqrt{1 - v^2/c^2}} \quad (5)$$

$$x'_A = \frac{x_A - vt_A}{\sqrt{1 - v^2/c^2}} \quad (6)$$

$$y'_A = y_A \quad (7)$$

$$z'_A = z_A \quad (8)$$

The relations (5)-(8) were found by the Danish physicist Hendrik Lorentz even before the work of Einstein. Therefore, they are given the name Lorentz transformations. Lorentz discovered these relations in his search for transformations that kept Maxwell's equations invariant when going from one stationary frame to another frame that moves with constant velocity relative to the first frame [7].

Einstein's derivation of Lorentz transformations was revolutionary in physics. His work depends on the assumption that light has the same speed relative to all stationary frames. As this assumption contradicts common sense, it is introduced as a postulate (the second of Einstein's two postulates).

In this article we show that by using a different approach, we can obtain the Lorenz transformations without using this postulate and that the constancy of the speed of light can be derived from the approach that we introduce.

## 2-The Interaction between an Object and Two Observers

In all situations in this article there is an object and an observer that are connected by means of light. Light represents the electromagnetic interaction between charges found at the object and those at the observer (the measuring instruments). This interaction leads to the electromagnetic forces created between the object and the observer that control all the physical processes resulting from this interaction including the act of measurement. This interaction between the object and the observer occurs at the speed of light, c.

When two observers a and b observe an object (Figure 2), two different interactions occur: The first between the object and the first observer and the second between the object and the second observer. In other words, there are two different rays that carry information from the object to the two observers, the first that proceeds from the object to observer a (ray 1 in Figure 2), and the second that proceeds from the same object to observer b (ray 2 in Figure 2) [8-10].

Any measurement of the object's properties (location for example) that are made will be determined only by the ray that reaches that specific observer. The other ray does not play any role. Therefore, any relation between measurements made by observer a and measurements made by observer b is obtained by a transformation that relates the front of ray 1 with the front of ray 2 and not by a transformation that transforms the front of the same ray into the frame of the other observer. We shall later develop Lorentz transformations based on this idea, but first we need to expand the idea of the interaction between an object and two observers by means of light. We do that with the help of Figures 3-5. Figure 3 contains two stationary frames, S and *S′*. Frame *S′* moves with a constant velocity ν in the positive direction along the x-axis of frame *S*. In frame *S′* there is an object which is found at rest at a location M defined by coordinates $(x'_M, y'_M)$. For simplicity we ignore the z direction. Let us assume that at $t = t' = 0$, where the origins *O* and *O′* of frames S and S', respectively, coincide, a spherical light wave is emitted from the object (Figure 3). The front of the wave will expand and move within frame *S′* in the positive direction of the x-axis (Figure 4). This wave front will reach an observer at *O′* by means of a ray of light that travels along line *MO′* with a velocity c and will reach the observer at time *t′* which is given by:

$$t' = \frac{\overline{MO'}}{c} = \frac{\sqrt{x'^2_M + y'^2_M}}{c} \quad (9)$$

In the course of the time, as the spherical wave front continues to expand it will reach an observer at *O* (the origin of frame S) by means of another ray of light which travels along line MO as described in Figure 5 and reaches the observer at time t given by:

$$t = \frac{OM}{c} \quad (10)$$

With the help of the previous description we can:

1. Explain the constancy of the speed of light relative to the source and the observer even if they move one relative to the other (section 3).
2. Derive Lorentz transformations (section 4).

## 3-The Constancy of the Speed of Light

We know that light conveys the interaction from one object to the other. Light is not an object that we can follow by means of another light. It is the interaction itself. Therefore, there is no meaning to any information conveyed from one object to another until the moment that the light reaches the other object. Hence, the speed of light is defined as a physical quantity that enables us to find the time that it takes the information (light) to travel from the first object to the second by the relation t=d/c, where d is the distance between them [11].

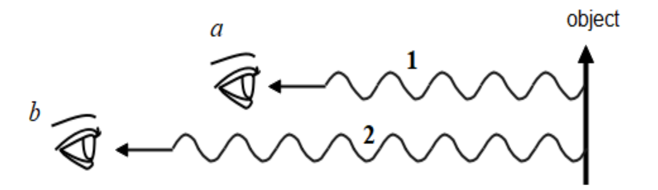


**Figure 2:** Two observers interacting with an object by means of two different rays of light.

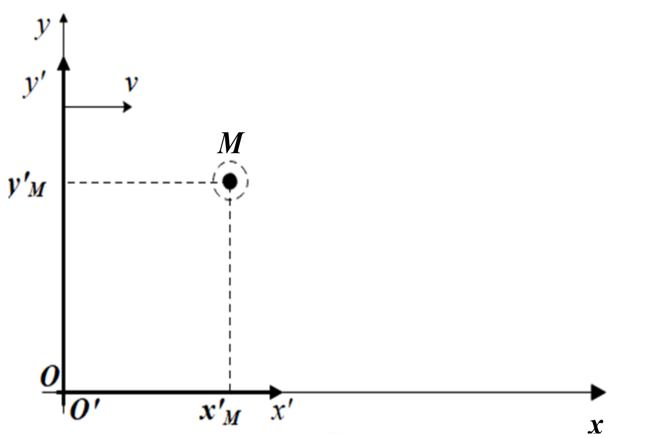


**Figure 3:** A spherical light wave is emitted from an object found in the *S*' frame at *t*=*t*'=0 where the origins of the two frames coincide.

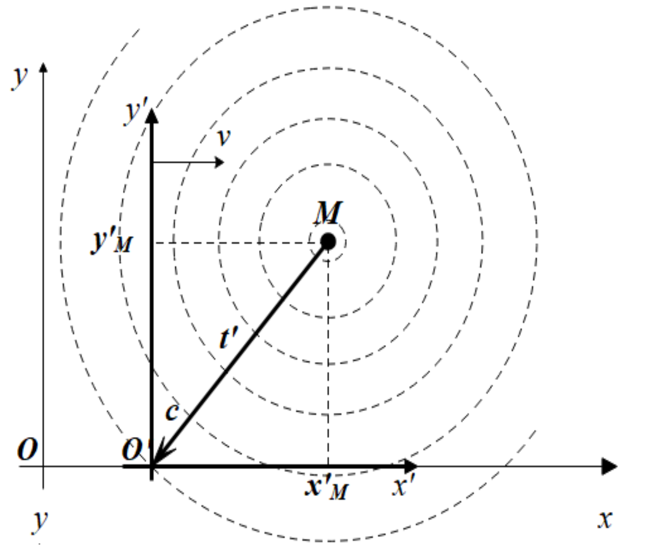


**Figure 4:** The light wave reaches an observer at by a ray that proceeds along line *MO*′.

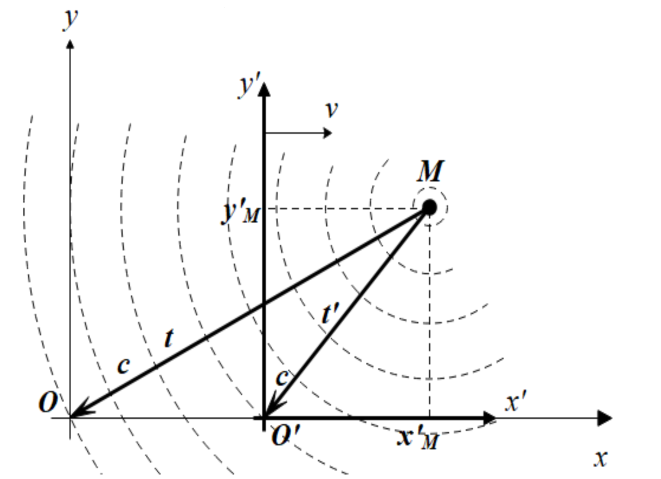


**Figure 5:** The light wave reaches an observer at O by a ray proceeds along line MO.

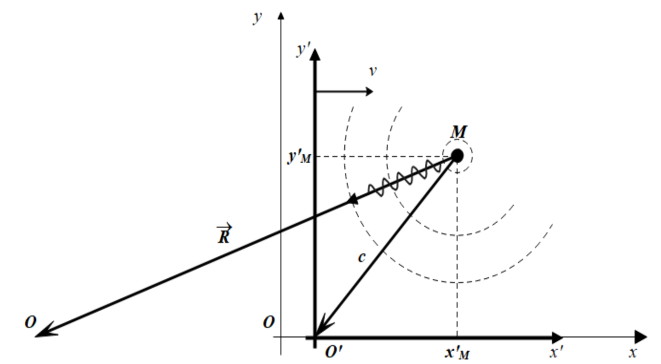


**Figure 6:** Light that travels from object M to the observer at O is represented as a vector attached to the object and moves with it. This vector has the same length and direction as the ray MO at the moment it reaches the origin O.

