Journal of Physical Mathematics
ISSN: 2090-0902

Djeghloul and Tahiri, J Phys Math 2014, 5:1
DOI: 10.4172/2090-0902.1000123

Research Artilce

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A Modified *N*=2 Extended Supersymmetry

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Abstract

A modification of the usual extended *N*=2 super symmetry algebra implementing the two dimensional permutation group is performed. It is shown that one can found a multiplet that forms an off-shell realization of this alternative extension of standard super symmetry.

Keywords: Extended *N*=2 Super symmetry; Non-linear algebra; Off-shell super multiplets.

Introduction

The present paper deals with the possibility of modifying the usual extended *N*=2super Poincare algebra via a suitable implementation of the symmetric group S_2. This construction has the interesting advantage that one can find a multiplet that realizes the modified extended super symmetry algebra off-hell. This multiplet involves the same fields as those of the standard double tensor multiplet [1] (see also [2] for explicit construction). It is worth mentioning that this construction is not a standard extension of super symmetry in the sense that it relies on a non-local invariance represented by the symmetric group S_2.This is what the term *modified* underlies. The obtained transformations still transform bosons in fermions and vice versa.

We will first show that a suitable modification of the *N*=2 super symmetry algebra is possible in the context of nonlinear extension of standard Lie algebras [3]. In this context, we introduce the symmetric group S_2within the standard extended *N*=2 super Poincare algebra [4]. This is what is explicitly performed in section two.

In section three, we show that the multiplet containing two Weyl fermions, two real scalar fields and two 2-form gauge potentials is an off-shell multiplet of the modified super algebra so that no auxiliary fields are needed. Finally we show that the construction of a nilpotent Becci-Rouet-Stora-Tyutin BRST operator can be considered.

Modified *N*=2 Super Symmetry Algebra

The possibility of modifying the extended *N*=2 super Poincare algebra is based on the observation that the free Lagrangian density and thus the free action of two scalar fields φ_i(i=1, 2) or two spinor fields ψ^a (a=1, 2) (here after a repeated index means a summation, indices a are never lowered and indices i are never raised)

$$L\varphi = -\partial_\mu \varphi_i^* \partial^\mu \varphi_i \tag{1}$$

$$L\psi = -i\overline{\psi}^a \overline{\sigma}^\mu \partial_\mu \psi^a, \tag{2}$$

It is manifestly invariant under a permutation operation 1⇔2. So the symmetric group S_2 [5] defines a discrete symmetry of these models. The action of the identity operator s^1 and the transposition operator S_2 can be written as

$$s^1\varphi_i = \delta_{ik}\varphi_k, s^1\psi^a = \delta^{ab}\psi^b,$$
$$s^2\varphi_i = \eta_{ik}\varphi_k, s^2\psi^a = \eta^{ab}\psi^b, \tag{3}$$

Where δ^{ab}={1 for a=b and 0 for a≠b}, δ^{ik}={1 for i=k and 0 for i≠k}, η^{ab}={0 for a=b and 1 for a≠b} and η_{ik}={1 for i=k and 0 for i≠k}.

Furthermore, we define the modified translation operator P_μ^a as a successive application of a permutation operator s^a defined by (3) and the four dimensional translation operator P_μ

$$P_\mu^a = s^a P_\mu, a = 1,2 \text{ and } \mu = 0,1,2,3 \tag{4}$$

P_μ^1 is the usual translation (since s^1 is just the identity) while P_μ^2 is the combination of a translation and the transposition operator. The action of P_μ^2 is then given by

$$\delta'_\kappa \varphi_1 = \kappa^\mu \partial_\mu \varphi_2, \delta'_\kappa \varphi_2 = \kappa^\mu \partial_\mu \varphi_1, \tag{5}$$

$$\delta'_\kappa \psi^1 = \kappa^\mu \partial_\mu \psi^2, \delta'_\kappa \psi^2 = \kappa^\mu \partial_\mu \psi^1, \tag{6}$$

