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A General Fluctuation Model for Nonlinear Dynamics, Bifurcation, Fractals, and Control Mechanisms

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Short Communication

Extracting information from data occupies a considerable proportion of studies. Many newly developed statistical methods and algorithms have been adapted to mine the hidden knowledge from datasets in many fields. A significant common limitation of these methods and algorithms is that they provide no interpretation of the results. This is because most of the models and parameters have no physical explicit meanings, but the most valuable information results from an interpretable structure.

Therefore, a general model that can both extract valuable information out of a wide variety of data and mimic physical structure is desirable to theoretical research and applications. Here we present such a general model: the nonlinear autoregressive integrated (NLARI) process [1]. Recently, there has been a growing interest in using the NLARI process across a wide range of fields for extracting valuable information of nonlinear dynamics, stability, bifurcations, fractals, and their generative and control mechanisms [2,3].

For $\kappa_1 = 1$ the NLARI process can be specified by,

$$X_t = \omega + (1+\theta_1) X_{t-1} - \theta_1 X_{t-2} + \theta_2 \frac{-\left(X_{t-\kappa_2} - \mu_{t-\kappa_2}\right)}{\exp\left(\left(X_{t-\kappa_2} - \mu_{t-\kappa_2}\right)^2\right)} + \varepsilon_t \quad (1)$$

where X_{t-j} lags X_t by j steps at time t for j=1,……., k_2 and t=1,…..n, k_1 and k_2 are time delays in resistance and restoration, $\mu_t=E(X_t|X_0,X_{-1},\ldots,X_{1-k2})$ is the mean of a set of data values for given initial values, $\mu_t=X_0+(\omega/\alpha)t$ if $\epsilon_t=\epsilon_t-E(\epsilon_t)$ is Gaussian noise, ϵ_t expresses an unpredictable disturbance or noise with mean $\omega=E(\epsilon_t)$ and variance $\sigma^2=var(\epsilon_t)$ as an external force, θ_1=1-α, θ_2=β, α is the resistance coefficient, and β is the restoration coefficient [1,4].

The NLARI process as a data generative process provides interpretable results on the physical mechanisms of the system. The model parameters have the explicit physical meanings, which is attributed to the fact that the NLARI process was derived by applying Newton's second law to stochastic (random) restoring systems [1]. The stability coefficient γ=β/(4-2α) determines the nonlinear dynamics, stability, and bifurcations of the NLARI deterministic system where γ>0 represents negative feedback as a restoring force to maintain equilibrium, γ<0 represents positive feedback as an anti-restoring force, but γ=0 means that the NLARI process degenerates into a nonstationary linear unit root process (random walk is the simplest unit root process, Brownian motion is its discrete-time version). The NLARI deterministic system for k_1=1 and k_2=1 exhibits an evolutionary route from a stable fixed point for γ ∈ (0, 1) and a stable limit cycle for γ ∈ (1, √e) through aperiodic oscillations for γ ∈ (√e, 3.07) to chaos for γ>3.07 [2,4]. The NLARI process exhibits typical fractals (long memory, self-similarity, a power law). The fractal level of NLARI is controlled by the wave indicators η_1=ω/α and η_2=σ/β: a larger slope indicator η_1 discloses a higher dependence, while a smaller amplitude indicator η_2 discloses a higher self-similarity. Both fractals and nonlinear dynamics are a property of the data generative process and their relationships have been clarified in previous report [2].

The integer-dimension fractals of NLARI suggest that fractal dimension may be a redundant concept. The NLARI process can simultaneously exhibit fractals and nonlinear dynamics controlled by the explicit fractal and bifurcation indicators of the data generative process without additional fractal and dynamic parameters [2,4]. Moreover, integer-dimension fractals avoid potential problems. Fractal dimension does not uniquely describe nor provide enough information to reconstruct it, although a continuous spectrum of exponents multifractal scaling was introduced in different ways to make up for these deficiencies [5,6]. Non-integer fractal dimensions lack a physical basis; thus it is less likely that integer dimension is the special case of non-integer dimensions embedded in real world. If not, all the integer-order differential equations that play a prominent role in theories of physics, chemistry, biology, astrology, geography, engineering, and economics should be fractalized [1].