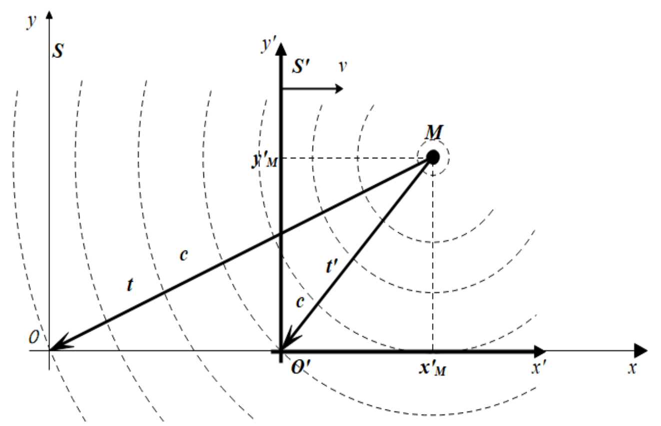


**Figure 7:** Light travels from object to the observers by two different rays.

Using this definition and with the help of the description in the previous section, we can explain the constancy of the speed of light. Turning back to Figure 5 we can show that the speed of the light is the same relative to an observer at O and also to the source M. In spite of the fact that each moves relative to the other. To do that we define line MO in Figure 5 as a vector denoted by $\vec{R}$ which is attached to object M and moves with it relative to frame S, (Figure 6). This vector has a constant direction and length: the direction and the length of MO (Figure 5). When light that was emitted from M reaches the other end of vector $\vec{R}$, this end will be exactly at O. Therefore, when light from M reaches O along line $\overline{MO}$, it travels the same length at the same time relative to the observer at *O* as it does relative to the source M. This means that light travels along MO with the same speed relative to the source and the observer [12-14].

Therefore, we see that the constancy of the speed of light results from the fact that light spread from the source in all directions (in straight lines as photons, or in rays produced by the expansion of the light as waves). As a result, light passes from an object to an observer along a line that is produced by the light spreading. Light proceeds along this line and travels for the same length and duration relative to the source and the observer, even if both move relative to each other (Figures 5 and 6).

## 4-New Approach for Derivation of Lorentz Transformations

### 4.1-The two-ray description

The details described in Figures 4-5 can be combined into one figure which (Figure 7). According to this figure, light emitted from object M at $t = t' = 0$ when *O* and *O*′ coincide, will reach observers at *O* and *O*′ at different times. It will reach the observer at *O* by a ray that proceeds along line MO at time $t = \overline{MO} / c$ and it will reach the observer at *O*′ by a ray that proceeds along line *MO*′ at time $t' = \overline{MO'} / c$, such that $t > t'$.

To handle the physical situation described in Figure 7, mathematically, it is easier to describe it by the following equivalent reverse process: when the two origins, *O* and *O*′ of the two stationary frames S and *S*′, respectively, coincide at $t = t' = 0$, two spherical light waves are emitted simultaneously from the origins of the two frames. That is, the first from the origin *O* of frame S and the second from origin *O*′ of frame *S*′ (Figure 8). In frame *S*′ light will reach object M by means of a ray that travels along line $O'M$ with velocity c at time $t' = O'M / c$, and in frame S, light will reach object M by means of a ray that proceeds along line OM with velocity c and will reach it at time t=OM/c.

In this reverse two-ray description the following properties hold:

1. The times t and $t'$ that it takes rays emitted from $O$ and $O'$ to reach object M are equal to the times that light emitted from the object M takes to reach origins $O$ and $O'$, respectively.
2. The location of the front of the ray travelling in frame S from $O$ to M at time $t$ is the location of the object in frame S, that is $\vec{r} = (ct)\hat{r}$, where $\hat{r}$ is a unit vector in the direction of OM, and $\vec{r}$ is the location vector of the object. The location of the front of the ray that travels in frame $S'$ from $O'$ to M at $t'$ is the location of the object in frame $S'$, that is $\vec{r}' = (ct')\hat{r}'$, where $\hat{r}'$ is a unit vector in the direction of $O'M$, and $\vec{r}'$ is the location vector of the object in this frame (Figure 8).

We shall see in the following subsections that:

1. The Lorentz transformation with respect to time, relates simultaneously the times t and $t'$, provided that the rays emitted from the object reach the two observers simultaneously. Obviously, these two rays were neither emitted from the object simultaneously nor emitted from the same location in frame S.
2. The Lorentz transformation for the coordinates of location, is a transformation which relates, simultaneously, between the locations of the fronts of the two rays emitted from the origins of the frames, S and $S'$ at $t = t' = 0$, toward the object.

### 4.2-The two-ray description and time transformation

As mentioned in the previous section, the measurements made by the two observers depend upon the light that reaches them from the object. This process is described more easily if we follow the rays that are simultaneously emitted from the observers at origins $O$ and $O'$ when they coincide at $t = t' = 0$, toward the object (Figure 9).

To find the transformation that relates the location of the object measured by one observer to the location of the same object as measured by the other observer, we must first find a transformation

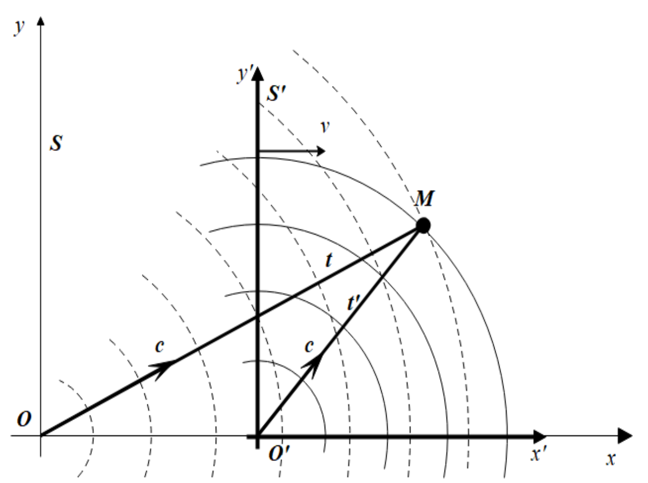


**Figure 8:** Two spherical light waves emitted simultaneously from origins O and $O'$ when they coincide at $t = t' = 0$.

that relates the time $t$ that the first ray (emitted from $O$) takes to reach the object, and the time $t'$ that the second ray (emitted from $O'$) takes to reach the object.

In order to do that we denote the location of object M, which is at rest in frame $S'$, by $(x', y')$, and the location of this object in frame S by (x,y) (Figure 9). The coordinates $(x', y')$ are the location of the front of ray $O'M$ at time $t'$, and the coordinates (x,y) are the location of the front of ray OM at time t.

According to this description, and with the aid of Figure 9, the following relations hold:

$$y' = y \tag{11}$$

$$x^2+y^2=c^2t^2 \tag{12}$$

$$x'^2 + y'^2 = c^2t'^2 \tag{13}$$

From the last three equations we get:

$$x^2 + \left(ct'^2 - x'^2\right) = c^2t^2 \tag{14}$$

Substituting $x = vt + x'$ (Figure 9), we get:

$$\left(vt + x'\right)^2 + \left(ct'^2 - x'^2\right) = c^2t^2 \tag{15}$$

From this equation we obtain:

$$\left(1 - \frac{v^2}{c^2}\right)t^2 - \frac{2vx'}{c^2}t - t'^2 = 0 \tag{16}$$

Solving the last equation for t we get:

$$t = \frac{\dfrac{vx'}{c^2} + \sqrt{\dfrac{v^2x'^2}{c^4} + t'^2\left(1 - \dfrac{v^2}{c^2}\right)}}{1 - v^2 / c^2} \tag{17}$$

Where

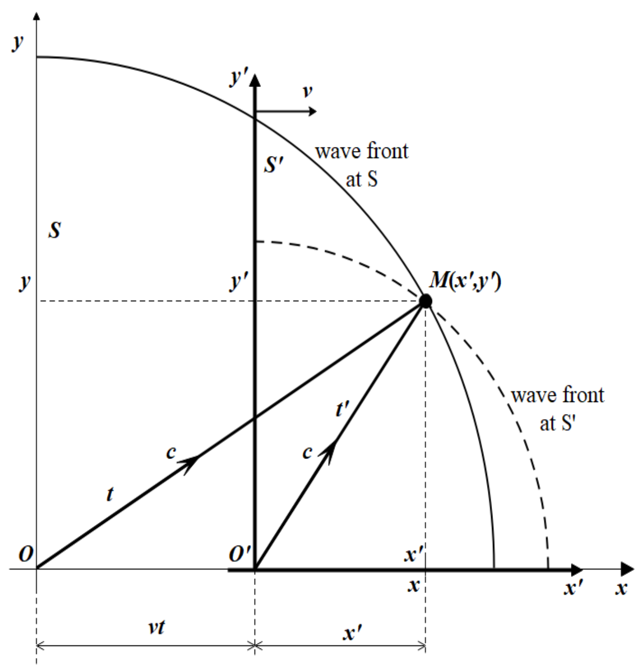


**Figure 9:** The fronts of the spherical light waves that were simultaneously emitted from the origins when they coincided at $t = t' = 0$ when each of them reached the object.