Where κ^μ is an infinitesimal real constant four-vector parameter. One can easily see that, as it is the case for the usual translation, this transformation leads also to an invariance of the Lagrangian densities (1) and (2). One explicitly finds that $\delta'_\kappa L_\varphi$ and $\delta'_\kappa L_\psi$ are total derivatives, i.e., $\delta'_k L_\varphi = -\partial_\nu(\kappa^\nu(\partial_\mu \varphi_2^* \partial^\mu \varphi_1 + \partial_\mu \varphi_1^* \partial^\mu \varphi_2))$ and $\delta'_k L_\psi = -i\partial_\nu(\kappa^\nu(\overline{\psi}^2 \overline{\sigma}^\mu \partial_\mu \psi^1 + \overline{\psi}^1 \overline{\sigma}^\mu \partial_\mu \psi^2))$.

Moreover, the infinitesimal transformations δ' defined by (5) and (6) Forman abelian algebra. For two successive transformation δ' of parameters κ and ζ weget $\delta'_\varsigma \delta'_\kappa X = \varsigma^\nu \kappa^\mu \partial_\nu \partial_\mu X$, where X stands for all the fields. This leads obviously to

$$(\delta'_\varsigma \delta'_\kappa = \delta'_\kappa \delta'_\varsigma)X = 0 \tag{7}$$

One can also remark that these transformations commute with usual translations, i.e.,

$$(\delta_a \delta'_k - \delta'_k \delta_a)\psi = 0 \tag{8}$$

Where, as usual, a translation δ of parameter a is defined by $\delta_a X = a^\mu \partial_\mu X$.

Finally, it is straight forward to check that the commutator of δ' with rotations R (i.e., transformations of the Lorentz group) closes on δ'. We have

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Received August 17, 2013; **Accepted** August 26, 2013; **Published** August 28, 2013

Citation: Djeghloul N, Tahiri M (2013) A Modified N=2 Extended Super Symmetry. J Phys Math 5: 123 doi: 10.41722090-0902.1000123

$$(R_\omega \delta'_\kappa - \delta'_\kappa R_\omega)X = \delta'_{\omega.\kappa} X, \tag{9}$$

Where $\omega \cdot k = -\omega^\mu{}_\nu k^\nu$ is the infinitesimal parameter of the resulting δ' transformation In deriving (9), we used the fact that a rotation *R* of infinitesimal parameter ω acts on any four- vector as $R_\omega V^\mu = - {}^\mu{}_\nu V^\nu$, on any spinor ψ as $R_\omega \psi = -\frac{1}{2}\omega_{\mu\nu}\sigma^{\mu\nu}\psi$ with $\sigma^{\mu\nu} = \frac{1}{4}(\sigma^\mu \bar{\sigma}^\nu - \sigma^\nu \bar{\sigma}^\mu)$ and leaves any scalar fields invariant.

Therefore, we can define a modified construction for the extended *N*=2 supersymmetry algebra relying on the nonlinear extension of a Lie algebra. In this context [3], the defining commutator contains, in addition to linear terms, terms that are multilinear in generators, i.e., $[T_a, T_b] = f^c_{ab} T_c + V^{cd}_{ab} T_c T_d$ for quadratically nonlinear algebras. As it was pointed out in [6], such nonlinear generalization has also to satisfy Jacobi identities. As extension of the standard supersymmetry construction where the anti-commutator of two extended supersymmetry transformations closes on translation, we postulate that it closes also on the composition of a translation P_μ and a transposition s^2, such that

$$\{Q_{i\alpha}, \bar{Q}_{j\dot{\alpha}}\} = 2\sigma^\mu_{\alpha\dot{\alpha}} \tau^a_{ij} p^a_\mu, \tag{10}$$

Where $\tau^a = (\tau^a_{ij})$ are the two 2×2 matrices given by

$$\tau^1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tau^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{11}$$

These two matrices form a representation of S_2 and satisfy the following relations

$$\tau^a_{ij}\tau^b_{jk} = \delta^{ab}\delta_{ik} + \eta^{ab}\eta_{ik}, \tag{12}$$

$$\tau^a_{ij}\tau^a_{kl} = \delta_{il}\delta_{jk} + \eta_{il}\eta_{jk}, \tag{13}$$

