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A Grill between Weak Forms of Faint Continuity

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Abstract

We are interested introducing new classes of faint continuity namely faint G-γ continuity, faint G-α-continuity and faint G-semi-continuity to obtain their characterizations and some of their properties. AMS Mathematics Subject Classification: (2000) 54D10, 54C10.

Keywords: G-γ-open sets; faint G-semi-continuity; Faintly G-γ-continuous; faint G-α-continuity

Introduction

The idea of grill topological space aimed at generalize the concept of topological space since it generates newtopology τ_G which helps to measure the things that was di cult to measure. Choquet [1] in 1947 was the first author introduced the idea of grill. It has been explored that there is some of similarity between Choquet;s concept and that ideals, nets and lters. In 2007, Roy and Mukherjee introduced the definition of τ_G which is related to two operators Φ and Ψ. They have determined the relation between τ and $\tau_{G.}$ A number of theories and characterization has been handeld in previous studies [2-4] whether respect to sets or functions. Hatir and Jafari [5] have the ability determine the defi nition of new categories of sets in a space which carries grill and topology and accordingly could obtain a new infection of continuity in terms of grill. Both Karthikeyan and Rajesh [6] have recently fixed the definition of a new category of functions which is called faintly G-precontinuous in a grill topological space. This paper aims at defining and studying new classes of functions called faintly G-γ continuous (resp. faintly G-α-continuous, faintly G-semi-continuous) between grill topological spaces. Some new descriptions and basic advantages of these classes of functions along with their relationships with certain types of function are investigated.

Preliminaries

"In what follows, by a space Y we shall mean space Y which carries topology τ. For AY, we shall adopt the usual notations Cl(A) and Int(A) to A, respectively denote the closure and interior of A in Y. Also, the power set of Y will be written as P(Y). A nonempty sub collecting G of a space Y is called a grill [2] if

$\phi \notin G$

A ϵ G and $A \subseteq B \subseteq Y$ implies that B ϵ G,

if A;B ⊆ Y and A [B 2 G implies that A 2 G or B 2 G."

Remark 2.1

(1) "The minimal grill is G={Y} in any space Y which carries topology τ. (2) The maximal grill is G=P(Y)\{ϕ} in any topological space (Y; τ)."

Since the grill depends on the two mappings Φand Ψ which is generated a unique grill topological space finer than on space Y denoted byτ_G on Y have been discussion [6].

Definition 2.2: "Let Y be a space which carries topology and G be a grill on Y. A subset A in Y is called [3]:

(1) G-open if A ⊆ Int(Φ(A));

(2) G-preopen if A ⊆ Int((ΨA));

(3) G-semi-open if A ⊆Ψ (Int(A));

(4) G-open if A ⊆ Int(Ψ (Int(A)));

(5) G-open if A ⊆ Int(Ψ (A))∪ Ψ(Int(A))."

"The family of all G-semi-open (resp. G-α-open, G-γ-open, G-preopen) sets of (Y,τ,G) is denoted by GSO(Y) (resp. GαO(Y), GγO(Y), GPO(Y)).

The family of all G-semi-open (resp. G-α-open, G-γ-open, G-preopen) sets of (Y,τ,G) containing a point y∈Y is denoted by GSO(Y; y) (resp. $G_\alpha O(Y; y)$, $G_\gamma O(Y; y)$, GPO(Y; y))".

Definition 2.3: "The intersection of all G-γ-closed (resp. G-α-closed, G-semi-closed) sets contained S ⊂ Y is called the G-γ-closure (resp. G-α-closure, G-semi-closure) of S and is denoted by γClG(S) (resp. ClG(S), sClG(S)) [7]".

Definition 2.4: "A function h: (Y:τ;G) → (Z;σ) is said to be G-γ-continuous at each point y ∈ Y if for each open set B of Z containing h(y), there exists A∈$G_\gamma O$(Y, y) such that h(A) ⊆ B. If h has this property at each point of y, then it is said to be G-γ-continuous [8]".

Definition 2.5: "A subset A of Y is said to be θ-open [9] if for each y ∈ A there exists an open set U such that y ∈ U⊂ Cl(U) ⊂ A".