$$t'^2 = \frac{x'^2 + y'^2}{c^2} \tag{18}$$

We have the following two special cases:

### Case 1: The object is found in frame $S'$ on the $x'$ axis

When we move the location of object M along arc $\widehat{MA}$ (Figure 10) such that t stays constant until we reach the $x'$ axis at point $A:(x'_A, 0)$, we get:

$$t'_A = \frac{x'_A}{c} \tag{19}$$

And from eqn. (17) we obtain:

$$t = t_A = \frac{vx'_A / c^2 + \sqrt{v^2 t'^2_A / c^2 + t'^2_A \left(1 - v^2/c^2\right)}}{1 - v^2/c^2} = \frac{vx'_A / c^2 + t'_A}{1 - v^2/c^2} \tag{20}$$

### Case 2: The object is found in frame $S'$ on the axis:

When we move the location of object M along arc $\widehat{MB}$ such that t stays constant until we reach the $y'$ axis at point $B:(0, y'_B)$, from eqn. (17) we get (Figure 10):

$$t = t_B = \frac{0 + \sqrt{0 + y'^2_B / c^2 \left(1 - v^2/c^2\right)}}{1 - v^2/c^2} = \frac{y'_B / c}{\sqrt{1 - v^2/c^2}} = \frac{t'_B}{\sqrt{1 - v^2/c^2}} \tag{21}$$

If we multiply eqns. (20) and (21) by c, and use the relations $x_A = ct_A$, $x'_A = ct'_A, y'_B = ct'_B$ and $y_C = ct_C$, we get:

$$x_A = \frac{vt'_A + x'_A}{1 - v^2/c^2} \tag{22}$$

$$y_C = \frac{y'_B}{\sqrt{1 - v^2/c^2}} \tag{23}$$

And with the common notations:

$$x = \frac{vt' + x'}{1 - v^2/c^2} \tag{24}$$

And

$$y = \frac{y'}{\sqrt{1 - v^2/c^2}} \tag{25}$$

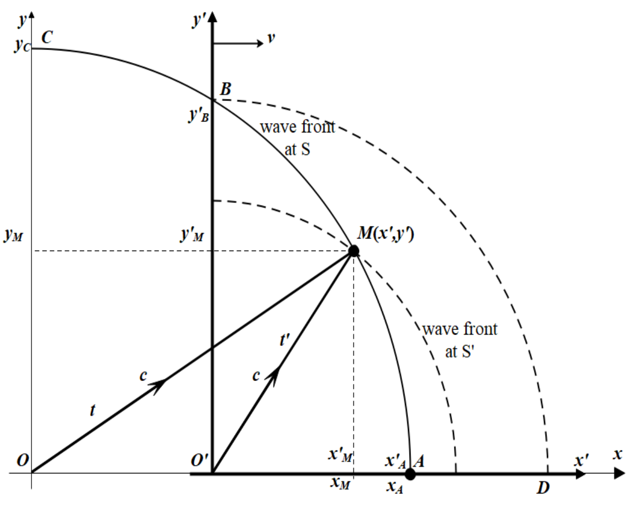


**Figure 10:** When we move along arc $\widehat{AB}$ the time t remains constant and when we move along arc $\widehat{CD}$ the time $t'$ remains constant.

Note that according to the description in Figure 10, the general transformation (17), and the two special cases given by eqns. (20) and (21) are actually transformations that relate $t'$, the time that it takes light to travel from the object to $O'$ to t, the time that it takes light to travel from the object to $O$, under the condition that the light was emitted simultaneously from the object towards the two origins. Because $t > t'$, light does not reach $O$ and $O'$ simultaneously. Therefore, the measurements made by the two observers in this case are not simultaneous.

### 4.3-The two-ray description, simultaneity and Lorentz transformations

In order to compare measurements of the location of the object made by the two observers, these measurements must be performed simultaneously. The meaning of simultaneity here is that the light emitted from the object reaches observers at $O$ and $O'$ at the same time. Obviously, light emitted from the object that reaches each observer simultaneously need not be emitted from the object at the same time nor be emitted from the same location relative to frame S since frame $S'$ (and with it the object) is moving.

There are two different approaches to finding the relation between t and $t'$ when measurements of the object made by the two observers are simultaneous:

**The first approach:** If two spherical light waves are emitted from origins $O$ and $O'$ when they coincide at $t = t' = 0$, then simultaneous determination of the locations of the fronts of the two waves requires that this measurement occur at the same time in frames S and $S'$. In other words, simultaneity requires that measurement of the location of the fronts of the two waves happen at times $t = t'$.

Turning back to Figure 10, we find that the fronts of waves $\widehat{BD}$ and $\widehat{CA}$ are not simultaneous because they appear at different times. In order that these two waves become simultaneous, we must determine them at the same time $(t = t')$. Since $y_C = ct$ and $y'_B = ct'$, then, simultaneity requires that $y_C = y'_B$. Therefore, according to relation (23), in order to obtain $y_C = y'_B$, we must gauge eqn. (23) by multiplying the right hand side of this equation by $\sqrt{1 - v^2/c^2}$. Because $x_A$ and $y_C$ are on the front of the same wave (Figure 10), we must also multiply the right hand side of eqn. (22) by the same factor. Therefore, the following transformations:

$$x_A = \frac{vt'_A + x'_A}{\sqrt{1 - v^2/c^2}} \tag{26}$$

And

$$y_B = y'_B \tag{27}$$

do fulfill simultaneity.

From eqn. (26) and with the help of the relation t=x/c we get:

$$t = \frac{vx'/c^2 + t'}{\sqrt{1 - v^2/c^2}} \tag{28}$$

**The second approach:** Let us assume that the object is found in frame $S'$ on the $x'$ axis at point A defined by $(x'_A, 0)$. Let us assume that when the object reaches some point E in frame S whose coordinates are $(x_E, 0)$, light is emitted from the object towards the observer at $O$ (Figure 11). Let $t_E$ be the time that it takes for light to travel from E to $O$ $(t_E = t_{OE})$ and $t'_A$ the time that it takes this light to reach $O'$. In order for light emitted from the object to reach the observers at $O$ and $O'$

simultaneously, light emitted from the object towards the observer at $O'$ must be delayed by time $\Delta t'$ relative to light emitted from the object towards $O$ (when it passed through E ), such that the relation:

$$\Delta t' + t'_A = t_E \tag{29}$$

holds. If we multiply the last relation by the speed of the light, c, we find that the delay $(\Delta t')$ is equivalent to that of the light emitted from some point F on the $x'$ axis toward $O'$ such that the following relation holds (Figure 11):

$$FO' = EO \tag{30}$$

This means that if light is emitted from points E and F simultaneously, it will reach $O'$ and $O$ simultaneously. The last relation is the condition that must be fulfilled in order for the two observers to make simultaneous measurements.

In order to translate condition (30) into a relation between times $t'_A$ and $t_E$, we follow the reverse process: two spherical wave fronts are emitted from $O$ and $O'$ simultaneously at $t = t' = 0$ when they coincide. Light emitted from $O$ reaches the object at A in time $t_A$, which is related to $t'_A$ (the time it takes the light to travel from $O'$ to A) by the following relation:

$$t_A = \frac{vx'_A / c^2 + t'_A}{1 - v^2 / c^2} \tag{31}$$

Drawing arc $\widehat{ABC}$ whose center is at $O$ (Figure 12), then, because $\overline{BO} = \overline{AO}$ we have:

$$t_{OB} = t_A = \frac{vx'_A / c^2 + t'_A}{1 - v^2 / c^2} \tag{32}$$

To determine the locations of points E and F which are referred to Figure 11 and that fulfill the condition of simultaneity, we draw two arcs: the first $\widehat{BF}$ in frame $S'$ with its centre at $O'$, and the second $\widehat{DE}$ in frame S with its centre at O. These two arcs must have the following properties:

1) They have the same radius (to maintain simultaneity)

2) Their radii are equal to $O'B$ in order to be able to relate time $t_E$ to $t'_A$.

The arcs $\widehat{BF}$ and $\widehat{DE}$ that fulfil these two conditions appear in (Figure 12). The intersection of these two arcs with the $x'$ and x axes, determine the location of points F and E, respectively (Figure 12).

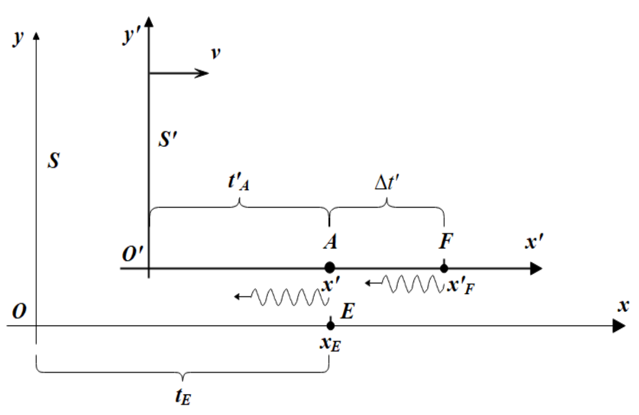


**Figure 11:** If *FO'=EO*, then light simultaneously emitted from F and E, reaches O' and O, respectively, simultaneously.