In view of what precedes on the commutation relations of this modified translation and the other generators of the Poincare algebra (translations and rotations), the other commutators of the as modified *N*=2super Poincare algebra read

$$[P^a_\mu, P^b_\nu] = 0 \text{ and } [M_{\alpha\beta}, P^a_\mu] = -\eta_{\alpha\mu} P^a_\beta + \eta_{\beta\mu} P^a_\alpha, \tag{14}$$

Where $M_{\alpha\beta}$ are the generators of the rotations and all other commutators are identical to those of usual extended *N*=2 super Poincare algebra. Moreover, it is straight forward to check that the modified super symmetry algebra (10) is consistent with all possible Jacobi identities of the whole algebra.

It is worth noting that P^2_μ satisfies, just as the usual translation, $P^2_\mu P^{2\mu} = m^2$ since permutation operators satisfy $(s^a)^2=1$ (a=1,2). The Casmir invariant operator P^2 expressed thus as $\frac{1}{2}\sum_a P^a_\mu P^{a\mu} = m^2$ shows as usual, that all members of the same multiplet representation are of same masses.

One can also see that the reduction to the simple *N*=1 case leads obviously to standard results since the permutation operations on a set of one object are trivial.

We will now show that one can find an off-shell representation of this algebra, i.e., a multiplet that realizes this modified *N*=2super symmetry algebra *off shell*.

An off-shell Representation

We first start with the same field contents that of the double tensor multiplet (as a generalization of the N=1 super symmetric multiplet of the gauge spinor super field [7]. We will show that such a multiplet forms a representation of the above introduced modified *N*=2 supersymmetric algebra (10) and more over, a consistent off-shell construction can be performed. This multiplet contains two Weyl fermions ψ and χ, two real scalar fields $\varphi_i i = 1,2$ and two real 2-formgauge potentials $B_{i\mu\nu}$, $\mu(\nu)$= 0, 1, 2, 3 and i=1,2. All conventions and notations are the same as in the previous section. In what follows we work in the two-component formalism and adopt the standard conventions of Wess and Bagger [8]. The Lagrangian density of this multiplet reads

$$L = -i\bar{\psi}\bar{\sigma}^\mu \partial_\mu \psi - i\bar{\chi}\bar{\sigma}^\mu \partial_\mu \chi - \frac{1}{2}\partial_\mu \varphi_i \partial^\mu \varphi_i + \frac{1}{2} H_{i\mu} H^\mu_i, \tag{15}$$

Where $H_{i\mu}$ are the Hodge-duals of the field strengths of the 2-form gauge potentials, i.e.

$$H^\mu_i = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\partial_\nu B_{i\rho\sigma}, \tag{16}$$

With $\varepsilon^{\mu\nu\rho T}(\varepsilon^{0123} = +1)$ being the four-dimensional Levi-Civita tensor.

To see that this is indeed a representation of the modified *N*=2 super symmetric algebra defined by (10), we first check that (15) is invariant, up to total derivatives, under the following modified extended *N*=2 super symmetric transformations

$$\delta\psi = i\sigma^\mu \bar{\xi}_1 \partial_\mu \varphi_1 + i\sigma^\mu \bar{\xi}_2 \partial_\mu \varphi_2 + \sigma^\mu \bar{\xi}_1 H_{1\mu} + \sigma^\mu \bar{\xi}_2 H_{2\mu}, \tag{17}$$

$$\delta\chi = i\sigma^\mu \bar{\xi}_1 \partial_\mu \varphi_2 + i\sigma^\mu \bar{\xi}_2 \partial_\mu \varphi_1 + \sigma^\mu \bar{\xi}_1 H_{2\mu} + \sigma^\mu \bar{\xi}_2 H_{1\mu}, \tag{18}$$

$$\delta\varphi_1 = \xi_1 \psi + \xi_2 \chi + h.c., \tag{19}$$

$$\delta\varphi_2 = \xi_1 \chi + \xi_2 \psi + h.c., \tag{20}$$

$$\delta H^\mu_1 = 2i\xi_1 \sigma^{\mu\nu}\partial_\nu \psi + 2i\xi_2 \sigma^{\mu\nu}\partial_\nu \chi + h.c., \tag{21}$$

$$\delta H^\mu_2 = 2i\xi_1 \sigma^{\mu\nu}\partial_\nu \chi + 2i\xi_2 \sigma^{\mu\nu}\partial_\nu \psi + h.c., \tag{22}$$

