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A Comparative Study of Centroid Ranking Method and Robust Ranking Technique in Fuzzy Assignment Problem

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Abstract

Allocation of subjects is an important task for every educational institution. Subject allocation is considered as a major factor to the teaching quality.

In this study, we used the Fuzzy allocation methods: centroid ranking method and Robust ranking method, for allocating the subjects in a department for the coming semester. We used the scores obtained by the faculty members, from the Chairman, Course Director and the students, based on the performance in the previous semester.

Keywords: Centroid ranking • Robust ranking • Magnitude ranking • Fuzzy assignment • Membership function

Introduction

Effective teaching and learning is critically important to all students and especially for those with special educational needs. In this, allocation of subjects plays a vital role.

Andrew and Collins [1] developed a procedure for assigning the subjects to the teachers. Hawood and Lawless [2] used a goal programming to solve the teacher assignment problem. Aldy Gunawan, K. M. Ng and H. L. Ong [3] used genetic algorithm for the teacher assignment problem.

Assignment problem with fuzzy parameters have been studied by several authors such as Balinski [4] and Chi-Jen-Lin [5] and Chen [6], Kuhn Liu and Gao [7], Sathi, Mukherjee and Kajla Basu [8].

Suppose there are 'n' people and 'n' jobs. Each job must be done by exactly one person; also each person can do, at most, one job. The problem is to assign jobs to the people so as to minimize the total cost of completing all the jobs. The general assignment problem can be mathematically stated as follows:

Minimize $Z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$

Subject to

$\sum_{i=1}^{n} x_{ij}$ x_{ij} = 1 *for* i=1,2, ...,*n* (*one job is doneby the* i^{th} *person, i* = 1,2,...,*n*) w = 1

$\sum_{i=1}^{n} x_{ij}$ =1 *for* j=1,2, ...,*n* (*only one person should be assigned the* j^{th} *job*, *j* = 1,2,...,*n*)

$$x_{ij} = \begin{cases} 1, \; if\ i^{th}\ person\ assigned\ j^{th}\ job \\ 0, \quad if\ \ not \end{cases}$$

Fuzzy assignment problem

Minimize $Z = \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{c}_{ij} x_{ij}$

$\sum_{j=1}^{n} x_{ij} = 1 \;\; for\ \ i = 1,2,\ldots\ldots n$ Subject to

$\sum_{j=1}^{n} x_{ij} = 1 \;\; for\ \ i = 1,2,\ldots\ldots n$

x_{ij} = 0 or 1

Triangular fuzzy number

A fuzzy number A is a triangular fuzzy number denoted by (a_1, a_2, a_3) and its membership function $\mu_A(x)$ is given below:

$$\mu_A(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1} & a_1 \le x \le a_2 \\ 1 & x = a_2 \\ \dfrac{a_3-x}{a_3-a_2} & a_2 \le x \le a_3 \\ 0 & otherwise \end{cases}$$

Centroid ranking method

The centroid of a triangle fuzzy number $\tilde{a} = (a,b,c;w)$ as $G_{\tilde{a}} = \left(\frac{a+b+c}{3}, \frac{w}{3}\right)$. The ranking function of the generalized fuzzy number $\tilde{a} = (a,b,c;w)$ which maps the set of all fuzzy numbers to a set of real numbers is defined as $R(\tilde{a}) = \left(\frac{a+b+c}{3}\right)\left(\frac{w}{3}\right)$.

Robust ranking technique

Robust ranking technique which satisfies compensation, linearity, additive properties and provides results which consist of human intuition [9-14]. If $\tilde{a}$ is a fuzzy number then the Robust ranking is defined by

$$R(\tilde{a}) = 0.5\int_0^1 (a_\alpha^L, a_\alpha^U)\, d\alpha$$

where (a_α^L, a_α^U) is the α- level cut of the fuzzy number $\tilde{a}$ and

$$(a_\alpha^L, a_\alpha^U) = \{((b-a)+a), (d-(d-c))\}$$

Magnitude ranking method

For an arbitrary triangular fuzzy number $\tilde{a} = (a_1, a_2, a_3)$ with parametric form $\tilde{a} = (a(r), \bar{a}(r))$, the magnitude of the triangular fuzzy number $\tilde{a}$ by

$$Mag(\tilde{a}) = \frac{1}{2}\int_0^1 (a_3 + 3a_1 - a_2) f(r)\, dr$$

Where the function *f(r)* is a non-negative and increasing function on $(0,1)$. In the real life applications *f(r)* can be chosen by the decision maker according to the situation.