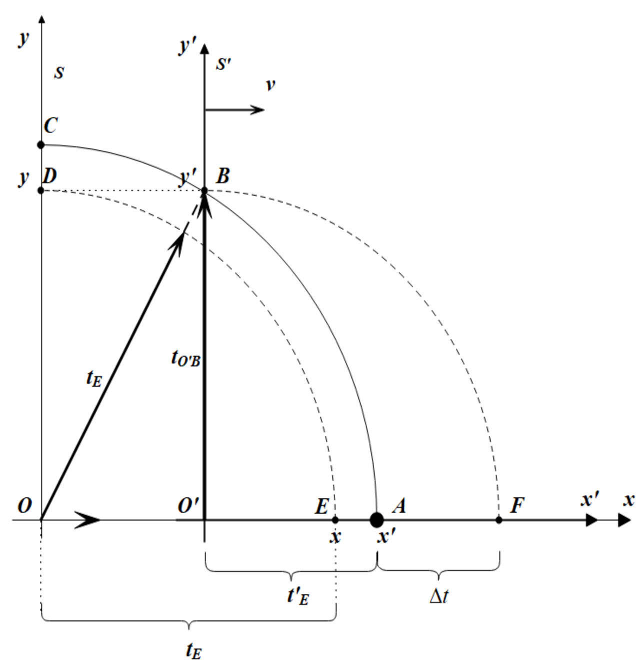


**Figure 12:** From the condition of simultaneity: *FO'=EO*, we get that $t_E = O'B$, from which we obtain eqn. (36).

According to relation (21) we have:

$$t_{OB} = \frac{t_{O'B}}{\sqrt{1 - v^2 / c^2}} \tag{33}$$

Since $t_{O'B} = t_{O'F}$, we obtain

$$t_{OB} = \frac{t_{O'F}}{\sqrt{1 - v^2 / c^2}} \tag{34}$$

from the last equation.

From the condition of simultaneity in eqn. (30) we find that $t_{O'F} = t_E$. Therefore, from the last equation we obtain:

$$t_E = t_{OB}\sqrt{1 - v^2 / c^2} \tag{35}$$

Substituting $t_{OB}$ from eqn. (32) we get:

$$t_E = \left(\frac{vx'_A / c^2 + t'_A}{1 - v^2 / c^2}\right)\sqrt{1 - v^2 / c^2} = \frac{vx'_A / c^2 + t'_A}{\sqrt{1 - v^2 / c^2}} \tag{36}$$

This relation between $t'_A$ and time $t(t_E)$ fulfils the condition of simultaneous measurement.

To find the relation between the location of the object in frame $S'$, which is $x'_A$ and its location $x_E$ which is measured in frame S, we multiply the last equation by c and use relations $x' = ct'$ and $x_E = ct_E$:

$$= \frac{vx'_A \;\; c + ct'_A}{\sqrt{1 - v^2 / c^2}} = \frac{vt'_A + x'_A}{\sqrt{1 - v^2 / c^2}} \tag{37}$$

For the y coordinate transformation, we have:

$$y_D = ct_E \tag{38}$$

From the last equation because $t_E = t'_{O'B}$, we obtain:

$$y_D = ct_{O'B} = y_B \qquad \Rightarrow y = y' \tag{39}$$

In the same way we obtain $z' = z$ .

Therefore, according to the previous analysis, the transformations for the time and the location coordinates measured by the two observers simultaneously are given by the following relations (the Lorentz transformations):

$$x = \frac{vt' + x'}{\sqrt{1 - v^2 / c^2}} \tag{40}$$

$$y = y' \tag{41}$$

$$z = z' \tag{42}$$

$$t = \frac{vx' / c^2 + t'}{\sqrt{1 - v^2 / c^2}} \tag{43}$$

## 5-The Two-ray Description, Velocity Transformation and the Constancy of the Speed of Light

Einstein proved that if an object with velocity $\vec{u}'$ whose components are denoted by $\left(u'_x, u'_y, u'_z\right)$ is moving in frame $S'$, then, with the aid of the Lorentz transformations, the components of the velocity of this object in frame S which are denoted by ($u_x$,$u_y$,$u_z$) are given by the following relations:

$$u_x = \frac{u'_x + v}{1 + u'_x v / c^2} \tag{44}$$

$$u_y = \frac{u'_y \sqrt{1 - v^2 / c^2}}{1 + u'_x v / c^2} \tag{45}$$

$$u_z = \frac{u'_z \sqrt{1 - v^2 / c^2}}{1 + u'_x v / c^2} \tag{46}$$

Let us discuss the following cases using the two-ray description explained in the previous sections.

### Case 1: A ray of light that travels in frame $S'$ in the positive direction along the $x'$ axis

In this case we have: $u'_x = c$ and $u'_y = u'_z = 0$ . According to the transformations (44)-(46) we obtain: $u_x = u'_x = c$ and $u_z$=$u_y$=0.

The result $u_x = u'_x = c$ leads to contradiction. But according to the physical meaning of the Lorentz transformations and the two-ray description introduced in this article, the result that $u_x = u'_x = c$ is expected, because the Lorentz transformations for the location coordinates in this case simultaneously connect the fronts of two different rays emanating at $t = t' = 0$ from $O$ and $O'$ when they coincide, and that travel in the positive direction. These fronts determine the location of the object in frames S and $S'$.

If the object is light propagated in frame $S'$ in the $x'$ direction, then the Lorentz transformations for the coordinates in this case convey the location of the front of the first ray in $S'$ to the location of the front of the second ray in S. Therefore, the Lorentz transformations for velocity relate the velocity of the front of the first ray moving in the $x'$ direction in frame $S'$ (which is c) to the velocity of the front of the second ray moving in the $x$ direction in frame S which also travels at velocity c.

### Case 2: A ray of light that proceeds in the $y'$ direction at frame $S'$

In this case we have $u'_y = c$ and $u'_x = u'_z = 0$. According to transformations (44)-(46) we obtain: $u_x$=v, $u_z$=0 and $u_y = \sqrt{c^2 - v^2}$ . This is the expected result, because when the first ray travels along the $y'$ axis in frame $S'$ towards some point, for example, B (Figure 12). Then the second ray in frame S proceeds along line OB toward this point (Figure 12) with the velocity component v moving in the positive $x$ direction. Therefore, the y component of the velocity for this ray is: $u_y = \sqrt{c^2 - v^2}$ .

## 6-General Lorentz Transformations

Suppose there is an object at rest at frame $S'$ at a location defined by $M\left(x', y'\right)$ as described in Figure 13. If we assume that at $t = t' = 0$ when the origins $O$ and $O'$ of frames S and $S'$ respectively coincide, a spherical light wave is emitted from the object, its light will reach an observer at $O'$ by a ray that travels along line $MO'$ at time $t'_M$ . The light will reach an observer at $O$ by another ray that travels along line MO at time $t_M$. We previously found that times $t_M$ and $t'_M$ are related by eqn. (17), which is:

$$t_M = \frac{\frac{vx'}{c^2} + \sqrt{\frac{v^2 x'^2}{c^4} + t_M'^2 \left(1 - \frac{v^2}{c^2}\right)}}{1 - v^2 / c^2} \tag{47}$$

Where the time $t'_M$ is given by the relation:

$$t'_M = \frac{\sqrt{x'^2 + y'^2}}{c} \tag{48}$$

The physical situation described above is equivalent to the following reversal: when the origins $O$ and $O'$ coincide at $t = t' =$ , two spherical light waves are emitted, the first from O, and the second from $O'$ (Figure 13). In this case the time t that it takes light in frame S to travel from O to M equals $t_M$ and the time $t'$ that it takes light in frame $S'$ to travel from $O'$ to M equals $t'_M$ (Figure 13). Therefore, the times t and $t'$ are both related by eqn. (47):

$$t = \frac{\frac{vx'}{c^2} + \sqrt{\frac{v^2 x'^2}{c^4} + t'^2 \left(1 - \frac{v^2}{c^2}\right)}}{1 - v^2 / c^2} \tag{49}$$

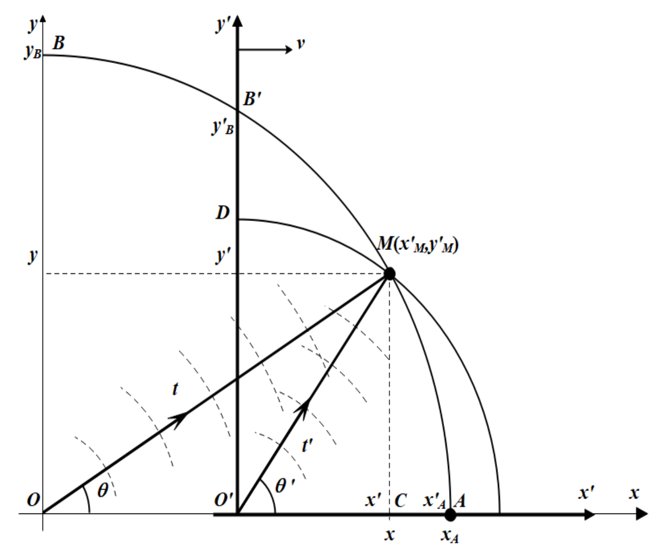


**Figure 13:** The three points M, ($x'$,0) and (0,$y'$) are not found on the front of the same wave in frame, therefore they are not simultaneous.