Then recasting the spinor fields ψ and χ as defined in previous section, such that $\psi^1=\psi$ and $\psi^2=\chi$ one can easily write the above transformations as

$$\delta\psi^a = i\sigma^\mu \bar{\xi}_i \tau^a_{ij} \partial_\mu \varphi_j + \sigma^\mu \bar{\xi}_i \tau^a_{ij} H_{j\mu}, \tag{23}$$

$$\delta\varphi_i = \tau^a_{ij}\xi_j \psi^a + \tau^a_{ij} \overline{\xi_j \psi}^a, \tag{24}$$

$$\partial H^\mu_i = 2i\tau^a_{ij}\xi_j \sigma^{\mu\nu}\partial_\nu \psi^a - 2i\tau^a_{ij}\overline{\xi_j \sigma}^{\mu\nu}\partial_\nu \bar{\psi}^a, \tag{25}$$

A direct computation leads explicitly

$$\delta L = -\partial_\mu(\psi^a \sigma^\mu \bar{\sigma}^\nu \xi_i \tau^a_{ij} \partial_\nu \varphi_j + i\bar{\psi}^a \bar{\sigma}^\mu \sigma^\nu \bar{\xi}_i \tau^a_{ij} H_{j\nu} - \partial_\mu(\tau^a_{ij}\xi_j \psi^a \partial^\mu \varphi_i - iH^\mu_i \tau^a_{ij}\xi_j \psi^a h.c.)$$

We are now able to compute the action of the commutator of two successive modified *N*=2 super symmetric transformations of parameters $\xi(\xi_1,\xi_2)$ and $\zeta(\zeta_1,\zeta_2)$ on each field of the multiplet. Starting with the scalar fields φ_i, we first get

$$\delta_\varsigma \delta_\xi \varphi_i = i(\xi_k \sigma^\mu \bar{\varsigma}_k + \bar{\xi}_k \bar{\sigma}^\mu \varsigma_k)\partial_\mu \varphi_i + i(\xi_k \sigma^\mu \eta_{kl} \bar{\varsigma}_l + \bar{\xi}_k \bar{\sigma}^\mu \eta_{kl} \zeta_l)\eta_{ij}\partial_\mu \varphi_j,$$
$$+(\xi_k \sigma^\mu \bar{\zeta}_k - \bar{\xi}_k \bar{\sigma}^\mu \varsigma_k) H_{i\mu} + i(\xi_k \sigma^\mu \eta_{kl} \bar{\varsigma}_l - \bar{\xi}_k \bar{\sigma}^\mu \eta_{kl} \zeta_l)\eta_{ij} H_{j\mu} \tag{26}$$

Using $\bar{\xi}_i \bar{\sigma}^\mu \varsigma_j = -\varsigma_j \sigma^\mu \bar{\xi}_i$, we see that the terms proportional to $\partial_\mu \varphi$ are anti symmetric under the substitution ξ ⇄ ς such that they are doubled in the commutator $(\delta_\varsigma \delta_\xi - \delta_\xi \delta_\varsigma)\varphi_i$, while the terms proportional to *H* are symmetric under the same substitution, thus they disappear when computing this commutator. Explicitly, we get

$$(\delta_\varsigma \delta_\xi - \delta_\xi \delta_\varsigma)\varphi_i = -2i(\varsigma_k \sigma^\mu \bar{\xi}_k - \xi_k \sigma^\mu \bar{\varsigma}_k)\partial_\mu \varphi_i - 2i(\varsigma_k \sigma^\mu \eta_{kl} \bar{\xi}_l - \xi_k \sigma^\mu \eta_{kl} \bar{\varsigma}_l)\eta_{ij}\partial_\mu \varphi_j, \tag{27}$$

