Review Article
Volume 11:4, 2020

Journal of
Physical Mathematics

ISSN: 2090-0902
Open Access

A Comparative Study of Some Stochastic Models in the Context of COVID-19 Pandemic

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Abstract

In this article it is tried to work out on the mathematics of stochastic version of Von-Bertalanffy power law model to find an explicit solution and the MLEs of the model parameters are also worked out. Then this model is applied to the COVID infection data (First wave data) of South Korea after observing the nature of growth and some comparisons are made with stochastic Gompertz model and stochastic Logistic model in this context and the stochastic Von-Bertalanffy power law model performs better than the other two at least here in this case.

Keywords: Von-Bertalanffy model • Gompertz model • Logistic model • White noise

Introduction

There are some variants of Corona virus that exists and scientists classified them intofour subgroups-1) 229E (alpha), 2) NL63 (alpha), 3) OC43 (beta), 4) HKU1 (beta) and thereare three rare ones also and they are-1) MERS-CoV, 2) SARS-CoV, 3) SARS-CoV-2 [1]. The third one under the rare category which is also known as COVID-19. It is mainly responsible for the pandemic situation that occurred all around the world. It is actually an infectious disease, majorly causing respiratory problems. It is observed that most of the infected persons by this virus has experienced a mild to moderate respiratory problemsand many of them has recovered under normal treatments, but for aged persons who were suffering from CVD (cardio vascular disease), severe respiratory problems, diabetes, cancer etc. i.e., persons with co-morbidity have faced serious trouble after getting infected (COVID positive) [2]. Since this is an infectious disease, it is natural to be interested about howit spreads. This virus (COVID-19) actually spreads itself *via* droplets of saliva or when a person infected by this virus sneezes or coughs, there is every possibility that surrounding persons may become infected by it [3]. From previous paragraph one can classifythis virus as a respiratory virus and it is important to have an idea about the possibleways of transmission of this respiratory virus. Sources indicate that there are mainly three ways of transmission of such respiratory virus, firstly it could be transmitted *via* direct contact with infected person, secondly through droplet transmission and thirdly, through airborne transmission of smaller droplets and particles that stays in air for fair amount of time and can travel a significant amount of distance [3], also some Italian scientists collected outdoor air pollution samples and after studying it, they found gene highly specific to COVID-19 in multiple samples [3] i.e., one may think it as airborne, though there is not much evidence of this fact. But one can consider it as partially air bone because of the fact that air current can help it to travel from one place to another and because of this reason, it is quite possible that some of the countries like South Korea, Sweden etc, haven't imposed any strict lockdown or curfew. So far from the experienceof patients who had an infection of this COVID-19 virus, it is described that there are some common symptoms observed when a person, possibly infected by this virus can be identified and the symptoms are fever(high temperature), cough, breathing problem, headache, sore throat, loss of smell or taste [1,3]. Observing the situation all aroundthe world, one can understand the severity of this situation, not because of the fact thattill now there is no such vaccine or specific treatments for the patients suffering from this disease. There are some clinical trials are going on to evaluate specific vaccine or treatmentof this disease [3]. This pandemic situation has a large adverse effect all around the world and the most one can observe it in world economy as well as in social life. Accordingto IMF report [4] the growth of world economy is under the worst recession since great depression in 1920s. It is not only world economy but it has huge impact on social life especially psychological side of the society. There are many research articles already available that focuses on this issue. According to the research this pandemic situation has significantimpact on psychological well-being of most of the exposed groups like children, college students, health workers etc, who are more likely to have post-traumatic stress, anxiety disorder, depression etc, it is also observed that the social distance and the precautionarymeasures has its impact on the relationship among people and their response towards others [5].