Denoting the length OM by r, $O'M$ by $r'$, the angle between r and the positive direction of the x-axis by θ, and the angle between $r'$ and the positive direction of the $x'$ axis by $\theta'$ (Figure 13), then we have that $x' = r'\cos\theta'$ and $t' = r'/c$. Substituting the last two relations in eqn. (49), we get with a few mathematical steps that:

$$t = \frac{vr'\cos\theta'/c^2 + t'\sqrt{1-\frac{v^2}{c^2}\sin^2\theta'}}{1-v^2/c^2} \quad (50)$$

If we multiply the last equation by c and use relations ct=r and $ct' = r'$, we obtain:

$$r = \frac{vt'\cos\theta' + r'\sqrt{1-\frac{v^2}{c^2}\sin^2\theta'}}{1-v^2/c^2} \quad (51)$$

According to the last two equations we obtain:

1. If the object in frame $S'$ is found on the $x'$ axis, then we have; $\theta' = 0$, $r' = x'$, r=x and $t' = x'/c$. In this case eqns. (50) and (51) reduce to:

$$t = \frac{vx'/c^2 + t'}{1-v^2/c^2} \quad (52)$$

$$x = \frac{vt' + x'}{1-v^2/c^2} \quad (53)$$

2. If the object is found on the $y'$ axis, then, $\theta'$=90°, $r'=y'$ and $r=y$. In this case eqn. (51) reduces to:

$$y = \frac{y'}{\sqrt{1-v^2/c^2}} \quad (54)$$

Under condition of simultaneity (as explained in Section 4), eqns. (52)-(54) become:

$$t = \frac{vx'/c^2 + t'}{\sqrt{1-v^2/c^2}} \quad (55)$$

$$x = \frac{vt' + x'}{\sqrt{1-v^2/c^2}} \quad (56)$$

$$y = y' \quad (57)$$

$$z = z' \quad (58)$$

According to the previous analysis, we conclude that the Lorentz transformations given by eqns. (55)-(58) are suitable only for objects found on any of the x, y, or z axes. For an object at any other location, $M(x',y')$ (Figure 13), eqns. (55)-(58) do not hold, because light does not reach an observer at O from $M(x',y')$ at the same time as it does from points $(0,y')$ and $(x',0)$. These points are not found at the front of the same wave whose center is at O. In this case, we need to find general transformations, by using the following relations:

$$x^2+y^2=c^2t^2 \quad (59)$$

$$y = y' \quad (60)$$

$$y'^2 + x'^2 = c^2t'^2 \quad (61)$$

and eqn. (49):

$$t = \frac{\frac{vx'}{c^2} + \sqrt{\frac{v^2x'^2}{c^4} + t'^2\left(1-\frac{v^2}{c^2}\right)}}{1-v^2/c^2} \quad (62)$$

From the last four equations we obtain:

$$x^2 = c^2\left(\frac{\frac{vx'}{c^2} + \sqrt{\frac{v^2x'^2}{c^4} + t'^2\left(1-\frac{v^2}{c^2}\right)}}{1-v^2/c^2}\right)^2 - \left(c^2t'^2 - x'^2\right) \quad (63)$$

With a few mathematical steps we obtain the following relation for x :

$$x = \frac{x' + \frac{v}{c}\sqrt{\frac{v^2x'^2}{c^2} + c^2t'^2\left(1-\frac{v^2}{c^2}\right)}}{1-v^2/c^2} \quad (64)$$

Where $t' = r'/c = \sqrt{x'^2 + y'^2}/c$.

Using relations $x' = r'\cos\theta' = ct'\cos\theta'$ and $t' = r'/c$, we can write eqn. (64) as:

$$x = \frac{x' + vt'\sqrt{1-\frac{v^2}{c^2}\sin^2\theta'}}{1-v^2/c^2} \quad (65)$$

Therefore, for an object found in frame $S'$ at location $M(x',y')$, we have the following transformations:

$$z = z' \quad (66)$$

$$y = y' \quad (67)$$

$$x = \frac{x' + vt'\sqrt{1-\frac{v^2}{c^2}\sin^2\theta'}}{1-v^2/c^2} \quad (68)$$

$$t = \frac{vx'/c^2 + t'\sqrt{1-\frac{v^2}{c^2}\sin^2\theta'}}{1-v^2/c^2} \quad (69)$$

where $t' = r'/c$, and t=r/c.

Relations (66)-(69) are not suitable for simultaneous measurements. To make them appropriate for simultaneous measurements, we proceed as described in Section 4. We need to find two points, E in frame S and F in frame $S'$ where light simultaneously emitted from them at $t = t' = 0$ when O and $O'$ coincide reaches $O$ and $O'$ simultaneously.

To find the location of these two points we proceed as follows:

1. We draw two arcs as shown in Figure 14: Arc $\widehat{BD}$ in frame $S'$ with its centre at $O'$ and arc $\widehat{HG}$ in frame S with the same radius but with its centre at O. We make the radii of these two arcs equal in order to fulfil the condition of simultaneity. In addition, we make these radii equal to $O'B$ in order to connect the times found along these arcs to the times t and $t'$ (Figure 14).
2. Any two points found on arcs $\widehat{BD}$ and $\widehat{HG}$ with the same y coordinate fulfill the condition of simultaneity. Therefore, point E is found on arc $\widehat{HG}$ and point F is found on arc $BD$.
3. Because light in frame S reaches an observer at O along line MO (Figure 14), point E must also be found on this line.
4. From the last two points, we conclude that point E is the intersection of arc $\widehat{HG}$ and line MO (Figure 14).
5. To find the location of point F, we draw a horizontal line from E to intersect arc $\widehat{BD}$. This intersection defines the point F.

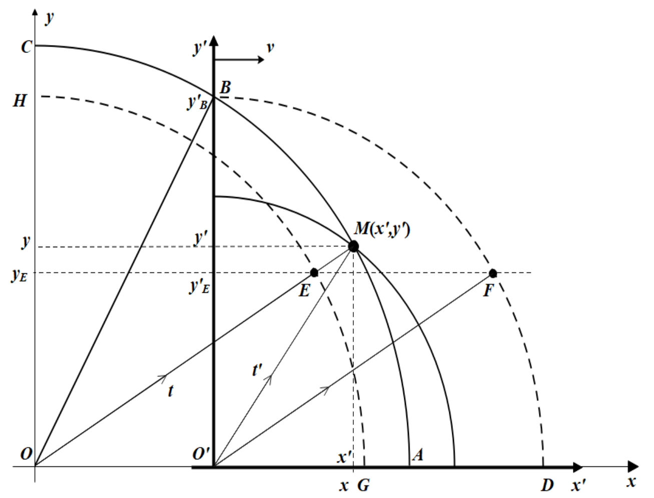


**Figure 14:** Light simultaneously emitted from points E and F reaching observers O and $O'$ simultaneously.

Light simultaneously emitted from E and F reaches O and $O'$ simultaneously. To express the time $t_E$ (the time from O to E in frame S), (Figure 14) we use the following relation:

$$t_{OB} = \frac{t_{O'B}}{\sqrt{1-v^2/c^2}} \tag{70}$$

Because $t_{O'B} = t_{O'F}$ (because *B* and F are found on the same arc in *S'*) we obtain:

$$t_{OB} = \frac{t_{O'F}}{\sqrt{1-v^2/c^2}} \tag{71}$$

Due to simultaneity, we have $t_E = t_{O'F}\left(t_{OE} = t_{O'F}\right)$. Therefore, we obtain:

$$t_E = t_{OB}\sqrt{1-v^2/c^2} \tag{72}$$

Because $t_{OB}=t_{OM}$ (B and M are found on the same arc in S), we obtain:

$$t_E = t_{OM}\sqrt{1-v^2/c^2} \tag{73}$$

Substituting $t_{OM}$ from eqn. (69), we obtain:

$$t_{OE} = \left(\frac{vx'/c^2 + t'\sqrt{1-\frac{v^2}{c^2}\sin^2\theta'}}{1-v^2/c^2}\right)\sqrt{1-v^2/c^2} \tag{74}$$

Therefore, the simultaneous relation:

$$t = \frac{vx'/c^2 + t'\sqrt{1-\frac{v^2}{c^2}\sin^2\theta'}}{\sqrt{1-v^2/c^2}} \tag{75}$$

Follows.