Which in regard to (10), shows that $(\delta_\zeta \delta_\xi - \delta_\xi \delta_\zeta)$ on the scalar fields φ_i closes off shell. We turn now to compute the commutator on the spinor fields. A direct evaluation of $\delta_\zeta\,\delta_\xi\,\psi^a$ shows that the terms in equations of motion of ψ^a cancel due to the contribution of the variation of $H_{i\mu}$, we then obtain

$$\delta_\varsigma\delta_\xi\psi^a_\alpha = -2i\varsigma_k\sigma^\mu\overline{\xi}_k\partial_\mu\psi^a_\alpha + -2i\varsigma_k\sigma^\mu\eta_{kl}\overline{\xi}_l\eta^{ab}\partial_\mu\psi^b_\alpha,$$
$$-2i\overline{\varsigma}_k\overline{\xi}_k\sigma^\mu_{\alpha\dot\alpha}\partial_\mu\overline{\psi}^{a\dot\alpha} - 2i\overline{\varsigma}_k\eta_{kl}\overline{\xi}_l\sigma^\mu_{\alpha\dot\alpha}\eta^{ab}\partial_\mu\overline{\psi}^{b\dot\alpha} \tag{28}$$

Where we used the identities $(\sigma^\mu\overline{\sigma}^\nu + \sigma^\nu\overline{\sigma}^\mu)^\beta_\alpha = -2\eta^{\mu\nu}\delta^\beta_\alpha, \sigma^\mu_{\alpha\dot\alpha}\overline{\sigma}^{\dot\beta\beta}_\nu = -2\delta^\beta_\alpha\delta^{\dot\beta}_{\dot\alpha}$ and the definition $\overline{\sigma}^{\mu\dot\alpha\alpha} = \varepsilon^{\dot\alpha\dot\beta}\varepsilon^{\alpha\beta}\sigma^\mu_{\beta\dot\beta}$ with $\varepsilon_{12} = \varepsilon^{21} = -1$ is the two-dimensional Levi-Civita tensor. Noticing that the factors of the terms involving the equations of motion of $\overline{\psi}^a$ are symmetric under the substitution $\overline{\xi} \leftrightarrow \overline{\varsigma}$ we end up with the following commutator

$$(\delta_\varsigma\delta_\xi - \delta_\xi\delta_\varsigma)\psi^a = -2i(\varsigma_k\sigma^\mu\overline{\xi}_k - \xi_k\sigma^\mu\overline{\varsigma}_k)\partial_\mu\psi^a$$
$$-2i(\varsigma_k\sigma^\mu\eta_{kl}\overline{\xi}_l - \xi_k\sigma^\mu\eta_{kl}\overline{\varsigma}_l)\eta^{ab}\partial_\mu\psi^b_\alpha, \tag{29}$$

Which close off-shell.

Finally, we check the closure on the fields H^μ_i. Using the identity $\overline{\xi}_i\overline{\sigma}^{\mu\nu}\overline{\sigma}^\rho\varsigma_j = -\varsigma_j\sigma^\rho\overline{\sigma}^{\mu\nu}\overline{\xi}_i$ and rearranging terms, we first find

$$\delta_\varsigma\delta_\xi H^\mu_i = -2(\xi_k\sigma^{\mu\nu}\sigma^\rho\overline{\varsigma}_k - \varsigma_k\sigma^\rho\overline{\sigma}^{\mu\nu}\overline{\xi}_k)\partial_\nu\partial_p\varphi_i - 2(\xi_k\sigma^{\mu\nu}\sigma^\rho\eta_{kl}\overline{\varsigma}_l - \varsigma_k\sigma^\rho\overline{\sigma}^{\mu\nu}\eta_{kl}\overline{\xi}_l)\eta_{ij}\partial_\nu\partial_\rho\varphi_j$$
$$+2i(\xi_k\sigma^{\mu\nu}\sigma^\rho\overline{\varsigma}_k + \varsigma_k\sigma^\rho\overline{\sigma}^{\mu\nu}\overline{\xi}_k)\partial_\nu H_{ip} + 2i(\xi_k\sigma^{\mu\nu}\sigma^\rho\eta_{kl}\overline{\varsigma}_l - \varsigma_k\sigma^\rho\overline{\sigma}^{\mu\nu}\eta_{kl}\overline{\xi}_l)\eta_{ij}\partial_\nu H_{jp}. \tag{30}$$