Definition 2.6: The θ-closure of A, denoted by $Cl_\theta(A)$, is defined to be the set of all y ∈ Y such that if for each y ∈ A there exists an open set U such that A ∩ Cl(U) 6≠ϕ for every open neighborhood U of Y. If A=$Cl_\theta(A)$, then A is called-closed. The complement of a θ-closed set is called θ-open. It follows that the collection of θ-open sets in a topological space (Y;τ) forms a topology τ_o on Y. The θ-interior of A is defined by the union of all θ-open sets contained in A and is denoted by $Int_\theta(A)$ [10]".

Definition 2.7: "A function h: (Y;τ)→ (Z;σ) is said to be faintly continuous [9] if h^{-1} (H) is open in Y for every θ-open set H of Z".

Definition 2.8: "A function h: (Y;τ;G)→ (Z;σ) is said to be faintly G-precontinuous [6] at a point y∈Y if for each θ-open set B of Z

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containing h(y), there exists A ∈ GPO(Y; y) such that h(A) ⊂ B. If h has this property at each point of Y, then it is said to be faintly G-precontinuous".

Some Fundamental Properties

In this section, we define some modern classes of functions called faint G-γ-continuity faint G-γ-continuity and faint G-semi-continuity. Depictions and essential properties of these concepts are studied.

Definition 3.1: A function h: (Y; τ;G)→ (Z;σ) is said to be faintly G-γ-continuous (resp. faintly G-α-continuous, faintly G-semi-continuous) if for each y ϵ Y and each θ-open set B of Z containing h(y), there exists A ϵ $G_\gamma O(Y; y)$ (resp. A∈ $G_\alpha O(Y; y)$, U ϵ GSO(Y; y)) such that h(A) ⊂ B.

Theorem 3.2

For a function h: (Y;τ;G) →(Z;σ), the following statements are equivalent:

(1) h is faintly G-γ-continuous;

(2) h^{-1} (H) is G-γ-open in Y for every θ-open set H of Z;

(3) h^{-1} (F) is G-γ-closed in Y for every θ-closed set F of Z;

(4) h: (Y; τ;G)→ (Z;σ_θ) is G-γ-continuous;

(5) $\gamma Cl_G(h^{-1}(W)) \subseteq h^{-1}(Cl_\theta(W))$ for every subset W of Z;

(6) $h^{-1}(Int_\theta(W)) \subseteq Int_G(h^{-1}(W))$ for every subset W of Z.

Proof: (1⇒ 2): Let H be a θ-open subset of Z and y ∈ h^{-1} (H). Since h(y) ∈ H, h is faintly G-continuous, then there exist Uy ∈ $G_\gamma O(Y; y)$ such that $h(U_y) \subset H$, it follows that $y \in U_y \subset h^{-1}(H)$. We obtain $h^{-1}(H)=\cup\{U_y: y \in h^{-1}(H)\}$. Since any union of G-γ-open sets is G-γ-open, h^{-1} (H) is G-γ-open in Y.

(2 ⇒ 1): Let y ∈ Y and H be a θ-open set of Z containing h(y). From (2), h^{-1} (H) is G-γ-open set of Y containing y. Let U=h^{-1}(H), then h(U) ⊂ H. This implies that h is faintly G-γ-continuous

(2 ⇒ 3): Let H be any θ-closed set of Z. Since Y \H is θ-closed in Z, by (3) $h^{-1}(Y\backslash H)=Y\backslash h^{-1}(H)$ is G-γ-closed and thus h^{-1} (H) is G-γ-open. The other implications are clear.

Theorem 3.3

For a function h: (Y; τ;G) → (Z; σ) the following statements are equivalent:

(1) h is faintly G-α-continuous;

(2) h^{-1} (H) is G-α-open in Y for every θ-open set H of Z;

(3) h^{-1} (F) is G-α-closed in Y for every θ-closed set F of Z;

(4) h: (Y,τ,G) → (Z, σ) is G-α-continuous;

(5) $\alpha Cl_G(h^{-1}(W)) \subseteq h^{-1}(Cl_\theta(W))$ for every subset W of Z.

(6) $h^{-1}(Int_\theta(W)) \subset \alpha\, Int_G(h^{-1}(W))$ for every subset W of Z.

Proof: The proof is similar to the proof of Theorem 3.2.