This reveals that simultaneity requires that the right hand side of eqn. (69) be multiplied by $\sqrt{1-v^2/c^2}$. Because the relation $x^2+y^2+z^2=c^2t^2$ is fulfilled for all points on the front of the same light wave emitted from O, we also need to multiply relations (66)-(68) by the same factor, $\sqrt{1-v^2/c^2}$. Therefore, from eqns. (66)-(68) we obtain the following general transformations that fulfill the condition of simultaneity:

$$z = z'\sqrt{1-v^2/c^2} \tag{76}$$

$$y = y'\sqrt{1-v^2/c^2} \tag{77}$$

$$x = \frac{x' + vt'\sqrt{1-\frac{v^2}{c^2}\sin^2\theta'}}{\sqrt{1-v^2/c^2}} \tag{78}$$

In addition, we find that:

$$t = \frac{vx'/c^2 + t'\sqrt{1-\frac{v^2}{c^2}\sin^2\theta'}}{\sqrt{1-v^2/c^2}} \tag{79}$$

Since r=ct and $r' = ct'$, by multiplying eqn. (79) by *c* we obtain that:

$$r = \frac{vt'\cos\theta' + r'\sqrt{1-\frac{v^2}{c^2}\sin^2\theta'}}{\sqrt{1-v^2/c^2}} \tag{80}$$

## 7-What about the Traditional Way of Obtaining Lorentz Transformations?

We know that the traditional textbook way of obtaining Lorentz transformations is as follows:

A light source is found at O, the origin of frame S. At $t = t' = 0$, when the origins O and $O'$ of frames S and $S'$ respectively, coincide, a spherical light wave is emitted from the light source found at O. Because of the spherical front of the wave in frame S, for any point (x,y,z) on the front of the wave for any time t, we have the following relation:

$$x^2+y^2+z^2=c^2t^2 \tag{81}$$

According to the second postulate of Special Relativity, the front of the wave in frame $S'$ is also spherical. Therefore, in frame $S'$, we have:

$$x'^2 + y'^2 + z'^2 = c^2t'^2 \tag{82}$$

Where $\left(x', y', z'\right)$ are the coordinates of the same point relative to frame $S'$ and $t'$ is the time relative to this frame.

To find the transformations for the location coordinates and the time, we assume the following form for general linear transformations:

$$x' = \alpha x + \varepsilon t \tag{83}$$

$$y' = y \tag{84}$$

$$z' = z \tag{85}$$

$$t' = \delta x + \eta t \tag{86}$$

To find the coefficients in the last equations, we proceed as follows:

For an observer at O, origin $O'$ moves with constant velocity ν in the positive direction such that for $x' = 0$, hence x=νt. Substituting in eqn. (83) we obtain 0=ανt+εt. From this equation we get:

$$\nu=-\varepsilon/\alpha \tag{87}$$

From the other side, an observer at $O'$ observes that origin O (x=0) moves with constant velocity ν in the negative direction, such that $x' = -vt'$. Substituting in eqn. (83) we obtain: $-vt' = 0 + \varepsilon t$ from which we obtain: $t' = -\varepsilon t / v$. Substituting the last relation in eqn. (86), we get:

$$\nu=-\varepsilon/\eta \tag{88}$$

**Citation:** Zoabi E (2019) A New Approach for the Derivation of Lorentz Transformations and Its Implications on Understanding the Special Theory of Relativity. J Phys Math 10: 301.



Comparing eqns. (87) and (88) we obtain:

$$\alpha = \eta \tag{89}$$

The next step is to substitute eqns. (83)-(86) into eqn. (82) with the condition α=ŋ. By comparing coefficients, we get the known Lorentz transformations.

There is a mistake in this approach to finding Lorentz transformations, which is that relation (89) holds only for $\theta' = 0$, that is, for objects found on the $x'$ axis (see eqns. (78) and (79)), where $y = y' = 0$. If an object is found at some other location $(x', y')$, the relation α=ŋ is not fulfilled. Therefore, the approach described above for finding Lorentz transformations is not correct.

## 8-Light Aberration

Suppose a light source in frame $S'$ at point $M(x', y')$ (Figure 15) whose light reaches an observer at $O'$ along line $MO'$ which makes an angle $\theta'$ with the positive $x'$ axis and reaches an observer at O in frame S along line MO which makes an angle θ with the positive x axis (Figure 15). In order to find the relation between θ and $\theta'$ we use the reversal process: Two spherical light waves simultaneously emitted from O and $O'$ at $t = t' = 0$ when they coincide reach the light source along line OM in frame S and along line $O'M$ in frame $S'$ (Figure 15).

We have:

$$\tan\theta = \frac{y}{x} = \frac{y}{\sqrt{r^2 - y^2}} \tag{90}$$

Substituting *y* from eqn. (77) and *r* from eqn. (80) we obtain:

$$\tan\theta = \frac{y'\sqrt{1 - v^2/c^2}}{\sqrt{\left(\dfrac{vt'\cos\theta' + r'\sqrt{1 - v^2/c^2\sin^2\theta'}}{\sqrt{1 - v^2/c^2}}\right)^2 - \left(y'\sqrt{1 - v^2/c^2}\right)^2}} \tag{91}$$

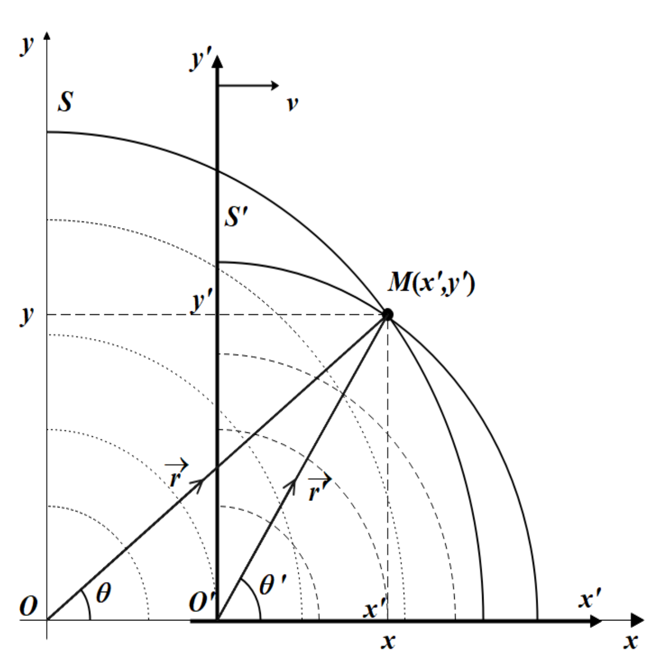


**Figure 15:** Light emitted from O reaches object M with angle θ and light emitted from $O'$ reaches the object with angle θ′.

Using relations $y'/r' = \sin\theta'$ and $r' = ct'$ and some mathematical steps we obtain:

$$\tan\theta = \frac{\sin\theta'\left(1 - v^2/c^2\right)}{\cos\theta' + \dfrac{v}{c}\sqrt{1 - v^2/c^2\sin^2\theta'}} \tag{92}$$

Using the relation $\cos\theta = \sqrt{\left(\tan^2\theta + 1\right)^{-1}}$ we obtain:

$$\cos\theta = \frac{\cos\theta' + \dfrac{v}{c}\sqrt{1 - v^2/c^2\sin^2\theta'}}{\sqrt{1 - v^2/c^2\sin^2\theta'} + \dfrac{v}{c}\cos\theta'} \tag{93}$$

And from the relation sin²θ+cos²θ=1 we obtain:

$$\sin\theta = \frac{\sin\theta'\left(1 - v^2/c^2\right)}{\sqrt{1 - \dfrac{v^2}{c^2}\sin^2\theta'} + \dfrac{v}{c}\cos\theta'} \tag{94}$$

From eqn. (92) we obtain for $\theta' = 0$ that θ=0, and for $\theta' = 90^\circ$ we obtain $\tan\theta = \sqrt{c^2 - v^2}/v$.

From eqn. (93) we obtain for $\theta' = 0$ that θ=0, and for $\theta' = 90^\circ$ we obtain cosθ=v/c.

We can also obtain eqns. (92) and (93) by using the relation:

$$\tan\theta = \frac{u_y}{u_x} = \frac{dy/dt}{dx/dt} = \frac{dy}{dx} \tag{95}$$

To express *dy* and *dx* we use eqns. (77) and (78). But, before we differentiate these two equations, we note that during the journey of light along lines $O'M$ and OM angels θ and $\theta'$ remain constant (Figure 15). Therefore, we differentiate eqn. (77) and eqn. (78) under the condition that the angles θ and $\theta'$ are constant. From eqns. (77) and (78) we obtain (for constants θ and $\theta'$):

$$dy = dy'\sqrt{1 - v^2/c^2} \tag{96}$$

$$dx = \frac{dx' + vdt'\sqrt{1 - \dfrac{v^2}{c^2}\sin^2\theta'}}{\sqrt{1 - v^2/c^2}} \tag{97}$$

Substituting the last two equations in eqn. (95) we obtain:

$$\tan\theta = \frac{dy'\left(1 - v^2/c^2\right)}{dx' + vdt'\sqrt{1 - \dfrac{v^2}{c^2}\sin^2\theta'}} = \frac{u'_y\left(1 - v^2/c^2\right)}{u'_x + v\sqrt{1 - \dfrac{v^2}{c^2}\sin^2\theta'}} \tag{98}$$

For light emitted from $O'$ in frame $S'$ traveling toward M, we have the following components of velocity: $u'_y = c\sin\theta'$ and $u'_x = c\cos\theta'$. Substituting these relations in eqn. (98) we obtain:

$$\tan\theta = \frac{c\sin\theta'\left(1 - v^2/c^2\right)}{c\cos\theta' + v\sqrt{1 - \dfrac{v^2}{c^2}\sin^2\theta'}} = \frac{\sin\theta'\left(1 - v^2/c^2\right)}{\cos\theta' + \dfrac{v}{c}\sqrt{1 - \dfrac{v^2}{c^2}\sin^2\theta'}} \tag{99}$$

which is the same as eqn. (92).