When evaluating the commutator $(\delta_\eta\delta_\xi - \delta_\xi\delta_\eta)H^\mu_i$, we can see that all terms proportional to $\partial_\nu\partial_\rho\varphi$ are of type $\xi_k(\sigma^{\mu\nu}\sigma^\rho + \sigma^\rho\overline{\sigma}^{\mu\nu})\overline{\varsigma}_l - (\overline{\xi} \leftrightarrow \overline{\varsigma})$ which is identical to $i\varepsilon^{\mu\nu\rho T}\xi_k\sigma_T\overline{\varsigma}_l$ so that all φ contributions in the resulting commutator vanish. At the same time $\partial_\nu H_{i\rho}$ contributions involve terms of type $\xi_k(\sigma^{\mu\nu}\sigma^\rho - \sigma^\rho\overline{\sigma}^{\mu\nu})\overline{\varsigma}_l - (\xi \leftrightarrow \varsigma)$ which is identical to $(\eta^{\mu\rho}\xi_k\sigma^\nu\overline{\varsigma}_l - \eta^{\nu\rho}\xi_k\sigma^\mu\overline{\varsigma}_l) - (\overline{\xi} \leftrightarrow \overline{\varsigma})$ so that we end up with the following commutator.

$$(\delta_\varsigma\delta_\xi - \delta_\xi\delta_\varsigma)H^\mu_i = -2i(\varsigma_k\sigma^\nu\overline{\xi}_k - \xi_k\sigma^\nu\overline{\varsigma}_k)\partial_\nu H^\mu_i - 2i(\varsigma_k\sigma^\nu\eta_{kl}\overline{\xi}_l - \xi_k\sigma^\nu\eta_{kl}\overline{\varsigma}_l)\eta_{ij}\partial_\nu H^\mu_j, \tag{31}$$

Where the identities $\sigma^\mu\overline{\sigma}^\nu\sigma^\rho - \sigma^\rho\overline{\sigma}^\nu\sigma^\mu = 2i\varepsilon^{\mu\nu\rho\tau}\sigma_\tau$, $\sigma^\mu\overline{\sigma}^\nu\sigma^\rho + \sigma^\rho\overline{\sigma}^\nu\sigma^\mu = 2(\eta^{\mu\rho}\sigma^\nu - \eta^{\nu\rho}\sigma^\mu - \eta^{\mu\nu}\sigma^\rho)$ and $\overline{\varsigma}_k\overline{\sigma}^\mu\sigma^\nu\overline{\sigma}^\rho\xi_l = -\xi_l\sigma^\rho\overline{\sigma}^\nu\sigma^\mu\overline{\varsigma}_k$ are used as well as the identity $\partial_\mu H^\mu_i = 0$ which follows from the definition(16). This ends the proof that the modified *N*=2 super symmetric transformations (23)-(25) form a super symmetric algebra that closes off shell. The *N*=2 multiplet $(\psi^a, \phi_i, H^\mu_i)$ is then an off-shell multiplet of the modified *N*=2 super symmetric algebra defined by (10). It is well known [2] that the double tensor multiplet model has also special gauge invariance. Similarly for the above-introduced multiplet, it is easy to check that the Lagrangian density (15) is also invariant upon the gauge transformation

$$\delta_\wedge B^{\mu\nu}_i = \partial^\mu\wedge^\nu_i - \partial^\nu\wedge^\mu_i \tag{32}$$

