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A Mathematical Model of the Maximum Rigidity Resulting from Diffusive Shock Acceleration

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Abstract

The primary cause of the high-energy particle acceleration of cosmic rays and solar-particle events is first-order Fermi diffusive shock acceleration. The importance of the maximum rigidity is that it governs the effective depth in the atmosphere that accelerated particles can reach. Because the rigidity is a function of the particle energy, a high rigidity implies a high energy and thus a more intense hadronic cascade and greater atmospheric penetration. Because of their high energies, cosmic rays can penetrate the Earth's surface and be measured underground. However, whether solar-particle events will produce significant radiation exposure at aircraft altitudes depends on the maximum rigidity resulting from the shock. First-order shock acceleration is governed by the speed of the shock, the local magnetic field the length of time the shock continues and the strength of the shock. These factors are combined in a simple equation which provides a simple model for the maximum rigidity resulting from the shock. The equation derived from these considerations accounts reasonably well for cosmic-ray and solar-particle maximum rigidities.

Keywords: Maximum rigidity; Particle acceleration; Hadronic cascade

Introduction

Diffusive shock acceleration takes place when the supersonic shock front from a high-energy event, such as a Type II supernova, or a coronal mass ejection (CME) from the sun, interacts with surrounding plasma. If the shock is supersonic, it accelerates "downwind" particles which pass back and forth across the shock front, gaining rigidity on each pass. The rigidity of a charged particle is the property that governs its behavior in an electric or magnetic field, and, of course, a shock front in plasma. The maximum rigidity resulting from such an event depends on the velocity of the front, the ambient magnetic field (which determines the diffusion coefficient of the medium) and the length of time the acceleration takes place. The maximum rigidity governs the ability of the particles accelerated by this process to penetrate the Earth's atmosphere.

Theory

The rigidity of a particle is defined by:

$$P=\frac{pc}{Ze}=300B\rho \tag{1}$$

where ***P*** is the rigidity in volts, p is the particle momentum in eV/c, ***C*** is the speed of light, Z is the atomic number of the particle and *e* is the charge of the particle in electronic units (for a proton, $Ze=1$), *B* is the local magnetic field in Gauss and $\boldsymbol{\rho}$ is the Larmor radius, the radius described by a particle in the magnetic field *B*.

This process can be described by the differential equation:

$$\frac{\partial\phi(P)}{\partial P}=-\phi(P)\left(\frac{1}{hP}+\frac{1}{R}\right) \tag{2}$$

where $\phi(P)$ is the rigidity spectrum, per (cm², V, sr, s), *P* is the rigidity, *h* is the non-escape probability and and *R* is the (average) maximum rigidity.

The solution (writing $1/h=\gamma$) is

$$\phi(P)=N(Z)P^{-\gamma}\exp(-P/R) \tag{3}$$

Here, $\phi(P,R,Z)$ is the number of particles (cm^{-2} sr^{-1} s^{-1} V^{-1}) at a rigidity *P* volts, and *N*(*Z*) is the fraction of φ of atomic number *Z*. Finally $\gamma_{diff}=1/h$ and $|\gamma_{diff}|=|\gamma_{\text{int}}|+1$ where γ_{int} is the index of the integral flux.

Eq. 3 closely resembles the Ellison-Ramaty equation [3] except that it is expressed in terms of the rigidity instead of momentum and energy.

Additionally [1,2]

$$M=\sqrt{\frac{\gamma^2-4}{\gamma-2}} \text{ and } \gamma=\frac{2(M^2+1)}{M^2-1} \tag{4}$$

where *M* is the Mach number of the shock.

Since the rigidity is a function of the particle energy, a high maximum rigidity implies a high energy (*cf.* Eq. 1). At relativistic energies, $R\simeq E/Ze$. Higher energy particles create a more intense hadronic cascade - more energetic secondary particles per particle-atmospheric nucleus collision. Thus the maximum rigidity (and hence, the maximum energy) governs the effective depth in the atmosphere that particles generated in the hadronic cascade can reach. In other words, *R*, the maximum rigidity, is the critical parameter in Eq. 3. If *R* is less than about 1300 MV, the particle spectrum is "soft" and will not penetrate deep into the atmosphere. Note that if *R* is very large, the integral of Eq. 3 (the total flux) must become very large to compensate for the exponential term.

Because of their high energies, cosmic rays can penetrate the

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Earth's surface and be measured underground. However, solar-energetic particles may not produce significant radiation levels at aircraft altitudes.

The diffusion coefficient is approximated by assuming that the diffusion length cannot be less than the Larmor radius of the particle. Energetic particles cannot respond to irregularities in a magnetic field smaller than the Larmor radius. Letting $\boldsymbol{\lambda}$ be the diffusion length, the diffusion coefficient κ is then

$$\kappa = \frac{1}{3}\lambda v \geq \frac{1}{3}\rho c \tag{5}$$

where v is the particle velocity and c is the speed of light (assuming that the high-energy particles are relativistic) and ρ is the Larmor radius.