The NLARI process is a general fluctuation model applicable for a wide range of fields. The NLARI process describes the nonlinear stochastic restoring systems for σ>0, 0<α<2, and β> 0, but also the special systems of a deterministic (σ ↓ 0), two unit roots (α ↓ 0), and linear (β ↓ 0) without the restoring force. The resistance force is dependent on the speed of an object and is typically expressed by a nonlinear function of the speed. The nonlinear resistance function can be approximated by a linear function when it moves with a slow speed or change. The NLARI process was derived by linearizing the nonlinear resistance function under the assumption of a slow speed [1]. Hence, the NLARI process is a model applicable for slow systems if real systems are divided into fast and slow systems by a fast and slow speed (or change) of the system. Fractals contain geometric and stochastic fractals involved in the 'static and dynamic' and 'deterministic and stochastic' concepts. A geometric shape in the real world may be formed quickly after a naturally occurring event, whereas most of them are the real-time trajectories plotted by a slow motion as the results of spatiotemporal evolution such as coastlines. That is, a static geometrical structure in the real world can be roughly regarded as the result of a dynamic motion in many slow systems. Thus, the NLARI process is a general fluctuation model for the slow systems, regardless of whether the system is deterministic or stochastic, static or dynamic, linear or nonlinear, null or non-null restoring or anti-restoring force. We have reason to expect the NLARI process will be used in a wide range of natural, engineering, and social fields.

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The NLARI model can be applied for various purposes, for example:

- Detect/Recognize/Identify/Assess the internal structure (α, β), external drives (ω, σ), and control mechanisms (γ, η_1, η_2) with observation data.
- Detect/Recognize/Identify/Assess the internal structure (α, β, γ) with external drives.
- Encode/Propagate/Predict/Simulate information with the NLARI in the stable fixed-point range as a transfer function.

The analysis of NLARI can be completed by the following procedures: For the parameters and control indicators:

Step 1: Estimate the regression line X_t=a+b_t+v_t by ordinary least squares (OLS) estimates with the reduced data (e.g., X_t=log(raw data) or X_t=raw data/1000) to obtain ˆa and ˆb. Denote $Y_t = X_t - \hat{a} - \hat{b}t$

Step 2: Let $\Delta Y_t = Y_t - Y_{t-1}$. Using OLS method, estimates

$$\Delta Y_t = \theta_1 \Delta Y_{t-1} + \theta_2 \frac{-Y_{t-1}}{\exp\left(Y_{t\ -1}^2\right)} + \varepsilon_t \tag{2}$$

To obtain the parameter $\hat{\theta} = \left(\hat{\theta}_1, \hat{\theta}_2\right) = \left(Y'Y\right)^{-1} Y'y$

where $\Delta Y_t = Y_t - Y_{t-1}, Y' = \left(Y'_{1,0} \ldots Y'_{1,n-1}\right), y' = \left(\Delta Y_1 \ldots \Delta Y_n\right)$

$$Y_{1,t-1.} = \left(\Delta Y_{t-1} - Y_{t-1} \exp\left(-Y_{t-1}^2\right)\right), \Delta Y_t = Y_t - Y_{t-1},$$

$$\hat{\omega} = \hat{b}\left(1 - \hat{\theta}_1\right), \hat{\alpha} = 1 - \hat{\theta}_1, \hat{\beta} = \hat{\theta}_2, \hat{\gamma} = \frac{1}{2} \frac{\hat{\theta}_2}{1 + \theta_1}, \hat{\eta}_1 = \hat{b}, \hat{\eta}_2 = \frac{\hat{\sigma}}{\hat{\theta}_2} \tag{3}$$

$$\hat{\sigma} = \sqrt{\frac{}{} \left(y - Y\hat{\theta}\right)' \left(y - Y\hat{\theta}\right)}$$

For long memory, self-similarity, and a power law:

Step 1: Compute $r_i = r_i(X) = \sum_{t=1}^{n-i} \left(X_t - \bar{X}\right)\left(X_{t+i} - \bar{X}\right)$ and $\bar{X} = (1/n) \sum_{t=1}^{n} X_t$ and obtain the autocorrelation function $\rho_i = r_i / r_0$ for i=0,….., h; and the sd_m similarity ratio $sd_{(j,jm)} = \sqrt{r_0(j, jm)}$ and the for j=1, · · · , l and the average similarity ratio $sd_m = (1/n) = \sum_{i=1}^{n} sd_{(j,jm)}$ for j=1,….,l and m=2…..,s.

Step 2: If ρ_i exhibits a slow decay with increasing lag i, the data support long memory.

Step 3: If both $sd_{(j,jm)}$ against size j tends to a horizontal line and the average similarity ratio sd_m obeys a power law $m^{-\delta}$, the data support self-similarity (for testing the NLARI process). For more details refer to He [7].

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