Where $\wedge^\mu_i$ are the space time dependent gauge parameters. Since, by construction (16), the Hodge-duals H^μ_i are obviously invariants upon such a transformation, we have, to make this gauge transformation appear, to replace H^μ_i by the corresponding gauge potentials $B^{\mu\nu}_i$ within the transformations (23)-(25). We find

$$\delta\psi^a = i\sigma^\mu\overline{\xi}_i\tau^a_{ij}\partial_\mu\varphi_j + \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\sigma_\mu\overline{\xi}_i\tau^a_{ij}B_{j\rho\sigma}, \tag{33}$$

$$\delta B^{\mu\nu}_i = -2\tau^a_{ij}\xi_j\sigma^{\mu\nu}\psi^a - 2\tau^a_{ij}\overline{\xi}_j\overline{\sigma}^{\mu\nu}\overline{\psi}^a, \tag{34}$$

While the transformations of the scalar fields (24) remains the same. We now show that the commutator $(\delta_\zeta\,\delta_\xi\text{-}\delta_\xi\,\delta_\zeta)$ on the gauge potentials $B^{\mu\nu}_i$ closes, as previously, off shell on the combination of translation and permutations but also on the above defined gauge transformation. After a similar computation to (31), we find

$$(\delta_\varsigma\delta_\xi - \delta_\xi\delta_\varsigma)B^{\mu\nu}_i = -2i(\varsigma_k\sigma^\lambda\overline{\xi}_k - \xi_k\sigma^\lambda\overline{\varsigma}_k)\partial_\lambda B^{\mu\nu}_i$$
$$-2i(\varsigma_k\sigma^\lambda\eta_{kl}\overline{\xi}_l - \xi_k\sigma^\lambda\eta_{kl}\overline{\varsigma}_l)\eta_{ij}\partial_\lambda B^{\mu\nu}_i + \partial^\mu\wedge^\nu_i - \partial^\nu\wedge^\mu_i, \tag{35}$$

Where the gauge parameters $\wedge$ are given by

$$\wedge^\mu_i = 2i\wedge^{\mu\nu}_{ijk}(\varsigma_j\sigma_\nu\overline{\xi}_k - \xi_j\sigma_\nu\overline{\varsigma}_k),\ \wedge^{\mu\nu}_{ijk} = \tau^a_{ij}\tau^a_{kl}(\eta^{\mu\nu}\varphi_l - B^{\mu\nu}_l). \tag{36}$$

In deriving (35) the identity $\varepsilon_{k\tau\rho\sigma}\varepsilon^{k\mu\nu\lambda} = -[\delta^\mu_\tau(\delta^\nu_\rho\delta^\lambda_\sigma - \delta^\nu_\sigma\delta^\lambda_\rho) - \delta^\nu_\tau(\delta^\mu_\rho\delta^\lambda_\sigma - \delta^\mu_\sigma\delta^\lambda_\rho) + \delta^\lambda_\tau(\delta^\mu_\rho\delta^\nu_\sigma - \delta^\mu_\sigma\delta^\nu_\rho)]$ issued. As it is generally the case in super symmetric gauge theories, these gauge parameters are field dependent. It is worth noting that comparatively to the standard approach [2] of the double tensor multiplet, in addition to the fact that in the context presented here the off shell construction is possible. The obtained gauge parameters (36) do noting volve explicitly the space time coordinates.

Even if the structure of the modified algebra (10) differs from the usual one, we can ,in view of the off-shell closure obtained above, consider the construction of a nilpotent BRST operator. Indeed, starting from the modified *N*=2 super symmetry transformations (23) - (25), and upon the usual replacement of the symmetry parameters by the corresponding host fields of opposite statistics, the corresponding BRST construction follows naturally.