Since the Larmor radius is P/300B (from Eq.1), where P is the particle rigidity in Volts and $\boldsymbol{B}$ is the magnetic field strength in Gauss., T_{cycle} is the time it takes for a particle to cycle back and forth across the shock front [1-3],

$$\kappa = \frac{Pc/3}{300B} \tag{6}$$

and

$$T_{cycle} = \frac{4}{c}\left(\frac{\kappa_1}{v_1} + \frac{\kappa_2}{v_2}\right) \simeq \frac{4(r+1)}{3}\frac{Pcv_1}{300B} \tag{7}$$

and,

$$R = \frac{P}{T_{cycle}} v_1 t \tag{8}$$

or, making a "last-flight" approximation,

$$R = \frac{7.5\times10^{-9} v_1^2 Bt}{r+1}. \tag{9}$$

r is the ratio of the upstream to the downstream velocity,

v_1 is the velocity of the shock front and t is the length of time the acceleration takes place.

Cosmic Radiation

Using Gaisser's [4] values for a ten-solar-mass Type II supernova, v=5×10^{8}cm/s, B=3μG, t=1000 years and r=4, then R=3.53×10^{13} Volts, within 20% of Gaisser's value of 3×10^{13} Volts*.

Taking the extreme case, of $v_1 \simeq c$, then R<1.3×10^{17} Volts. 10^{17}V is though to be the boundary between galactic and extra-galactic cosmic rays. Other mechanisms for accelerating particles to high energies are discussed by T. M. Butt [5].

Solar Particle Events

Solar particle events result from a CME from the sun. If the resulting shock front is supersonic, the CME will accelerate particles in the interplanetary medium to high energies.

In the interplanetary medium there are considerable uncertainties in the parameters that make up Eq. 8 such as the local magnetic field, the ratio of the upstream to downstream velocity and the shock-front velocity.

1. The local magnetic field. The local magnetic field has a value of about one gauss at the surface of the sun and is expected to vary between10^{-5} to 2×10^{-4} at one AU [6,7].

2. The ratio of the upstream to the downstream velocities. The ratio of the upstream to the downstream velocities relative to the shock front depends on the strength of the shock. Since the shocks are supersonic, it is assumed that the shocks are strong shocks with a ratio of 4:1 as above.

3. The shock-front velocity. The shock-front velocity can be inferred from the Mach number.

Despite these uncertainties, Eq. 8 provides a useful mathematical framework to describe the phenomenon.

Assuming that the interplanetary medium, hydrogen plasma, is an ideal monatomic gas, the speed of sound is given by:

$$S = \sqrt{\frac{5}{3}\frac{kT}{m}} \tag{10}$$

where $\boldsymbol{k}$ is Boltzmann's constant, $\boldsymbol{T}$ is the temperature of the plasma in kelvins and m is the mass of the hydrogen nucleus. Assuming a temperature of 2 million kelvins, the speed of sound relative to the solar wind is 1.66×10^7 cm/s.

GLE 42

The ground level event of September 29-30, 1989, GLE42, was the largest event since the February 1956 flare. It will be considered here because of the detailed analysis of the data [8].

Near the surface of the sun, the Mach number was 5.5 [2]. The Mach number of cosmic rays is a nearly constant 2.5 over a wide range of energies and rigidities [2]. The shock-front velocity is then $M\times S$, and v=9.123 × 10^7 cm/s. It is assumed the acceleration time was ten hours, the time between the onset of the event and the ejection of the CME from the corona [2]. Substituting these parameters into Eq. 8, we get 45 GV for R, which is within 40% of the measured value of 33 GV [9,10].

After the acceleration continued for 95 hours, the Mach number was 1.5 so that M × S is equal to 2.488 × 10^{7}cm/s. Setting B=1 × 10^{-4}G (*cf.* 1 above) and using V=M×S we get 32 MV or 0.54 eV. Clearly, the magnetic field at the shock front must have been much greater, more than 0.04 G, after 100 hours, since the measured spectrum exceeds 1343 MV (700 MeV) [8]. However, the magnetic field is expected to be considerably disturbed during a solar particle event (for instance, Lozitsky [11]) or the shock front may have been closer to the sun at this time. It should also be noted that the counting rate at neutron monitors with high vertical cutoffs, such as the Huancayo monitor (13 GV), part of the data set analyzed by O'Brien and Sauer [8], were elevated during the early stages of this event.

R must significantly exceed 445 MV (100 MeV) to produce a ground-level event, and to produce a significant dose rate at flight levels, a ground-level event is usually required (Kataoka, et al. [12]). The parameters necessary to produce such an event are found in Eq. 8.

To produce an event as large as the 1859 Carrington event [13,14] which was about 17 times GLE 42, a very large maximum rigidity is required.

Conclusion

This paper presents a simple mathematical expression for the maximum rigidity generated by a supersonic shock front, which had some limited success in the case of GLE 42. The magnitude of the rigidity resulting from such a shock front depends on the velocity of the front, the local magnetic field, the ratio of the upstream to the downstream velocity and the acceleration time. The maximum rigidity of a shock front incident on the Earth, determines the effective depth in the atmosphere the radiation resulting from the shock will penetrate. To produce a radiation dose at flight levels the maximum rigidity should significantly exceed 445 MV.

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