Theorem 3.4

For a function h: (Y, τ,G) → (Z, σ), the following statements are equivalent:

(1) h is faintly G-semi-continuous;

(2) h^{-1} (H) is G-semi-open in Y for every θ-open set H of Z;

(3) h^{-1} (F) is G-semi-closed in Y for every θ-closed set F of Z;

(4) h: (Y; τ;G)→ (Z; σ_θ) is G-semi-continuous;

(5) $sCl_G(h^{-1}(W)) \subset h^{-1}(Cl_\theta(W))$ ∀ W of Z.

(6) $h^{-1}(Int_\theta(W)) \subseteq sInt_G(h^{-1}(W))$ ∀ W of Z.

Proof: The proof is similar to the proof of Theorem 3.2.

Theorem 3.5

Every G-γ-continuous function is faintly G-continuous.

Proof: It is clear from Definitions 3.1 and Theorem 3.2.

The converse of Theorem 3.5. is not true in general as it can be seen in the following example.

Example 3.6: Let Y={1; 2; 3; 4}, τ={Y,ф,{1},{3},{1,3} and G={Y, {2 },{1,2},{2,3},{2,4},2,{1,2,3},{1,2,4},{2,3,4}}. The function h: (Y, τ,G) → (Y, τ) where h(1)=2, h(2)=3, h(3)=4, h(4)=1 is faintly G-γ-continuous function but not G-γ-continuous function.

Corollary 3.7: Each faintly continuous function is faintly G-γ-continuous function (Figure 1).

Remark 3.8

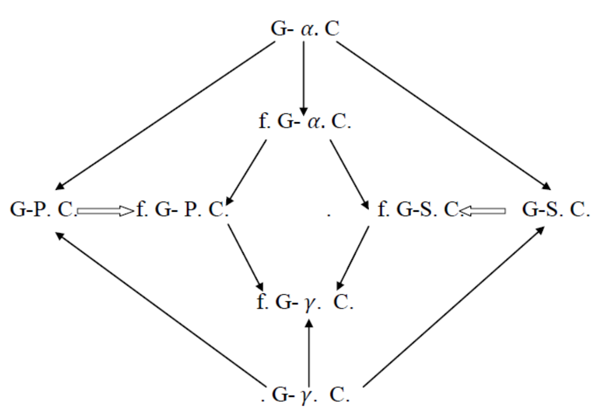
(1) The following figure shows the relationship among these new classes of functions and other corresponding types.

(2) the converse are not true in general as shown by the following examples.

Example 3.9: Let Y=Z={1, 2, 3}, τ={Y,ф, {1}, {2}, {1, 2}}, σ={Z,ф, {1}, {2}, {1, 2}, {1, 3}} and G=P(Y)\{ф}. The identity function h: (Y, τ,G) →(Z, σ) is faintly G-γ-continuous function but not faintly G-precontinuous function since h^{-1}\{1, 3\} GPO(Y, τ) where {1, 3} 2 σ_θ.

Example 3.10: Let Y=Z={1, 2, 3}, τ={Y, ф, {1}, {2}, {1, 2}}, σ={Z, ф, {1}, {2}, {1, 2}, {1, 3}} and G={Y, {1}, {1, 2}, {1, 3}}. The identity function h: (Y, τ,G) → (Y, σ) is faintly G-semi-continuous function but not G-semi-continuous function.

Example 3.11: Let Y=Z={a, b, c},τ={Y,ф, {a}, {b}, {a, b}}, σ={Z,ф,



G-α.c: Grill-α-Continuous, f.G-α.c: Faintly Grill α-Continuous, G-p.c: Grill-Precontinuous, f.G-p.c: Faintly Grill-Precontinuous, G-γ.c: Grill-γ-Continuous, f.G-γ.c: Faintly Grill-γ-Continuous, G-s.c: Grill-Semi-Continuous and f.G-s.c: Faintly Grill-Semi-Continuous.

Figure 1: G-γ-continuous function.

{a}, {b}, {a, b}, {a, c}} and G={Y, {a}, {a, b}, {a, c}}. The function h: $(Y,\tau,G) \to (Y;\sigma)$ where h(a)=h(b)=a, h(b)=b is faintly G-precontinuous function but not faintly G-α-continuous function.

Example 3.12: Let Z=Y={1, 2, 3}, τ={Y,ϕ, {1}, {2}, {1, 2}}, σ={Z,ϕ, {1}, {2}, {1, 2}, {1, 3}} and G=P(Y)\{ϕ} The function h: $(Y,\tau,G) \to (Y;\sigma)$ where h(1)=h(3)=1, h(2)=2 is faintly G-precontinuous function but not G-precontinuous function.