Defining the BRST operator Δ on the fields $\psi^a, \varphi_i, B^{\mu\nu}_i$ as

$$\Delta\psi^a = i\sigma^\mu\overline{\xi}_i\tau^a_{ij}\partial_\mu\varphi_j + \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\sigma_\mu\overline{\xi}_i\tau^a_{ij}B_{j\rho\sigma} + c^\rho\partial_\rho\psi^a + \kappa^\rho\eta^{ab}\partial_\rho\psi^b, \tag{37}$$

$$\Delta\varphi_i = \tau^a_{ij}\xi_j\psi^a + \tau^a_{ij}\overline{\xi}_j\overline{\psi}^a + c^\rho\partial_\rho\varphi_i + \kappa^\rho\eta_{ij}\partial_\rho\varphi_j, \tag{38}$$

$$\Delta B^{\mu\nu}_i = -2\tau^a_{ij}\xi_j\sigma^{\mu\nu}\psi^a - 2\tau^a_{ij}\overline{\xi}_j\overline{\sigma}^{\mu\nu}\overline{\psi}^a + c^\rho\partial_\rho B^{\mu\nu}_i + \kappa^\rho\eta_{ij}\partial_\rho B^{\mu\nu}_i + \partial^\mu\wedge^\nu_i - \partial^\nu\wedge^\mu_i \tag{39}$$

And on the ghosts fields $\xi_i(\overline{\xi}_i), c^\mu, \kappa^\mu$ and $\wedge^\mu_i$ as

$$\Delta\xi_i = 0, \tag{40}$$

$$\Delta c^\mu = -2i(\xi_k\sigma^\mu\overline{\xi}_k), \tag{41}$$

$$\Delta\kappa^\mu = -2i(\xi_k\sigma^\mu\eta_{kl}\overline{\xi}_l) \tag{42}$$

$$\Delta\kappa^\mu = -2i(\xi_k\sigma^\mu\eta_{kl}\overline{\xi}_l), \tag{43}$$

$$\Delta\wedge^\mu_i = 2iT^a_{ij}T^a_{kl}(\eta^{\mu\nu}\varphi_j - B^{\mu\nu}_j)(\xi_k\sigma_\nu\overline{\xi}_l), \tag{44}$$

It is straight forward to show its off-shell nil potency, i. e , $\Delta^2 X = 0, \forall X$.

Conclusion

The main result of this work is that an alternative *N*=2 extension of standard supersymmetry is possible. This is done in the context of nonlinear extensions of standard Lie algebra by a suitable introduction of the symmetric group S_2.The additional nonlinear term being a composition of translation and transposition. The obtained algebra being a quadratically nonlinear extension of the standard *N*=2 super symmetric algebra is however a non usual construction. Indeed, this latter contains structurally the permutation transformations that are obviously non-local, while, in deriving the general realization of supersymmetry algebra, only continuous groups (in particular Lie groups) are usually considered (see e.g. [8]).This kind of construction will be analyzed in detail elsewhere.

The presented result is different from the standard extended *N*=2 super symmetry, i.e., the anti commutator of two modified super symmetric transformations must close (at least on shell) on a mix of translation and permutations, but leads to a consistent algebraic construction. The reduction to the N=1 case leads to usual super symmetry due to the triviality of the group S_1 which contains only the identity. Such a modified extended *N*=2 super symmetric algebra (10) admits as representation a multiplet that contains the same fields as the double tensor multiplet (which is in particular, relevant to type II B super string vacua [9]). We have shown that an off-shell construction is possible, i.e., without relying on field equation. This result has to be compared with the usual double tensor multiplet for which, inspite of the fact that the bosonic and fermionic degrees of freedom balance at both on-shell and off-shell levels, the off-shell construction fails.

Moreover, if a systematic procedure can be considered in order to give the off-shell version of any given open gauge (local) theory [10], no such systematic approach is available in the context of global (rigid) symmetries such as extended matter super symmetry, even if specific models exist where the construction of off-shell realization is possible, i.e., the so-called 0(2n) super multiplets [11] (see also [12] for a modern review). We believe that the approach developed here in which the symmetric group S_2 (or equivalently the group Z_2) shows up within the nonlinear extension of the usual *N*=2 super symmetry algebra can offer a new perspective for investigating the off-shell structure of extended super symmetric models (e.g *N*=4 super symmetric models).

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