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Analog of the Peter-Weyl theorem for Lorentz group and Y-Map in loop quantum gravity

Leo Perlov

University of Massachusetts, USA

The simplicity constraints, introduced by John Barrett and Louse Crane allow to consider the quantum gravity as a 4-dimensional topological model called BF-model plus some constraints on the form of the bivectors used in BF model. Those constraints are called the simplicity constraints. The simplicity constraints is what makes the 4-dim topological model to become Einstein's Quantum Gravity. The solutions of the simplicity constraints are the parameters of the Lorentz group principal series representation: $k = j$, $\rho = j\tau$, $j \in \mathbb{Z}$, $\tau \in \mathbb{C}$, or the corresponding Lorentz group matrix coefficients with those parameters. In my recent work published in the Math Physics 2015 I was able to use the simplicity constraints solution to derive the analog of the Peter-Weyl theorem for the non-compact Lorentz group (100 years after Peter and Weyl did it for the compact groups). It is very well known that the main theorem of the group representation theory – the Peter-Weyl theorem works only for the compact groups such as $SU(2)$. That's why it was not possible to use it for the Lorentz group. The nicety and usefulness of the Peter-Weyl theorem is that any square integrable function on the compact group can be expanded into its matrix coefficient functions and such expansion is convergent. I succeeded to prove that the square integrable function on $SL(2, \mathbb{C})$ can be expanded in its matrix coefficients with the parameters corresponding to the simplicity constraint solutions ($j, j \tau$) and such expansion is convergent. The proof of convergence is strictly mathematical and rigorous. This result shows that the simplicity constraint solutions are significant as they pick up the basis for expansion of any square integrable function on $SL(2, \mathbb{C})$. Second it allows to define a convergent map, Y-Map between the square integrable functions on $SU(2)$ and the functions on $SL(2, \mathbb{C})$ by using the matrix coefficients of both expansions. Thus one can embed the solutions on $SU(2)$ into the 4-dim Lorentz space. I believe that the ability to expand the functions on $SL(2, \mathbb{C})$ into the series will become a very useful tool for physicists. Aside from that, the strict mathematical result points us to the fact that the solutions of the simplicity constraints are more than just a model. The most recently (April 2017) published paper investigates all finite dimensional EPRL solutions of the simplicity constraints and their connections with the Barbero-Immirzi parameter spectrum. The finite dimensional solutions correspond to the non-unitary evolution, which is allowed in the background free quantum gravity.

Biography

Leo Perlov has completed his MS degree from Technion (Israel Institute of Technology). He is a Researcher at the Department of Physics in the University of Massachusetts. The recently published papers include: Physics Letters B 2015 L. Perlov Wheeler-DeWitt Equation for 4D Supermetric and ADM with Massless Scalar Field as Internal Time; Math Physics 2015 L. Perlov Analog of the Peter-Weyl Expansion for Lorentz Group; Annale Poincare 2017 L. Perlov and M Bukatin All Finite Dimensional Lorentz Representations contained in simplicity constraint solutions.

Leonid.Perlov@umb.edu

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