Definition 3.13: A function h: $(Y,\tau) \to (Z,\sigma)$ is named quasi-θ-continuous [8] if h^{-1} (H) is θ-open in (Y,τ) ∀ θ-open set H of (Z, σ).

Theorem 3.14

If the function h: $(X,\tau,G) \to (Y,\sigma)$ is faintly G-γ-continuous and g: $(Y;\sigma) \to (Z;\eta)$ is quasi-θ-continuous, then $g \circ h: (X,\tau,G) \to (Z,\eta)$ is faintly G-γ-continuous function.

Proof: Let W be a θ-open set of (Z,η). Then $g^{-1}(W)$ is θ-open in (Y,σ) and hence $(g \circ h)^{-1}(W)=h^{-1}(g^{-1}(W))$ is G-γ-open in (X,τ,G). This shows that g o h is faintly G-γ-continuous function. If (Y,σ) is a regular space, we have $\sigma=\sigma_\theta$.

Theorem 3.15

If a function h: $(Y,\tau,G) \to (Z,\sigma)$ is faintly G-γ-continuous and (Z,σ) is a regular space, then h is G-γ-continuous.

Proof: Let H be an open set of Z. Since Z is regular, H is θ-open in Z and h is faintly G-γ-continuous, then by Theorem 3.2, we have h^{-1} (H) is G-γ-open and hence h is G-γ-continuous.

Definition 3.16: A space (Y,τ,G) is named G-γ-connected if Y cannot be written as a union of two nonempty disjoint G-γ-open sets.

Theorem 3.17

Let h: $(Y,\tau,G) \to (Z,\sigma)$ be a faintly G-γ-continuous surjection and (Y,τ,G) is a G-γ-connected space, then Z is a connected space.

Proof: Suppose that (Z,σ) is not connected. Then there exist two nonempty open sets H_1 and H_2 of (Z,σ) such that $H_1 \cap H_2=\phi$ and $H_1 \cup H_2=Z$. Then, we have $h^{-1}(H_1) \cap h^{-1}(H_2)=\phi$ and $h^{-1}(H_1) \cup h^{-1}(H_2)=Y$. Since h is surjection, $h^{-1}(H_1)$ and $h^{-1}(H_2)$ are non empty subsets of Y. Since H_i is clopen, H_i is θ-open ∀i=1; 2. Because of h is faintly G-γ-continuous, $h^{-1}(H_i) \in G_\gamma O(Y)$. So, (Y,τ,G) is not G-γ-connected. This is a confict and (Z,σ) is connected.

Definition 3.18: A collection $\{G_\lambda : \lambda \in \nabla\}$ is called a G-γ-open cover of a subset A of a space Y which carries topology τ with grill G if $A \subset \{G_\lambda : Y\backslash G\lambda \in G_\gamma O(y), \lambda \in \nabla$:

Definition 3.19: Let Y be a space which carries topology τ with grill G, then it is said to be:

(1) G-γ-T_1 (resp. θ-T_1 [11]) if for each pair of different points y and z of Y, there exists G-γ-open (resp. θ-open) sets A and B containing y and z, respectively such that $z \notin A$ and $y \notin B$.

(2) G-γ-T_2 (resp. θ-T_2 [11]) if for each pair of different points y and z of Y, there exists disjoint G-γ-open (resp. θ-open) sets A and B in Y such that $y \in A$ and $z \in B$.

Theorem 3.20

Let h: $(Y,\tau,G) \to (Z,\sigma)$ be faintly G-γ-continuous injection and Z be a θ-T_1-space, then Y is a G-γ-T_1-space.

Proof: Let Z be a θ-T_1-space. For any two different points y and z in Y, then there exist $A;W \in \sigma_\theta$ such that $h(y) \in A$, $h(z) \notin A$, $h(y) \notin W$ and $h(z) \in W$ for Z is a θ-T_1-space. Because of h is faintly G-γ-continuous, h^{-1} (A) and h^{-1} (W) are G-γ-open subsets of (Y,τ,G) such that h^{-1} (A) and h^{-1}(W) containing y and z, respectively $z \notin h^{-1}(A)$ and $y \notin h^{-1}(W)$. Then Y is G-γ-T_1.