Like most of the biological growth phenomena, here also the growth dynamics of number of infected patients also shows sigmoidal pattern. Now as it is known that atpresent there is no such specific vaccine or treatment for this disease so, to prevent the infection majority of the countries has taken few precautionary steps like social distancing, home quarantine, curfew etc. On the other hand, few countries had their belief in herd immunity theory which states that it is a form of indirect protection from infection which occurs when a big part of the population has become immune to the infection [6] and the data set that is applied here belongs to the country which didn't imposed any lockdown or curfew as a precautionary measure to control the growth. Now as it is already mentionedthat the growth dynamics here shows sigmoidal pattern like most of the biological growth process shows, so it can be described by a model expressed by the following differential equation,

$$dX(t) = [a(X(t))^{\lambda} - bX(t)]dt \tag{1}$$

This model is known as Von-Bertalanffy power law model [7,8].

Where a and b are known as anabolic rate coefficient and catabolic rate coefficients. Buthere in this case we will treat a and b as growth rate factor and growth resistant factor. Here the growth resistant factor acts either in the form of herd immunity or someinbuilt immunal force supported by the nature.

Stochastic Version of Von-Bertalanffy Power Lawmodel

This model is mostly used in describing growth phenomena in the field of Biology but hereit we will try to construct it's stochastic version and fit it to the dataset (First wave) of COVID infections of South Korea. In

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Received 13 May 2021; **Accepted** 27 May 2021; **Published** 03 June 2021

the field of Biology or reliability engineering, environment or some other factor may have some possible impact over the system and which could possibly the reason behind oscillatory movements that appears in intermediate phase ofthe process. So, keeping this in mind and to describe it, we introduce a multiplicative noise term like many authors [9-11] did to form models. So, introducing multiplicative noise term the model of equation 1 leads to its stochastic version as,

$$dX(t) = [a(X(t))^{\lambda} - bX(t)]dt + \lambda X(t)dw(t) \quad (2)$$

Where a and b are known as growth rate factor and growth resistant factor and λ is the diffusion coefficient. w (t) is the standard wieners process and the differential is to be understood in Ito's sense [12].

It is clear from the above equation that it is a stochastic differential equation in Ito's form, so, it will be more suitable to solve it using Ito's lemma [10,13-16].

Let us apply the following transformation,

$$F(x,t) = x^{1-\lambda}$$

then,

$$F^{J}(x, t) = (1-\lambda)^{x-\lambda}$$

and

$$F^{JJ}(x, t) = -\lambda(1-\lambda)x^{-\lambda-1}$$

Applying Ito's lemma we get,

$$f = (1-\lambda)(a - bx^{1-\lambda}) - \frac{\sigma^2\lambda(1\ \lambda)}{2}X^{1-\lambda}$$

i.e,

$$f = a(1-\lambda) - (b(1-\lambda) + \frac{\sigma^2\lambda(1-\lambda)}{2}X^{1-\lambda}$$

And

$$g = \sigma(1-\lambda)x^{1-\lambda}$$

Therefore, the stochastic differential equation reduces to the following,

$$dY(t) = [(b(\lambda-1) - \frac{\sigma^2\lambda(1-\lambda)}{2}Y(t) + a(\lambda-1)]dt - \sigma(\lambda-1)Y(t)dw(t)$$

Which is a stochastic differential equation in standard form and standard methods areavailable to solve it. So, using standard method the solution of y(t) becomes,

$$Y(t) = exp([\frac{b(\lambda-1)}{2} - \frac{\sigma^2\lambda(1-\lambda)}{2}]t - \sigma(\lambda-1)w(t))[y(0) + a(\lambda-1)$$

$$\int_0^t exp([\frac{\sigma^2\lambda(1-\lambda)}{2} - \frac{b(\lambda-1)}{2}]s + \sigma(\lambda -$$

Transforming to the original variable we get,

$$X(t) = [exp([\frac{b(\lambda-1)}{2} - \frac{\sigma^2\lambda(1-\lambda)}{2}]t - \sigma(\lambda-1)w(t))[(x(0))^{1-\lambda} + a(\lambda-1)$$

$$\int_0^t exp([\frac{\sigma^2\lambda(1-\lambda)}{2} - \frac{b(\lambda-1)}{2}]s$$

This gives the analytical expression of X(t).