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Methods

The Let there are four faculty members A, B, C, D and we have to assign four subjects I, II, III, IV to each of them. Each faculty member has obtained the scores by the Chairman, Course Director and the students of the department based on the performance in previous semester.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{bmatrix} I & II & III & IV \\ (1,\ 4,\ 7) & (3,\ 6,\ 9) & (7,\ 10,\ 13) & (2,\ 5,\ 8) \\ (8,\ 11,\ 14) & (5,\ 8,\ 11) & (4,\ 7,\ 10) & (6,\ 9,\ 12) \\ (9,\ 12,\ 15) & (0,\ 3,\ 6) & (1,\ 4,\ 7) & (4,\ 7,\ 10) \\ (7,\ 10,\ 13) & (8,\ 11,\ 14) & (2,\ 5,\ 8) & (3,\ 6,\ 9) \end{bmatrix}$$

By centroid ranking method

$R(\tilde{a})=\left(\frac{a+b+c}{3}\right)\left(\frac{w}{3}\right)$ Let $w=1$

$$R(1,4,7)=\left(\frac{1+4+7}{3}\right)\left(\frac{1}{3}\right)=\frac{12}{9}=1.33$$

$$R(3,6,9)=\left(\frac{3+6+9}{3}\right)\left(\frac{1}{3}\right)=\frac{18}{9}=2$$

$$R(2,5,8)=\left(\frac{2+5+8}{3}\right)\left(\frac{1}{3}\right)=\frac{15}{9}=1.66$$

$$R(8,11,14)=\left(\frac{8+11+14}{3}\right)\left(\frac{1}{3}\right)=\frac{33}{9}=3.66$$

$$R(5,8,11)=\left(\frac{5+8+11}{3}\right)\left(\frac{1}{3}\right)=\frac{24}{9}=2.66$$

$$R(4,7,10)=\left(\frac{4+7+10}{3}\right)\left(\frac{1}{3}\right)=\frac{21}{9}=2.33$$

$$R(6,9,12)=\left(\frac{6+9+12}{3}\right)\left(\frac{1}{3}\right)=\frac{27}{9}=3$$

$$R(9,12,15)=\left(\frac{9+12+15}{3}\right)\left(\frac{1}{3}\right)=\frac{36}{9}=4$$

$$R(0,3,6)=\left(\frac{0+3+6}{3}\right)\left(\frac{1}{3}\right)=\frac{9}{9}=1$$

$$R(1,4,7)=\left(\frac{1+4+7}{3}\right)\left(\frac{1}{3}\right)=\frac{12}{9}=1.33$$

$$R(4,7,10)=\left(\frac{4+7+10}{3}\right)\left(\frac{1}{3}\right)=\frac{21}{9}=2.33$$

$$R(7,10,13)=\left(\frac{7+10+13}{3}\right)\left(\frac{1}{3}\right)=\frac{30}{9}=3.33$$

$$R(8,11,14)=\left(\frac{8+11+14}{3}\right)\left(\frac{1}{3}\right)=\frac{33}{9}=3.66$$

$$R(2,5,8)=\left(\frac{2+5+8}{3}\right)\left(\frac{1}{3}\right)=\frac{15}{9}=1.66$$

$$R(3,6,9)=\left(\frac{3+6+9}{3}\right)\left(\frac{1}{3}\right)=\frac{18}{9}=2$$

After putting these values, we get the assignment problem

$$\begin{array}{cc} & \begin{array}{cccc} I & II & III & IV \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 1.33 & 2 & 3.33 & 1.66 \\ 3.66 & 2.66 & 2.33 & 3 \\ 4 & 1 & 1.33 & 2.33 \\ 3.33 & 3.66 & 1.66 & 2 \end{bmatrix} \end{array}$$

By Hungarian method, the optimal assignment is as follows:

$A \rightarrow III$

$B \rightarrow IV$

$C \rightarrow I$

$D \rightarrow II$

The optimum value is 13.99

By Robust ranking method

R(1,4,7)

