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A Group Theory Approach to Symmetry in Optics and Photonics

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Introduction

Group theory (GT) provides a rigorous framework for studying symmetries in a variety of physics disciplines, including quantum field theories and the standard model, as well as fluid mechanics and chaos theory. To date, the use of such a powerful tool in optical physics has been limited. However, in recent years, several quantum-inspired symmetry principles (such as parity-time invariance and supersymmetry) have been introduced for the first time in optics and photonics. Despite the intense activity in these new research areas, only a few works have made use of the power of group theory. Motivated by this state of affairs, we present a brief overview of the application of GT in optics, carefully selecting examples that demonstrate the utility of this tool in both continuous and discrete settings [1].

Symmetry principles are critical in modern physics. Interest in symmetry concepts can be traced back to the ancient Greeks' early works on platonic solids. One of the first investigations that brought the concept of symmetry to the forefront of physical science was Emmy Noether's discovery that conservation laws and continuous symmetries are linked; for example, the conservation of energy, linear and angular momenta are a direct result of temporal, spatial, and rotational symmetries, respectively. Herman Weyl introduced the concept of gauge invariance almost simultaneously in an attempt to unify gravity and electromagnetism.

Except for a few notable examples, the tremendous progress in optics over the last few decades has benefited little from group theoretical techniques (see Ref. and references therein, as well as Refs. However, new research directions have emerged recently that exploit how quantum-inspired symmetries can be used to engineer novel optical structures. These include, for example, PT symmetry (and, more broadly, non-Hermiticity) and supersymmetry [2].

In this section, we will look at how group theory can be used to find special solutions to the scalar Helmholtz equation, with a focus on propagation invariant beams. In general, light propagation is described by Maxwell equations and their constitutive relations. For monochromatic fields in homogeneous, isotropic, and linear media, this takes the form of the vector Helmholtz equation. To that end, first-order symmetries of differential equations with either ordinary or partial derivatives are first-order differential operators. L transforms differential equation solutions into other solutions; that is, if is a differential equation solution, then L is also a solution. If L and L 0 are first-order symmetries, then any arbitrary linear combination L + L0 is also a first-order symmerty (and thus they form a vector space), and the product (composition of differential operators) Because LL0 is a second-order differential operator that maps solutions into solutions, it has second-order symmetry. Higher-order symmetries can be constructed in the same way [3].

If we reverse the propagation direction from z to y, we can recover the so-called half-Bessel, half-Mathieu, and Weber accelerating optical beams from the above solutions in circular-, elliptic-, and paraboliccylindrical coordinates, in that order. Furthermore, nonparaxial accelerating waves have been constructed using spherically symmetric solutions to the scalar Helmholtz equation in parabolic, oblate and prolate spheroidal and spherical coordinates. MS Discrete photonics has emerged as a new paradigm for engineering optical structures with unique properties over the last two decades. These systems (often built with waveguide arrays as shown schematically) serve as testbeds for observing some intriguing phenomena predicted theoretically in the context of condensed matter, such as Bloch oscillations, dynamic localization, Anderson localization, and, more recently, topological insulators. Furthermore, the mathematical analogy between discrete arrays and quantum optics has recently been studied.

Discussion

When we consider the Helmholtz equation, we will create differential equations with prescribed (well-known) symmetry algebras and solve those using group-theoretical methods. For this purpose, we will use the number operator (which produces a linear propagation constant ramp) and the step-up and step-down operators (or Susskind-Glogower operators, which produce the coupling between adjacent waveguides) as building blocks, which can be modulated by functions that depend on both the propagation distance and the number operator to produce the desired symmetry algebra [4].

Closed form solutions are difficult to find in general when some of the system's parameters vary with the propagation distance z. While the system can, of course, be solved numerically, analytical or semianalytical solutions can still provide deeper insight. The following sections demonstrate how group theoretical techniques can be used to help achieve this goal.

Previously discussed photonic systems, but with a semiinfinite array. This intriguing feature was discovered by noting that in the harmonic oscillator's symmetry algebra, there are both compact operators with discrete spectrum (featuring periodic oscillations) and non-compact operators with continuous spectrum (featuring free propagation). These two examples show how group theory can be used to not only find new solutions, but also to categorise photonics systems and predict new dynamics. Given the preceding discussion, and the increasing complexity of new photonic systems enabled by unprecedented simulation power and dense packing technologies, these new insights derived from group theory are unavoidable for studying and designing large photonic systems, as well as building complex optical devices with new functionalities [5].

Conclusion

It is important to note that group theory can provide insights that lead to the development of general frameworks for dealing with specific problems. Importantly, group theory enables us to categorise and analyse systems with similar symmetries in a unified manner regardless of dimensionality. Optical arrays with SU(2) symmetry, for example, will exhibit coherent oscillation dynamics that can be represented on the Poincare sphere. As a result, the behaviour of these arrays can be engineered using three independent rotations, an insight that would not have been possible without the use of group theory. Another application of group theory is the identification of a very unusual family

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of optical arrays known as Glauber-Fock photonic lattices, which share the same symmetry group as the harmonic.

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Conflict of Interest

There are no conflicts of interest by author.

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