Theorem 3.21

Let h: $(Y,\tau,G) \to (Z;\sigma)$ be a faintly G-γ-continuous-injection function and Z be a θ-T_2-space, then Y is a G-γ-T_2-space.

Proof: For any two several points y and z of Y and since Z is a θ-T_2-space, then there exists $h(y) \in A$ and $h(z) \in B$ for disjoint θ-open sets A and B in Z. h^{-1} (A) and h^{-1} (B) are G-γ-open subset of (Y,τ,G) such that h^{-1} (A) and h^{-1} (B) containing y and z, respectively by h is faintly G-γ-continuous injection. Therefore, $h^{-1}(A) \cap h^{-1}(B)=\phi$ for $A \cap B=\phi$. Then Y is G-γ-T_2.

Let h: $(Y,\tau,G) \to (Z;\sigma)$ be a function. A function g: $Y \to Y \times Z$, defined by g(y)=(y; h(y)) for every $y \in Y$, is called the graph function of h.

Theorem 3.22

A function h: $(Y,\tau,G) \to (Z,\sigma)$ is faintly G-γ-continuous if the graph function g: $Y \to Y \times Z$ is faintly G-γ-continuous.

Proof: Let $y \in Y$ and H be a θ-open set of Z containing h(x). Then $Y\times H$ is θ-open in $Y\times Z$ [6, Theorem 5] and contains g(y)=(y, h(y)). Therefore, there exists $A \in G_\gamma O(Y, y)$ such that $g(A) \subseteq Y\times H$. This implies that $h(A) \subseteq H$. Thus, h is faintly G-γ-continuous.

Definition 3.23: A graph g(h) of a function h: $(Y,\tau,G) \to (Z;\sigma)$ called θ-G-closed if for every $(y;z) \in (Y\times Z)\backslash g(h)$, there exist $A \in G_\gamma O(Y, y)$ and $B \in \sigma_\theta$ containing z such that $(A \times B) \cap g(h)=\phi$.

Lemma 3.24: A graph g(h) of a function h: $(Y,\tau,G) \to (Z,\sigma)$ is said to be θ-G-closed in Y×Z if and only if for each $(y;z) \in (Y \times Z)\backslash g(h)$, there exist $A \in G_\gamma O(Y, y)$ and $B \in \sigma_\theta$ containing z such that $h(A) \cap B=\phi$.

Proof: It is clear from Definition 3.24.

Theorem 3.25

If h: $(Y,\tau,G) \to (Z,\sigma)$ is faintly G-γ-continuous function and (Z; σ) is θ-T_2, then g(h) is θ-G-closed.

Proof: Since $(y;z) \in (Y \times Z) \backslash g(h)$, then $h(y) \neq z$. Because of Z is θ-T_2, there exist θ-open sets A and B in Z such that $h(y) \in A$; $z \in B$ and $B \cap A=\phi$. Since h is faintly G-γ-continuous, $h^{-1}(A) \in G_\gamma O(Y, y)$. Let $U=h^{-1}(A)$, we have $h(U) \subseteq A$. Therefore, we obtain $h(A) \cap B=\phi$. This shows that g(h) is θ-G-closed.

Theorem 3.26

If h: $(Y,\tau,G) \to (Z,\sigma)$ is faintly G-γ-continuous function and (Z,σ) is regular space, then the following statements are satisfied:

(1) Faintly G-γ-continuous function coincide with G-γ-continuous function,

(2) Faintly G-α-continuous function coincide with G-α-continuous function,

(3) Faintly G-semi-continuous function coincide with G-semi-continuous function.

Proof: (1) Let B be any open set in Z. Since Z is regular, B is θ-open in Z. Because of h is faintly G-γ-continuous function, by Theorem 3.2, we have h^{-1} (B) is G-open and h is G-continuous.

Proving (2, 3) are Similar to (1).

Conclusion

The study of faintly grill topological spaces is very important. It is generalization of faintly topological spaces. So we introduced neoteric classes of functions called faintly G-γ-continuous, faint G-α-continuous and faint G-semi-continuous in grill topological spaces that helps us in many applications such as computer and information systems. Furthermore relationships between different classes are introduced. Also, some of their basic properties of different types of functions between grill topological spaces are obtained.

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