$A \rightarrow III$

$B \rightarrow IV$

$C \rightarrow I$

$D \rightarrow II$

The membership function of the triangular fuzzy number (1,4,7) is

$$\mu(x)=\begin{cases} \frac{x-1}{3} & 1\le x\le 7 \\ 1, & x=4 \\ \frac{7-x}{3} & 4\le x\le 7 \\ 0, & otherwise. \end{cases}$$

The α — cut of the fuzzy number (1,4,7) is $\left(a_\alpha^L,a_\alpha^U\right)=\left(3\alpha+1,7-3\alpha\right)$.

$$R(1,4,7)=0.5\int_0^1(a_\alpha^L,a_\alpha^U)d\alpha=0.5\int_0^1(3\alpha+1+7-3\alpha)d\alpha=0.5\int_0^1 8\,d\alpha=4$$

R(3,6,9)

The membership function of the triangular fuzzy number (3,6,9) is

The α — cut of the fuzzy number (3,6,9) is

The α — cut of the fuzzy number (3,6,9) is $(a_\alpha^L,a_\alpha^U)=(3\alpha+3,9-3\alpha)$

$$\mu(x)=\begin{cases} \frac{x-3}{3}, & 3\le x\le 6 \\ 1, & x=6 \\ \frac{9-x}{3}, & 6\le x\le 9 \\ 0, & otherwise. \end{cases}$$

$$R(3,6,9)=0.5\int_0^1(a_\alpha^L,a_\alpha^U)d\alpha=0.5\int_0^1(3\alpha+3,9-3\alpha)d\alpha=0.5\int_0^1 12\,d\alpha=6$$

R(7,10,13)

The membership function of the triangular fuzzy number (7,10,13) is

$$\mu(x)=\begin{cases} \frac{x-7}{3}, & 7\le x\le 10 \\ 1, & x=10 \\ \frac{13-x}{3}, & 10\le x\le 13 \\ 0, & otherwise. \end{cases}$$

The α — cut of the fuzzy number (7,10,13) is $\left(a_\alpha^L,a_\alpha^U\right)=\left(3\alpha+7,13-3\alpha\right)$.

$$R(7,10,13)=0.5\int_0^1(a_\alpha^L,a_\alpha^U)d\alpha=0.5\int_0^1(3\alpha+7,13-3\alpha)d\alpha=0.5\int_0^1 20\,d\alpha=10$$

R(2,5,8)

The membership function of the triangular fuzzy number (2,5,8) is

$$\mu(x)=\begin{cases} \frac{x-2}{3}, & 2\le x\le 5 \\ 1, & x=5 \\ \frac{8-x}{3}, & 5\le x\le 8 \\ 0, & otherwise. \end{cases}$$

The α — cut of the fuzzy number (2,5,8) is $\left(a_\alpha^L,a_\alpha^U\right)=\left(3\alpha+8,14-3\alpha\right)$.

$$R(2,5,8)=0.5\int_0^1(a_\alpha^L,a_\alpha^U)d\alpha=0.5\int_0^1(3\alpha+2,8-3\alpha)d\alpha=0.5\int_0^1 10\,d\alpha=5$$

R(8,11,14)

The membership function of the triangular fuzzy number (8,11,14) is

$$\mu(x)=\begin{cases}\dfrac{x-8}{3}, & 8\le x\le 11\\ 1, & x=11\\ \dfrac{14-x}{3}, & 11\le x\le 14\\ 0, & otherwise.\end{cases}$$

The α — cut of the fuzzy number (8,11,14) is $\left(a_\alpha^L,a_\alpha^U\right)=\left(3\alpha+8,14-3\alpha\right)$.

$$R(8,11,14)=0.5\int_0^1(a_\alpha^L,a_\alpha^U)d\alpha=0.5\int_0^1(3\alpha+8,14-3\alpha)d\alpha=0.5\int_0^1 22\,d\alpha=11$$

R (5,8,11)

The membership function of the triangular fuzzy number (5,8,11) is

$$\mu(x)=\begin{cases}\dfrac{x-5}{3}, & 5\le x\le 8\\ 1, & x=8\\ \dfrac{11-x}{3}, & 8\le x\le 11\\ 0, & otherwise.\end{cases}$$

The α — cut of the fuzzy number (5,8,11) is $\left(a_\alpha^L,a_\alpha^U\right)=\left(3\alpha+5,11-3\alpha\right)$.

$$R(5,8,11)=0.5\int_0^1(a_\alpha^L,a_\alpha^U)d\alpha=0.5\int_0^1(3\alpha+5,11-3\alpha)d\alpha=0.5\int_0^1 16\,d\alpha=8$$

