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# A Curious Connection Between Fermat's Number and Multiple Factoriangular Numbers

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## Abstract

In the seventeenth century Fermat defined a sequence of numbers Fn=22n +1 for n ≥ 0 known as Fermat's number . If Fn happens to be prime then Fn is called Fermat prime. All the Fermat's number are of the form n!k+ Σnk for some fixed value of k and n. Further we will prove that after F4 no other Fermat prime exist upto 1050 .

**Keywords:** Fermat's Number • Prime Number • Multiple Factoriangular Numbers • Fermat Prime

## Introduction

**Fermat Number:** A positive number of the form $F_n = 2^{2n} + 1$ where n is non-negative integer.

First few Fermat's number are3, 5, 17, 257, 65537…

Pierre de Fermat conjectured that all numbers

(1.1) $F_n = 2^{2n} + 1$ for m = 0, 1, 2, . . .

Are prime. Nowadays we know that the first five members of this sequence are prime and that (see [2])

(1.2) $F_n$ is composite for 5 ≤ m ≤ 32.

The status of $F_{33}$ is for the time being unknown, i.e., we do not know yet whether it is prime or composite [1,2].

The numbers $F_n$ are called Fermat numbers. If $F_n$ is prime, we say that it is a Fermat prime.

Fermat numbers were most likely a mathematical interest before 1796. When C. F. Gauss mentioned that there is a remarkable relation between the Euclidean construction (i.e., by ruler and compass) of regular polygons and the Fermat numbers, interest in the Fermat primes skyrocketed. In particular, he proved that if the number of sides of a regular polygonal shape is of the form $2^k F_{m1} \cdot F_{mr}$, where k ≥ 0,r ≥ 0, where $F_{mi}$ are distinct Fermat primes, then this polygonal shape can be made by using compass ruler . The converse statement was proved later by Wantzel in [3,4].

There exist many necessary and sufficient conditions concerning the primality of

$F_n$. For instance, the number $F_n (n > 0)$ is a prime if and only if it can be written

as a sum of two squares in essentially only one way, namely $F_n = (2^{2n-1})^2 + 1^2$.

Recall also further necessary and sufficient conditions: the well-known Pepin's test, Wilson's Theorem, Lucas's Theorem for primality, etc., see [4].

**Multiple Factoriangular number** [5] : A generalization of Factoriangular number is known as multiple Factoriangular numbers and are defined as

$F_t(n,k) = (n!)^k + \Sigma n^k$

Where $T_n(k) = \Sigma n^k = 1^k + 2^k \dots + n^k$.

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**Table 1.** A connection between multiple Factoriangular numbers and Fermat number.

| N | $F_t(2,2^n-1)$ | Prime factorization of $F_t(n,15)$ | Number of digits | Sum of squares of prime, integer, natural numbers |
|---|---|---|---|---|
| 0 | 3 | Prime | 1 | |
| 1 | 5 | Prime | 1 | $2^2 + 1^2$ |
| 2 | 17 | Prime | 2 | $4^2 + 1^2$ |
| 3 | 257 | Prime | 3 | $16^2 + 1^2$ |
| 4 | 65537 | Prime | 5 | $256^2 + 1^2$ |
| 5 | 4294 967297 | 641 × 6 700417 | 10 | $65536^2 + 1^2$ |
| 6 | 18 446744 073709 551617 | 274177 × 67 280421 310721 | 20 | $4046\ 803256^2 + 1438\ 793759^2$ |
| 7 | 340 282366 920938 463463 374607 431768 211457 | 59649 589127 497217 × 5704 689200 685129 054721 | 39 | $18\ 446744\ 073709\ 551616^2 + 1^2$ |
| 8 | 115792 089237 316195 423570 985008 687907 853269 984665 640564 039457 584007 913129 639937 | 238 926361 552897 × 93 461639 715357 977769 163558 199606 896584 051237 541638 188580 280321 | 78 | $339\ 840244\ 399005\ 511779\ 394711\ 120340\ 266111^2 + 17\ 340632\ 172455\ 487023\ 654788\ 790090\ 010704^2$ |

In this paper, we establish a connection between multiple Factoriangular numbers and Fermat number Table 1.

By the common observation we see that the sequence of number so formed is well known Fermat Number Sequence and it follow the properties described in [2,4].

Now

$F_t(2, 2^n-1) = (2!)^{2n-1} + \Sigma\, 2^{2n-1} = 2^{2n} + 1$.

**Corollary:** All the Fermat prime are multiple Factoriangular primes.

## Conclusion

We end up with the conclusion that the only primes we get in different sequences of multiple Factoriangular numbers till $10^{50}$ are the Fermat Prime $F_0$, $F_1$, $F_2$, $F_3$, $F_4$. Also Sequence of Fermat Number are a special case of multiple Factoriangular number by fixing n=2, k= $2^n$-1.

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