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Quaternion algebra on 4D superfluid quantum space-time

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First, we define the momentum density as follows

$$\vec{p} = \gamma(v)\rho_M \vec{v}\cdot c + \nabla\phi \approx \rho_M \vec{v}\cdot c + \nabla\phi \qquad (1)$$

and the energy density

$$\varepsilon = \sqrt{\rho_M^2 c^4 + p^2} - \frac{\partial\phi}{c\partial t} = \gamma(v)\rho_M c^2 - \frac{\partial\phi}{c\partial t} \approx \rho_M c^2 + \rho_M \frac{v^2}{2} - \frac{\partial\phi}{c\partial t} \qquad (2)$$

accurate to an arbitrary scalar field ϕ dimension of which is [energy×length-2]. Here we multiply the momentum density by the constant c, which is the speed of light. It was done for order that the both variables, (1) and (2), would have equal dimensionality of pressure Pa = kg·m^{-1}·s^{-2} [1]. Here is the Lorentz factor. Next, we will consider for simplicity only nonrelativistic case.

We introduce four quaternion matrices η_0, η_x, η_y, η_z of size 4×4 [2]. They submit to the following communication relations

$$\eta_x\eta_y = -\eta_y\eta_x = -\eta_z, \quad \eta_y\eta_z = -\eta_z\eta_y = -\eta_x, \quad \eta_z\eta_x = -\eta_x\eta_z = -\eta_y, \quad \eta_x^2 = \eta_y^2 = \eta_z^2 = -\eta_0 \qquad (3)$$

These matrices have one-to-one correspondence with three the Pauli matrices and the unit matrix constituting the basis of the group SU(2). Let us now define the differential operators:

$$\mathcal{D} = \mathrm{i}c^{-1}\partial_t\eta_0 + \partial_x\eta_x + \partial_y\eta_y + \partial_z\eta_z, \quad \mathcal{D}^T = \mathrm{i}c^{-1}\partial_t\eta_0 + \partial_x\eta_x^T + \partial_y\eta_y^T + \partial_z\eta_z^T = \mathrm{i}c^{-1}\partial_t\eta_0 - \partial_x\eta_x - \partial_y\eta_y - \partial_z\eta_z$$

Observe that

$$\mathcal{D}\mathcal{D}^T = \mathcal{D}^T\mathcal{D} = (-c^2\partial_t^2 + \partial_x^2 + \partial_y^2 + \partial_z^2)\eta_0 = \Box \qquad (4)$$

Let us define the energy-momentum tensor $T = \mathrm{i}\varepsilon\eta_0 + p_x\eta_x + p_y\eta_y + p_z\eta_z - \mathcal{D}^T\phi$ with added an extra term $\mathcal{D}^T\phi$ The Lorentz gauge transformation reads

$$-(1/4)\mathrm{trace}\,\mathcal{D}T = c^{-1}\partial_t\varepsilon + \partial_x p_x + \partial_y p_y + \partial_z p_z + \Box\phi = 0 \qquad (5)$$

Now we can compute the force density tensor

$$F_{\Omega\Upsilon} = \mathcal{D}\cdot T = \begin{pmatrix} 0 & \Omega_x - \mathrm{i}\Upsilon_x & \Omega_y - \mathrm{i}\Upsilon_y & \Omega_z - \mathrm{i}\Upsilon_z \\ -\Omega_x + \mathrm{i}\Upsilon_x & 0 & -\Omega_z + \mathrm{i}\Upsilon_z & \Omega_y - \mathrm{i}\Upsilon_y \\ -\Omega_y + \mathrm{i}\Upsilon_y & \Omega_z - \mathrm{i}\Upsilon_z & 0 & -\Omega_x + \mathrm{i}\Upsilon_x \\ -\Omega_z + \mathrm{i}\Upsilon_z & -\Omega_y - \mathrm{i}\Upsilon_y & \Omega_x - \mathrm{i}\Upsilon_x & 0 \end{pmatrix} \qquad (6)$$

Here

$$\vec{\Omega} = [\nabla \times \vec{p}] \quad \text{and} \quad \vec{\Upsilon} = \frac{\partial \vec{p}}{c\partial t} + \nabla\varepsilon \qquad (7)$$

Finally, we find four equations

$$\boxed{\begin{array}{ll} (\nabla\cdot\vec{\Omega}) = 0, & \text{(a)} \\ [\nabla\times\vec{\Upsilon}] - \frac{1}{c}\frac{\partial}{\partial t}\vec{\Omega} = 0 & \text{(b)} \end{array}} \quad \boxed{\begin{array}{ll} [\nabla\times\vec{\Omega}] + \frac{1}{c}\frac{\partial}{\partial t}\vec{\Upsilon} = -\frac{4\pi}{c}\vec{\Im} & \text{(c)} \\ (\nabla\cdot\vec{\Upsilon}) = 4\pi\wp & \text{(d)} \end{array}} \qquad (8)$$

analogous to the Maxwell's EM equations. Eq. (a) says that $\vec{\Omega}$ is the orbital force density Eq. (b) is the vorticity equation. Eq. (c) computes the wave dynamics of the exchange of energies of two force densities $\vec{\Omega}$ and $\vec{\Upsilon}$ Here $\vec{J} = \mathrm{i}c\wp\,\eta_0 + \Im_x\eta_x + \Im_y\eta_y + \Im_z\eta_z$ is the 4D current density. Eq. (d) extracts a scalar field represented by the action function S. This equation, in pair with the continuity equation, leads to appearance of the Schrodinger equation as soon as we introduce the complex wave function written in the polar form [3].

Recent Publications:

1. V I Sbitnev (2016) Hydrodynamics of the physical vacuum: dark matter is an illusion. Modern Physics Letters A DOI: 10.1142/S0217732315501849.
2. V I Sbitnev (2018) Hydrodynamics of superfluid quantum space: particle of spin-1/2 in a magnetic field. Quantum Studies: Mathematics and Foundations 5(2):297-314.
3. V I Sbitnev (2018) Hydrodynamics of Superfluid Quantum Space: de Broglie interpretation of the quantum mechanics. Quantum Studies: Mathematics and Foundations 5(2):257-271.

Biography

Valeriy I Sbitnev has PhD in 1987 from Moscow State University. He is senior researcher in Saint-Petersburg Nuclear Physics Institute, Kurchatov NRC. He has published more than 50 papers in reputed journals.

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