## 9-The Doppler Effect

Based on eqn. (79) we can obtain a general equation for the Doppler effect. According to this equation if the time passing in frame $S'$ is one period which we denote by $T'$, then the period, T, in frame S is given by the relation:

$$T = \frac{\frac{vx'}{c^2} + T'\sqrt{1-\frac{v^2}{c^2}\sin^2\theta'}}{\sqrt{1-v^2/c^2}} \quad (100)$$

Since $x' = r'\cos\theta' = cT'\cos\theta'$, from the last equation we obtain:

$$T = \frac{\frac{v}{c}\cos\theta' + \sqrt{1-\frac{v^2}{c^2}\sin^2\theta'}}{\sqrt{1-v^2/c^2}}T' \quad (101)$$

From the last equation we find the following relation for frequencies:

$$f = \frac{\sqrt{1-v^2/c^2}}{\frac{v}{c}\cos\theta' + \sqrt{1-\frac{v^2}{c^2}\sin^2\theta'}}f' \quad (102)$$

Multiplying eqn. (101) by $c$ and using the relations λ=cT and $\lambda' = cT'$ we obtain:

$$\lambda = \frac{\frac{v}{c}\cos\theta' + \sqrt{1-\frac{v^2}{c^2}\sin^2\theta'}}{\sqrt{1-v^2/c^2}}\lambda' \quad (103)$$

From eqns. (101)-(103) we obtain the following cases:

1. When a light source is found on the $x'$ axis in frame $S'$, then $\theta = \theta' = 0$. In this case from eqns. (101)-(103) we obtain the relations:

$$T = \sqrt{\frac{1+v/c}{1-v/c}}T' \quad (104)$$

$$f = \sqrt{\frac{1-v/c}{1+v/c}}f' \quad (105)$$

$$\lambda = \sqrt{\frac{1+v/c}{1-v/c}}\lambda' \quad (106)$$

2. When a light source is found on the $y'$ axis in frame $S'$, we have $\theta' = 90^\circ$. In this case from eqns. (101)-(103) we obtain the relations:

$$T = T' \quad (107)$$

$$f = f' \quad (108)$$

$$\lambda = \lambda' \quad (109)$$

According to the last result we do not have a transverse Doppler effect for $\theta' = 90^\circ$. In order to detect this result experimentally, the detector in frame S must be directed relative to the positive $x$ axis with an angle which is obtained from eqn. (93) by substituting $\theta' = 90^\circ$. In this case we obtain:

$$\cos\theta_0 = v/c \ \Rightarrow \theta_0 = \cos^{-1}(v/c) \quad (110)$$

3. Then with the help of eqn. (103), we obtain that for $0<\theta<\theta_0$, where $\theta_0$ is given by eqn. (110), we have $\lambda > \lambda'$, where $\lambda'$ is the wave length of the light in the rest frame. In this case light is red-shifted. If $\theta_0<\theta<180°$, we have $\lambda < \lambda'$. In this case light is blue-shifted.

4. If the line connecting receiver O in frame S with the source is perpendicular to the velocity ν, we have θ=90. This is the case for the transvers Doppler effect. In this case according to eqn. (93), we have

$$0 = \frac{\cos\theta' + \frac{v}{c}\sqrt{1-v^2/c^2\sin^2\theta'}}{\sqrt{1-v^2/c^2\sin^2\theta'} + \frac{v}{c}\cos\theta'} \quad (111)$$

From this equation we obtain:

$$\sin\theta' = \frac{1}{\sqrt{1+v^2/c^2}} \quad (112)$$

and therefore:

$$\cos\theta' = \frac{v/c}{\sqrt{1+v^2/c^2}} \quad (113)$$

Substituting eqns. (112) and (113) into eqns. (101)-(103) we obtain the relations for the transverse Doppler effect (θ=90°):

$$T = \frac{1+v^2/c^2}{1-v^2/c^2}T' \quad (114)$$

$$f = \frac{1-v^2/c^2}{1+v^2/c^2}f' \quad (115)$$

$$\lambda = \frac{1+v^2/c^2}{1-v^2/c^2}\lambda' \quad (116)$$

## 10-Time Dilation

Suppose an event in frame $S'$ starting at $(x', y')$ at time $t_1'$ and ending at time $t_2'$, then according to eqn. (79), we have:

$$t_1 = \frac{vx'/c^2 + t_1'\sqrt{1-\frac{v^2}{c^2}\sin^2\theta'}}{\sqrt{1-v^2/c^2}} \quad (117)$$

$$t_2 = \frac{vx'/c^2 + t_2'\sqrt{1-\frac{v^2}{c^2}\sin^2\theta'}}{\sqrt{1-v^2/c^2}} \quad (118)$$

By subtracting the last two equations we obtain:

$$\Delta t = \frac{\Delta t'\sqrt{1-\frac{v^2}{c^2}\sin^2\theta'}}{\sqrt{1-v^2/c^2}} \quad (119)$$

where $\Delta t = t_2 - t_1$ is the duration of the event in frame S, and $\Delta t' = t_2' - t_1'$ is the duration of the event in frame $S'$ (the rest frame of the event).

According to the last equation, the time dilation depends on the location of the event in frame $S'$. We have the following two cases:

1) If the event happens on the $x'$ axis in frame $S'$, then $\theta'$ =0. In this case according to eqn. (119) we have:

$$\Delta t = \frac{\Delta t'}{\sqrt{1-v^2/c^2}} \quad (120)$$

2) If the event happens on the $y'$ axis in frame $S'$, then θ′=0. In this case according to eqn. (119) we obtain:

$$\Delta t = \Delta t' \quad (121)$$

According to eqn. (93), when $\theta' = 90^\circ$, we have cosθ=ν/c. Therefore, this case is suitable for $\theta=\cos^{-1}(\nu/c)$.

3) For θ=90, we have $\sin\theta' = \frac{1}{\sqrt{1+v^2/c^2}}$ (see eqn. (112)). By substituting the last relation into eqn. (119), we obtain:

$$\Delta t = \frac{\Delta t'}{1-v^2/c^2} \quad (122)$$

## 11-Length Contraction

In order to find expressions for length contraction, we need to write the transformation (78) in the reverse form as follows:

$$x' = \frac{x - vt\sqrt{1-\frac{v^2}{c^2}\sin^2\theta}}{\sqrt{1-v^2/c^2}} \qquad (123)$$

In addition, we have:

$$z = z'\sqrt{1-v^2/c^2} \qquad (124)$$

$$y = y'\sqrt{1-v^2/c^2} \qquad (125)$$

(see eqns. (76)-(77)).

We have the following cases:

1) If a ruler with length $l_0$(the length in the rest frame) is found along the $y'$ or the $z'$ axis in frame $S'$, then, according to relations (124) and (125) we obtain that the length of the ruler measured in frame is:

$$\ell = \ell_0\sqrt{1-v^2/c^2} \qquad (126)$$

2) If ruler is found along the $x'$ axis in frame $S'$, then, according to relation (123), we obtain that the length of the ruler measured in frame S is also:

$$\ell = \ell_0\sqrt{1-v^2/c^2} \qquad (127)$$

## 12-General Velocity Transformations

Suppose an object at location defined by $M(x',y',z')$ at time $t'$ in frame $S'$ and has a velocity $\vec{u}'$ denoted by components $(u'_x,u'_y,u'_z)$. In order to find the components ($u_x.u_y,u_z$) of the velocity in frame S which are simultaneously measured by another observer found in this frame, we must first differentiate eqns. (76)-(79). After differentiation, we obtain:

$$dx = \frac{dx'\left(\sqrt{1-\frac{v^2}{c^2}\sin^2\theta'} + \frac{v^3}{c^3}\sin^2\theta'\cos\theta'\right) + vdt'\left(1-\frac{v^2}{c^2}\sin^2\theta'\right)}{\sqrt{1-\frac{v^2}{c^2}\sin^2\theta'}\sqrt{1-v^2/c^2}} \qquad (128)$$