R (4,7,10)

The membership function of the triangular fuzzy number (4,7,10) is

$$\mu(x)=\begin{cases}\dfrac{x-4}{3}, & 4\le x\le 7\\ 1, & x=7\\ \dfrac{10-x}{3}, & 7\le x\le 10\\ 0, & otherwise.\end{cases}$$

The α — cut of the fuzzy number (4,7,10) is $\left(a_\alpha^L,a_\alpha^U\right)=\left(3\alpha+4,10-3\alpha\right)$.

$$R(4,7,10)=0.5\int_0^1(a_\alpha^L,a_\alpha^U)d\alpha=0.5\int_0^1(3\alpha+4,10-3\alpha)d\alpha=0.5\int_0^1 14\,d\alpha=7$$

R (6,9,12)

The membership function of the triangular fuzzy number (6,9,12) is

$$\mu(x)=\begin{cases}\dfrac{x-6}{3}, & 6\le x\le 9\\ 1, & x=9\\ \dfrac{12-x}{3}, & 9\le x\le 12\\ 0, & otherwise.\end{cases}$$

The α — cut of the fuzzy number (6,9,12) is $\left(a_\alpha^L,a_\alpha^U\right)=\left(3\alpha+6,12-3\alpha\right)$

$$R(6,9,12)=0.5\int_0^1(a_\alpha^L,a_\alpha^U)d\alpha=0.5\int_0^1(3\alpha+6,12-3\alpha)d\alpha=0.5\int_0^1 18\,d\alpha=9$$

R (9,12,15)

The membership function of the triangular fuzzy number (9,12,15) is

$$\mu(x)=\begin{cases}\dfrac{x-9}{3}, & 9\le x\le 12\\ 1, & x=12\\ \dfrac{15-x}{3}, & 12\le x\le 15\\ 0, & otherwise.\end{cases}$$

The α — cut of the fuzzy number (9,12,15) is $\left(a_\alpha^L,a_\alpha^U\right)=\left(3\alpha+9,15-3\alpha\right)$.

$$R(9,12,15)=0.5\int_0^1(a_\alpha^L,a_\alpha^U)d\alpha=0.5\int_0^1(3\alpha+9,15-3\alpha)d\alpha=0.5\int_0^1 24\,d\alpha=12$$

R (0,3,6,)

The membership function of the triangular fuzzy number (0,3,6,) is

$$\mu(x)=\begin{cases}\dfrac{x-0}{3}, & 0\le x\le 3\\ 1, & x=3\\ \dfrac{6-x}{3}, & 3\le x\le 6\\ 0, & otherwise.\end{cases}$$

The α — cut of the fuzzy number (0,3,6,) is $\left(a_\alpha^L,a_\alpha^U\right)=\left(3\alpha+0,6-3\alpha\right)$.

$$R(0,3,6)=0.5\int_0^1(a_\alpha^L,a_\alpha^U)d\alpha=0.5\int_0^1(3\alpha+0,6-3\alpha)d\alpha=0.5\int_0^1 6\,d\alpha=3$$

After putting these values, we get the assignment problem

$$\begin{array}{c c}
 & \begin{array}{cccc} I & II & III & IV \end{array}\\
\begin{array}{c} A\\ B\\ C\\ D \end{array} & \begin{bmatrix} 4 & 6 & 10 & 5\\ 11 & 8 & 7 & 9\\ 12 & 3 & 4 & 7\\ 10 & 11 & 5 & 3 \end{bmatrix}
\end{array}$$

By Hungarian method, the optimal assignment is as follows:

$A\to III$

$B\to IV$

$C\to I$

$D\to II$

The optimum value is 42.

Result

After using both the method we got the optimum value.

By using centroid ranking method, the optimum value is

$A\to III$

$B\to IV$

$C\to I$

$D\to II$

The optimum value is 13.99

By using Robust ranking method, the optimum value is

The optimum value is 42.

Discussion and Conclusion

In an educational institution, allocation of subjects to the faculty members plays an important role. In this paper, we compare two methods of fuzzy assignment problem; Centroid ranking method and Robust ranking method, to allocate the subjects to the faculty members. Here, we have used the scores given to the faculty members by the Chairman, Course Director and the students of the department in the previous semester.

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