MLEs of the Growth Rate Factor and Growth Resistant Factor

Now to obtain the MLE of a and b on the basis of continuous records available one may assume that the diffusion coefficient terms are known. So, this part does not involve any unknown parameters. Using Girsanov theorem [17] the infill log-likelihood becomes,

$$l(a,b,x) = A - \frac{1}{2}B$$

Where,

$$A = \int_T^0 \frac{a(x(t))^{\lambda} - bx(t)}{\sigma^2(x(t))^2}dx(t)$$

$$B = \int_T^0 \frac{(a(x(t))^{\lambda} - bx(t))^2}{\sigma^2(x(t))^2}dx(t)$$

This gives the estimates of a and b as,

$$\hat{a} = \frac{\frac{lnx(T)I_1}{T} - \frac{-(x(T))^{\lambda-1}}{\lambda-1}}{I^2{}_T 1-1}$$

and

$$\hat{b} = [\frac{\frac{lnx(T)}{T}I_1 - \frac{(x(T))^{\lambda-1}}{1-\lambda}}{I_2\frac{1}{T}-1}]\frac{I_2}{T} - \frac{lnx(T)}{T}$$

Where,

$$I_1 = \int_0^T (x(t))^{\lambda+1}dt$$

And

$$I_2 = \int_0^T (x(t))^{\lambda+1}dt$$

Now, before proceed to the final section, one thing must be mentioned here is that the performance of this stochastic Von-Bertalanffy power model is to be compared with the performance of stochastic Gompertz model [15,18] and stochastic Logistic model with respect to the results of fit of this model to the mentioned data set.

Analysis of Real-Life Data

Here, a real-life data set of nos. of infected persons in South Korea by COVID-19 at different time points are taken (data source: world meter). In this data set no. of infected persons at every 7th day are given. Data points are available from Feb 15, 2020 to Apr25, 2020. It is known from different sources that South Korea didn't impose any curfew or lockdown situation. Here, we tried to fit the mentioned models to the data set and their respective results are given as, The results of fit of stochastic Logistic model [15,18].

$\hat{a} \approx 1,$

$std.error \approx 0.573,$

$\hat{b} \approx -3.3044,$

$std.error \approx 0.00004,$

$AIC \approx 280.5285$

When this data set is fitted to stochastic Gompertz model [2,18,19], then we have,

$\hat{a} \approx 2.175,$

$std.error \approx 1.37,$

$\hat{b} \approx -0.0097,$

$std.error \approx 0.0034,$

$AIC \approx 238.6263$

Table 1. AIC model.

Model	est(a)	est(b)	AIC
Stochastic Logistic model	1	-3.3044	280.5285
Stochastic Gompertz model	2.175	-0.0097	238.6263
Stochastic Von-Bertalanffy model	11.473	-0.027	212.45

When this data set is fitted to stochastic Von-Bertalanffy power law model, then we have,

$\hat{a} \approx 11.473,$

$std.error \approx 5.47,$

$\hat{b} \approx -0.027,$

$std.error \approx 0.0155,$

$AIC \approx 212.45$

As it is known that model with smaller AIC would be considered as best fitted model to the data, so, looking at the AIC's of different models, one can say that the stochastic Von-Bertalanffy power law model fits better than the other two models that are used todescribe sigmoidal growth in many situations (Table 1).

Conclusion

As it appears here that the stochastic Von-Bertalanffy power law model performs better than the stochastic versions of the Gompertz model and the Logistic model, one can perform this study to different areas of biological growth or reliability engineering field or financial data modeling or in some other cases and observe the results for different values of λ. It may happen that for some values or a small interval of values of λ this stochastic Von-Bertalanffy power law model describes the particular data set better than the other two and except those values of λ the results may be reversed. Also, one may be interested to see what happens if the noise term is introduced with multiplicative power involving λ, i.e., the point here we are trying make is that there are many possible scopes where some interesting findings may come out.

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How to cite this article: Anirban Goswami, Proloy Banerjee, Shreya Bhunia and Sudipta Basu. "A Comparative Study of Some Stochastic Models in the Context of COVID-19 Pandemic." *J Phys Math* 11 (2021): 335