$$dt = \frac{\left(\frac{1}{c^2}\sqrt{1-\frac{v^2}{c^2}\sin^2\theta'} + \frac{v}{c^3}\sin^2\theta'\cos\theta'\right)vdx' + dt'\left(1-\frac{v^2}{c^2}\sin^2\theta'\right)}{\sqrt{1-\frac{v^2}{c^2}\sin^2\theta'}\sqrt{1-v^2/c^2}} \qquad (129)$$

$$dz = dz'\sqrt{1-v^2/c^2} \qquad (130)$$

$$dy = dy'\sqrt{1-v^2/c^2} \qquad (131)$$

Using the last equations, we obtain:

$$u_x = \frac{u'_x\left(\sqrt{1-\frac{v^2}{c^2}\sin^2\theta'} + \frac{v^3}{c^3}\sin^2\theta'\cos\theta'\right) + v\left(1-\frac{v^2}{c^2}\sin^2\theta'\right)}{\left(\sqrt{1-\frac{v^2}{c^2}\sin^2\theta'} + \frac{v}{c}\sin^2\theta'\cos\theta'\right)\frac{vu'_x}{c^2} + \left(1-\frac{v^2}{c^2}\sin^2\theta'\right)} \qquad (132)$$

$$u_y = \frac{u'_y\left(1-v^2/c^2\right)\sqrt{1-\frac{v^2}{c^2}\sin^2\theta'}}{\left(\sqrt{1-\frac{v^2}{c^2}\sin^2\theta'} + \frac{v}{c}\sin^2\theta'\cos\theta'\right)\frac{vu'_x}{c^2} + \left(1-\frac{v^2}{c^2}\sin^2\theta'\right)} \qquad (133)$$

$$u_z = \frac{u'_z\left(1-v^2/c^2\right)\sqrt{1-\frac{v^2}{c^2}\sin^2\theta'}}{\left(\sqrt{1-\frac{v^2}{c^2}\sin^2\theta'} + \frac{v}{c}\sin^2\theta'\cos\theta'\right)\frac{vu'_x}{c^2} + \left(1-\frac{v^2}{c^2}\sin^2\theta'\right)} \qquad (134)$$

For θ′=0 we obtain from eqns. (132)-(134):

$$u_x = \frac{u'_x + v}{vu'_x/c^2 + 1} \qquad (135)$$

$$u_y = \frac{u'_y\left(1-v^2/c^2\right)}{vu'_x/c^2 + 1} \qquad (136)$$

$$u_z = \frac{u'_z\left(1-v^2/c^2\right)}{vu'_x/c^2 + 1} \qquad (137)$$

For θ′=90° we obtain from eqns. (132)-(134):

$$u_x = \frac{u'_x + v\sqrt{1-v^2/c^2}}{vu'_x/c^2 + \sqrt{1-v^2/c^2}} \qquad (138)$$

$$u_y = \frac{u'_y\sqrt{1-v^2/c^2}}{vu'_x/c^2 + 1} \qquad (139)$$

$$u_z = \frac{u'_z\sqrt{1-v^2/c^2}}{vu'_x/c^2 + 1} \qquad (140)$$

## Summary, Discussion and Results

From the explanations and the mathematical development in this article we have obtained the following results:

1. Information about an event reaches two different observers by two different rays of light and these two rays determine the measurements of the location and time of the event that are made by the two observers. Transformations of the measurements made by the two observers are achieved by the transformation of the front of one ray into the other.

2. To find the transformations for these measurements, it is easier to describe the situation in point 1 by the following equivalent reverse process: when the two origins, O and $O'$ of the two stationary frames S and S', respectively, coincide at $t = t' = 0$, two light rays are emitted simultaneously from O and $O'$ towards the object.

3. The Lorentz transformation for time relates the time that it takes the two rays emitted from an object to reach each of the two observers simultaneously. The Lorentz transformation of the location coordinates relates, simultaneously, the location of the fronts of these two rays. These locations are those of the object as measured by the two observers at the appropriate times.

4. Light (the electromagnetic interaction) has the same speed relative to the object and the observer (even if they move relative to each other) because the electromagnetic interaction expands as spherical waves and travels from the source to the observer along a line, such that the same length is traversed at the same time relative to the object and the observer (Figure 6).

5. When Lorentz transformations for velocity are applied for light travelling in the x' direction we find that light has the same speed, *c*, in frames S and $S'$ because the transformation in this case conveys the velocity of the front of the light ray that travels in the $x'$ direction in frame $S'$ (which is *c*) to the velocity of the front of the ray that proceeds in the *x* direction in frame S which is also *c*.

6. The Lorentz transformations given by eqns. (55)-(58) are suitable only for objects found on the, x, y or z axes. These transformations are not correct for objects found at other locations $(x',y')$ due to the fact that light in this case does not reach the observer at O from point $(x',y')$ at the same time as it does from points $(0,y')$ and $(x',0)$. These points are not found on the front of the same wave whose origin is at *S*. In this case we use transformations (76)-(80).

7. Based on the general transformations for location and velocity we obtained new general relations for light aberration and for the Doppler effect which are given by eqns. (92)-(93), and eqns. (101)-(103).

8. For the transverse Doppler effect (θ=90) we obtained eqns. (114)-(116) for the period, frequency and wavelength. If we measure these quantities at angle $\theta_0$ relative to the velocity where $\theta_0$ is given by $\theta_0=\cos^{-1}(v/c)$, no Doppler effect will be detected.

9. For $0<\theta<\theta_0$, where $\theta_0$ is given by $\theta_0=\cos^{-1}(v/c)$, we have $\lambda > \lambda'$, where $\lambda'$ is the wave length of the light in the rest frame. In this case light is red-shifted. If $\theta_0<\theta<180$, $\lambda < \lambda'$. In this case light is blue-shifted.

10. Time dilation depends upon the location of the event in frame *S*'. For an event found on the $x'$ axis, the time dilation is given by eqn. (120). For an event found on the $y'$ axis in frame *S*', there is no time dilation, and for θ=90, the time dilation is given by eqn. (122).

11. We obtained that an object contracts by the same factor $\sqrt{1-v^2/c^2}$ in all directions.

## Conclusions

1. The constancy of the speed of light relative to the observer and the source, when they are in motion relative to each other, is caused by the fact that light spread from the source in straight lines in all directions, as a result, light reaches the observer along straight line that is created by light expansion such that it passes the same length at the same time relative to both the observer and the source.

2. Since the constancy of the speed of light relative to the observer and the source, when they are in motion relative to each other, is a property caused by the expansion character of light and the path it takes to reach the observer, we conclude that the speed of light is not absolute. Light may, in some cases have different speeds relative to different observers. This happens when the light ray is not travelling toward the observer. The possibility that light may have different speeds relative to different observers does not affect the physics of the transformation of the information to these observers, because the information passes to them via the ray that reaches them and this ray, as we explained travels relative to them with a velocity of *c*.

3. The results of the special theory of relativity which are achieved by using the Lorentz transformations are caused by the measurement process which is informed by light. The determination of the location of an object is made by different observers at different times by different rays, therefore the location of object seems different relative to different observers. Hence, the results of Special Relativity concerning locations and times are due to optical effects. Because locations and times measured by observers are the only information they receive, these measurements are absolute reality for them.

4. As explained above in Point 1, the constancy of the speed of light is derived from the fact that light (the interaction) expands in all directions (in straight lines as photons, or rays if we consider light as waves) and reaches the observer along a line produced by this expansion. We conclude that the Lorentz transformations and the results of the Special Theory of Relativity cannot be examined by laser beams because they do not spread.

### References

1. Das A (1993) The Special Theory of Relativity. A Mathematical Exposition, New York: Springer.
2. Friedman Y (2004) Physical Applications of Homogeneous Balls. Progress in Mathematical Physics 40: 1-21.
3. Krane K (1996) Modern Physics (2thedn) New York: John Wiley & Sons, Inc.
4. Lorentz HA, Einstein A, Minkowski H, Weyl H (2017) The principle of relativity. Mineola, NY: Dover Publications, Inc.
5. Tipler P, Llewellyn R (2002) Modern Physics (4thedn), New York: W. H. Freeman & Co.
6. Callender C (2011) The Oxford Handbook of the Philosophy of Time (1stedn), Oxford Press, Oxford, U.K.
7. Hawking S, Penrose R (1995) The Nature of Space and Time. Princeton U Press, Princeton, NJ USA.
8. Covneney P, Highfield R (1992) The Arrow of Time. Science and Society 56: 501-504.
9. Grable DR (2003) Information Characteristics for the Curriculum. ACM SIGCSE Bulletin 35: 74-77.
10. Grable DR (2008) Let There Be Light: The Physical Nature of Information. Amazon Kindle, USA.
11. Quznetsov G (2014) Final Book on Fundamental Theoretical Physics. American Research Press.
12. Shannon CE, Weaver W (1964) The Mathematical Theory of Communication. U Illinois Press, Urbana, IL, USA.
13. Carrol J, Long D (1989) Theory of Finite Automata. Prentice Hall, Englewood Cliffs, NJ, USA.
14. Quznetsov G (2013) Logical foundation of Fundamental Theoretical Physics. Lambert Academic, Saarbrucken, Germany.